

778.6
A R I T H M E T I C,

RATIONAL and PRACTICAL

W H E R E I N

The properties of NUMBERS are clearly pointed out,
the THEORY of the science deduced from first prin-
ciples, the methods of OPERATION demonstra-
tively explained, and the whole reduced to PRAC-
TICE in a great variety of useful RULES.

Consisting of THREE PARTS, viz.

I. VULGAR ARITHMETIC.

II. DECIMAL ARITHMETIC.

III. PRACTICAL ARITHMETIC.

By JOHN MAIR, A.M.

P A R T III.

PRACTICAL ARITHMETIC.

EDINBURGH:

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For A. KINCAID and J. BELL.

MDCCLXVI.

27



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ARITHMETIC,

RATIONAL and PRACTICAL.

PART III.

PRACTICAL ARITHMETIC; or, Arithmetic
Vulgar and Decimal, reduced to PRACTICE,
in a great variety of useful rules.

CHAP. I.

FELLOWSHIP.

FELLOWSHIP, called also *Company*, or *Partnership*, is when two or more persons join their stocks, and trade together, dividing the gain or loss proportionally among the partners.

Fellowship is either without or with time, called also *single* or *double*.

I. *Fellowship without time.*

Questions in fellowship without time are wrought by the following proportion.

As the total stock

To the total gain or loss,

So each man's particular stock

To his share of the gain or loss.

Quest. 1. A and B make a joint stock : A puts in 12 l. and B 8 l.; they gain 5 l.: What is each man's share?

	L.	Stock.	Gain.	Stock.
A's stock	12	A. If 20 : 5 :: 12		
B's stock	8			5
Tot. stock	20			20)60
		Stock. Gain. Stock.	A's gain	3 l.
B. If 20 : 5 :: 8				
	8			L.
	20)40		A's gain	3
B's gain	2 l.		B's gain	2
			Total gain	5 proof.

Quest. 2. A, B, and C, make a joint stock : A puts in 78 l. B 117 l. and C 234 l.; they gain 265 l.: What is each man's share?

Stock

	L.	Stock.	Gain.	Stock.
Stock of	A 78	A. If 429 :	265 ::	78
	B 117			78
	C 234			<u>2120</u>
Total stock	429			1855

$$\begin{array}{r} 429 \overline{) 20670} (48 \text{ l.} \\ \underline{1716} \end{array}$$

3510

3432

78

20

$$\begin{array}{r} 429 \overline{) 1560} (3 \text{ s.} \\ \underline{1287} \end{array}$$

273

12

$$\begin{array}{r} 429 \overline{) 3276} (7 \text{ d.} \\ \underline{3003} \end{array}$$

273

4

$$\begin{array}{r} 429 \overline{) 1092} (2 \text{ f.} \\ \underline{858} \end{array}$$

(234)

A 2

Stock.

Stock. Gain. Stock.
B. If 429 : 265 :: 117

$$\begin{array}{r}
 117 \\
 \hline
 1855 \\
 265 \\
 \hline
 265 \\
 \hline
 429)31005(72 \text{ l.} \\
 \underline{3003.} \\
 975 \\
 858 \\
 \hline
 117 \\
 20 \\
 \hline
 429)2340(5 \text{ s.} \\
 \underline{2145} \\
 195 \\
 12 \\
 \hline
 429)2340(5 \text{ d.} \\
 \underline{2145} \\
 195 \\
 4 \\
 \hline
 429)780(1 \text{ f.} \\
 \underline{429} \\
 (351)
 \end{array}$$

Stock. Gain. Stock.
C. If 429 : 265 :: 234

$$\begin{array}{r}
 234 \\
 \hline
 1060 \\
 795 \\
 \hline
 530 \\
 \hline
 429)62010(144 \text{ l.} \\
 \underline{429.} \\
 1911 \\
 1716 \\
 \hline
 1950 \\
 1716 \\
 \hline
 234 \\
 20 \\
 \hline
 429)4680(10 \text{ s.} \\
 \underline{429} \\
 390 \\
 12 \\
 \hline
 429)4680(10 \text{ d.} \\
 \underline{429} \\
 390 \\
 4 \\
 \hline
 429)1560(3 \text{ f.} \\
 \underline{1287} \\
 (273)
 \end{array}$$

	L.	s.	d.	f.	Rem.
Ans. Gain of {	A 48	3	7	2	234
	B 72	5	5	1	351
	C 144	10	10	3	273
Proof	265	0	0	0	858

Note 1. When in any question there happen to be remainders, they must be reduced equally low, so as to be all of one name; and then their sum will be either equal

equal to the divisor, or exactly double, triple, &c. of it: and accordingly 1, 2, 3, &c. carried from the sum of the remainders, and added to the particular gains, will make up the total gain; or the divisor will always divide the sum of the remainders exactly, and the quot added to the particular gains will give the total gain.

Note 2. When the partners have equal shares of stock or capital, their shares of gain, loss, or neat proceeds, is found readily by dividing the total gain, loss, &c. by the number of partners, thus.

Suppose A, B, and C, were equally concerned in a voyage to Virginia, and that the neat proceeds turned out to 575 l. each partner's share will be L. 191 : 13 : 4, cast up as under.

$$\begin{array}{r} \text{L. } s. \text{ d.} \\ 3)575(191 \text{ } 13 \text{ } 4 \end{array}$$

Note 3. When the fractions denoting the partners shares of stock or capital have all the same denominator, their shares of gain, loss, &c. is quickly found, by dividing the total gain, &c. by the denominators, and multiplying each quot by its respective numerator, thus.

Suppose A, B, C, and D, buy a ship for a sum of money, whereof A, B, and C, pay $\frac{1}{2}$ each, and D $\frac{3}{4}$ or $\frac{1}{2}$; and that her freight on a voyage amounts to 260 l. the partners shares are found as follows.

$$6)260$$

$$\text{L. } 43 : 6 : 8 \text{ to A.}$$

$$43 : 6 : 8 \text{ to B.}$$

$$43 : 6 : 8 \text{ to C.}$$

$$130 : 0 : 0 \text{ to D.}$$

Proof 260

Note 4. The operation may sometimes be shortened or facilitated by first finding the gain, loss, or neat proceeds, *per cent.* or *per pound*, and then working by the rules of practice, thus.

Suppose A, B, and C, join in an adventure to Barbadoes,

badoes, which, with all charges, amounted to 2000 l.; whereof A paid 400 l. B 600 l. and D 1000 l.; they have returns in goods, which being disposed of, the neat proceeds amounted to L. 4333 : 6 : 8 : What should each partner get?

$$\text{If } 2000 : 4333 \quad 6 \quad 8 :: 100$$

$$20 : 4333 \quad 6 \quad 8 :: 1$$

$$10 : 216\frac{1}{2} \quad 13 \quad 4 :: 1$$

20

13

12

40

216 13 4 per cent.

4

$$400 = 866 \quad 13 \quad 4$$

$$200 = 433 \quad 6 \quad 8$$

$$600 = 1300$$

$$1000 = 2166 \quad 13 \quad 4$$

L. s. d.

A gets 866 13 4

B gets 1300

C gets 2166 13 4

Proof 4333 6 8

Again, suppose A, B, and C, join in an adventure to Jamaica, and ship off, on their joint credit, goods to the value of 3000 l.; and, to complete the cargo, the partners, from their own warehouses, ship off such goods as they had proper for the voyage, viz. A goods to the value of 100 l. B 200 l. C 300 l.; the neat proceeds of their returns in sugar and rum amounted to 6930 l.: What share of this belongs to each partner?

$$3600 : 6930 :: 1$$

$$360 : 693 :: 1$$

$$120 : 231 :: 1$$

$$40 : 77 :: 1 : \frac{77}{40} \text{ per l.}$$

Shares.

L. s.

$$\text{A's stock } 1100 \times 77 = 84700, \div 40 = 2117 \quad 10$$

$$\text{B's stock } 1200 \times 77 = 92400, \div 40 = 2310$$

$$\text{C's stock } 1300 \times 77 = 100100, \div 40 = 2502 \quad 10$$

Proof 6930

Decimally.

Decimally.

Quest. 3. A, B, and C, make a joint stock of 735 l. whereof

	L.	s.	d.	L.
A puts in	277	13	4	= 277.6
B ———	163	6	8	= 163.3
C ———	294			= 294.
Total stock	735			735.0

They gain 49 l.; required each man's share.

	L.	L.
If 735 : 49 ::	277.6 : 18.51 A	
	163.3 : 10.88 B	
	294. : 19.6 C	

49.00 proof.

But the easiest way is to find the gain or loss *per l.*; and by this multiply each man's stock. This common multiplier is found by the rule of three, or simply by dividing the total gain or loss by the sum of the stocks; thus :

L. L. L. L.
As 735 : 49 :: 1 : .06 common multiplier.

	L.	L.	L.	L.	s.	d.
And	277.6	×	.06	=	18	10 2.6
	163.3	×	.06	=	10	17 9.3
	294.	×	.06	=	19	12 0

49.00 proof. 49 00 0.0

Again, suppose A put in 15 l. B 24 l. and C. 33 l. and gain 63 l.; what is each man's share of said gain?

A 15 l.
B 24
C 33

72)63.0(.875 common multiplier.

	L.	s.	d.	
15 × .875	=	13.125	=	13 2 6 A's share.
24 × .875	=	21	=	21 0 0 B's share.
33 × .875	=	28.875	=	28 17 6 C's share.

63

proof.

MORE

MORE EXAMPLES.

Quest. 4. A, B, and C, make a joint stock: A puts in 460 l. B 510 l. and C 480 l.; they gain 340 l.: What is each partner's share?

	L.	s.	d.	f.	Rem.
<i>Ans.</i> Gain of	A 107	17	2	3	85
	B 119	11	8	2	110
	C 112	11	0	1	95
Proof.	340	00	0	0	290

Quest. 5. Three persons trade together: A puts in 230 l. B 529 l. and C 344 l. 10 s.; they gain 520 l.: What is that to each?

	L.	s.	d.	Rem.
<i>Ans.</i> Gain of	A 108	7	7 $\frac{3}{4}$	271
	B 249	5	7	844
	C 162	6	9	1092

Quest. 6. A, B, and C, make a joint stock of 3256 l.; whereof A puts in 1026 l. B 985 l. and C the rest; by misfortune they lose 2000 l.: What part of that must each bear?

	L.	s.	d.	Rem.
<i>Ans.</i>	A 630	4	5	928
	B 605	0	8 $\frac{3}{4}$	1240
	C 764	14	10	1088

Quest. 7. Four partners, A, B, C, and D, built a ship, which cost 1730 l.; and the freight for her first voyage amounted to 370 l.; of which A's share was 74 l. B's 111 l. C's 148 l. and D's 37 l.: What was each partner's stock?

	L.
<i>Ans.</i>	A's stock was 346
	B's - - - 519
	C's - - - 692
	D's - - - 173
Total stock	1730 proof.

Quest. 8. A, B, and C, in company, gain 36 l.: A put in 20 l. B 30 l. C a sum unknown; C took up 16 l. of the gain: What did A and B gain? and what did C put in?

Ans.

L

Ans. { A's gain 8
B's gain 12
C's stock 40

Quest. 9. A, B, C, and D, in partnership, had a joint stock of 600 l. and gained a certain sum; of which A, B, and C, took up 60 l.; B, C, and D, 90; A, C, and D, 80; A, B, and D, 70: What was the stock and gain of each partner?

	Stock.	Gain.
	L.	L.
<i>Ans.</i> {	A's 60	10
	B's 120	20
	C's 180	30
	D's 240	40

II. Fellowship with time.

In fellowship with time, the gain or loss is divided among the partners, both in proportion to the stocks themselves, and also in proportion to the times of their continuance in company: for the same stock continued a double time, procures a double share of gain; and continued a triple time, procures a triple share of gain; that is, the shares of gain or loss are as the products of the several stocks multiplied into their respective times: and accordingly questions belonging to this rule are wrought by the following proportion.

As the sum of the products of the several stocks into their respective times

To the total gain or loss,

So the product of each man's stock into his time

To his share of the gain or loss.

Quest. 1. A put into company 40 l. for 3 months, B 75 l. for 4 months; they gain 70 l.: What share must each man have?

A $40 \times 3 = 120$ third term for A's share.

B $75 \times 4 = 300$ third term for B's share.

420 first term.

L.	L.
A. If 420 : 70 :: 120	B. If 420 : 70 :: 300
<u>120</u>	<u>300</u>
42 0)840 0(20 L.	42 0)2100 0(50 L.
84	210
<u>(0)</u>	<u>(0)</u>

	L.
A's gain	20
B's gain	<u>50</u>

Total gain 70 proof.

Quest. 2. A put into company 560 l. for 8 months, B 279 l. for 10 months, and C 735 l. for 6 months; they gained 1000 l.: What share of the gain must each have?

A $560 \times 8 = 4480$ third term for A's share.

B $279 \times 10 = 2790$ third term for B's share.

C $735 \times 6 = 4410$ third term for C's share.

11680 first term.

	L.	L.	s.	d.	f.	Rem.
A. If 11680 : 1000 :: 4480 :	383	11	2	3		208
B. If 11680 : 1000 :: 2790 :	238	17	4	3		80
C. If 11680 : 1000 :: 4410 :	377	11	4	1		880
Proof	1000	00	0	0		1168

Decimally.

Quest. 3. A, B, and C, enter into partnership, viz.

	L.	s.	d.	
A puts in	58	10	0	for 8 months 3 weeks.
B ———	65	15	0	for 6 months 4 days.
C ———	48	17	6	for 12 months 2 weeks.

They

They gain 125 l. 14 s.: What is each man's share?

	L.	Mon.	Prod.
A's stock	58.5	8.75	= 511.875
B's stock	65.75	6.1428	= 403.8891
C's stock	48.875	12.5	= 610.9375

Sum of the products 1526.7016

$$\text{As } 1526.7016 : 125.7 :: \begin{cases} 511.875 : 42.1449 \text{ A} \\ 403.8891 : 33.2539 \text{ B} \\ 610.9375 : 50.3011 \text{ C} \end{cases}$$

Total gain nearly 125.6999 proof.

But here too the easiest method is to find the gain or loss *per l.* and then make this a common multiplier of the several products of the stocks and times. Thus,

$$\text{As } 1526.7016 : 125.7 :: 1 : .08233 \text{ common multiplier.}$$

A	B	C
511.875	403.8891	610.9375
Multiplier 33280. inverted	33280.	33280.
409500	323111	488750
10237	8077	12218
1535	1211	1832
153	121	183
42.1425	33.2520	50.2983

The shares of the gain are nearly the same as before.

MORE EXAMPLES.

Quest. 4. A, B, and C, hire a pasture for 24 l.: A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months: What share of the rent must each pay?

B 2

Ans.

			L.	s.
	{	A's share of rent	9	12
<i>Ans.</i>	{	B's share	3	12
	{	C's share	10	16

Proof 24

Quest. 5. A, B, and C, agreeing to trade together, A puts in 529 l. for 4 months, B 329 l. for 7 months, and C 900 l. for 2 months; they gain 540 l.: What is each man's share?

		<i>E.</i>	<i>s.</i>	<i>d.</i>	<i>Rem.</i>
<i>Ans.</i>	{ A	183	14	8	2304
	{ B	199	19	5	1222
	{ C	156	5	10 $\frac{3}{4}$	2583

Quest. 6. A and B enter into partnership for a year; accordingly A put in the first of January 50 l.; but B could not put any money in till the first of May: What sum must B then put in to be intitled to an equal share of the gain at the year's end? *Ans.* 75 l.

Quest. 7. A, B, and C, agree to trade in company for 12 months: A put in at first 364 l. and at the end of 4 months he put in 40 l. more; B put in at first 408 l. and at the end of 7 months he took out 86 l.; C put in at first 148 l. and at the end of 3 months he put in 86 l. more, and at the end of other 5 months he put in 100 l. more; at the year's end their gain was 1436 l.: What is each man's share?

		L.	s.	d.	Rem.
Ans.	{ A's gain	556	3	6	6192
	{ B's gain	529	16	9	5496
	{ C's gain	349	19	8	416

Note. Questions in double fellowship are proper exercises for the learner, but seldom occur in real business; differences in point of time being usually adjusted by an interest-account.

CHAP. II.

FACTORAGE.

Factors are agents or doers, residing abroad, or beyond seas, who act for their employers; and for their trouble are generally allowed, in name of commission or fees, so much *per cent.* The computations they have occasion for are such as occur in the solution of the following or like questions.

Quest. 1. A B's sales *per* the Swan amount to 675 L. 18 s. 8 d.: What is the commission at 8 *per cent.*?

Decimally.

L.	s.	d.
If 100 : 8 :: 675	18	8
		8
L. 54.07	9	4
	20	
s. 1.49		
	12	
d. 5.92		
	4	
f. 3.68		

L.
If 100 : 8 :: 675.93
L. 54.0748
s. 1.493
d. 5.920
f. 3.68

Quest. 2. A factor buys, and ships off to his employer, goods to the value of L. 1334:11:5 $\frac{1}{4}$: What is the commission at 5 *per cent.*?

If

Decimally.

L.	s.	d.	L.
If 100 : 5 :: 1334	11	5 $\frac{1}{4}$	If 100 : 5 :: 1334.571875
		5	5
L. 66.72	17	2 $\frac{1}{4}$	L. 66.72859375
	20		20
s. 14.57			s. 14.57187500
	12		12
d. 6.86			d. 6.862500
	4		4
f. 3.45			f. 3.4500

Quest. 3. The prime cost of a parcel of consigned goods, per invoice, is L. 37 : 8 : 1 $\frac{1}{2}$; these the factor sells at 75 per cent. advance on the invoice: What is their value in the sale?

Decimally.

L.	s.	d.	L.
If 100 : 75 :: 37	8	1 $\frac{1}{2}$	If 100 : 175 :: 37.40625
	20		175
	748		18703125
	12		26184375
	8977		3740625
	4		L. 65.4609375
	35910		20
	75		s. 9.2187500
	17955		12
	25137		d. 2.62500
4)26932.50			4
12)6733			f. 2.500
20)561			
28	1	1	advance.
37	8	1 $\frac{1}{2}$	prime cost.
65	9	2 $\frac{1}{2}$	value in the sale.

Quest.

Quest. 4. The neat proceeds of a sale is 620 l. 12s. 6½ d.: To what value ought the factor to make returns, so as to clear the debt, and have 5 per cent. commission?

	L.	s.	d.
If 105 : 100 :: 620	12	6½	
	20		
	12412		
	12		
	148950		
	4		
	595802		
	100		
	4)		
105)59580200(567430			
525			
708	12)	141857	2 f.
630	2 0)	1182 r	5 d.
780		591	1 s.
735			
452			
420			
320			
315			
(50)			

Decimally.

Decimally.

L.

If 105 : 100 :: 620.627083

105)62062.7083(591.0734

525.....

20

956

s. 1.4680

945

12

112

d. 5.616

105

4

770

f. 2.464

735

358

315

	L.	s.	d.	
Value of returns	591	1	5½	483
Commission	29	11	1	420
Proof	620	12	6½	(13)

Quest. 5. A factor buys for his employer goods to the value of L. 7864 : 16 : 8 : What will his commission come to at 2½ per cent.?

2½

$2\frac{1}{2}$ per cent is $\frac{1}{40}$ of 100; and therefore,

By Practice,

L. s. d. L. s. d.
4|0)7864 16 8(196 12 5 *Ans.*

Rem. 24

Decimally.

20

49|6(

Rem. 16

12

20|0(

If 100 : 2.5 :: 7864.83

2.5

3932416

15729666

L. 196.62083

20

s. 12.4166

12

d. 5.000

Quest. 6. If I grant a draught on London for 4835 l. and be allowed $1\frac{3}{8}$ per cent. in name of premium or commission, what will that amount to?

L.

1 per cent. = $\frac{1}{100}$ = 48.35

$\frac{3}{8}$ = $\frac{1}{4}$ of that = 12.0875

$\frac{1}{8}$ = $\frac{1}{2}$ of that = 6.04375

L. 66.48125

20

s. 9.6250

12

d. 7.500

Ans. L. 66 : 9 : 7 $\frac{1}{2}$.

Some, instead of paying a premium, chuse to take the bill payable so many days after date or sight; and in this way, 73 days are reckoned equal to 1 per cent.

VOL. III.

C

Quest.

Quest. 7. A factor ships off, by his employer's order, goods to the value of L. 97 : 5 : 8 : What will his commission at 2 per cent. amount to?

$$2 \text{ per cent.} = \frac{1}{50} \text{ of } 100.$$

By Practice,

50)9|7 5 8(1 18 10 $\frac{3}{4}$ Ans.

Rem. 47

20

94|5(

Rem. 45

12

54|8(

Rem. 48

4

19|2(

(42)

Decimally.

L.

If 100 : 2 :: 97.283

2

L. 1.94566

20

s. 18.913

12

d. 10.960

4

f. 3.84

MORE EXAMPLES.

Quest. 8. A factor ships off for his employer goods to the value of L. 1629 : 4 : 3 $\frac{1}{2}$: What will his commission at 5 per cent. amount to? Ans L. 81 : 9 : 2 $\frac{1}{2}$.

Quest. 9. A factor owes his employer L. 262 : 17 : 7 $\frac{1}{2}$: How much ought he to remit, in order to clear the debt, and have 2 $\frac{1}{2}$ per cent. commission on the sum remitted? Ans. L. 256 : 3 : 6 $\frac{3}{4}$.

Quest. 10. A factor negotiates a bill of L. 76 : 10 : 9 : What commission may be charge on this head at 1 $\frac{1}{2}$ per cent. Ans. L. 1 : 2 : 11 $\frac{1}{2}$.

CHAR.

C H A P. III.

T A R E and T R E T, &c.

G*ross weight* is the weight both of the commodity, and of that which contains it, whether cask, hoghead, barrel, bag, sack, or wrapper.

Tare is an allowance on weighable goods, made by the King to the importer, or by the seller to the buyer, in consideration of the outside package; such as, cask, bag, box, chest, wrappers, &c.

Tret is an allowance of 4 lb. on 104 lb. on garbled goods, or such as are sold by the pound-weight, granted for break, waste, or dust mixed with the goods.

Cloff or *clough*, called also *draught*, is another small allowance on some weighable goods, usually 2 lb. on every 3 C. to turn the scale, or make the weight hold out, when the goods are re-weighed. This is claimed chiefly, or only, by the citizens of London.

Suttle or *suttle-weight* is what remains after the tare is deduced from the gross: and is so called only when tret is allowed; for when there is no tret, there is no *suttle*.

Neat weight is the weight of the goods after all given allowances are deduced. Thus, if tare, tret, and cloff, be allowed, the neat weight is what remains after these three allowances are deduced. If tare and tret only be allowed, the neat weight is what remains after these two allowances are deduced. If tare only be allowed, the neat weight is what remains after that allowance is deduced.

I. *Tare.*

Tare admits of three varieties: for sometimes the hoghead, cask, or barrel, is weighed before the goods are packed, and the tare inserted in the invoice along with the gross weight; or the tare is so much on the whole. Again, the allowance for tare is sometimes so

much *per* frail, *per* firkin, *per* hogshead, *per* chest, *per* bag, &c. Lastly, the allowance for tare is sometimes so much *per* C.

1. When the tare is inserted in the invoice, or is so much on the whole, the neat weight is found by subtracting the tare from the gross.

EXAMPLE I.

Suppose a factor in Jamaica buys for the use of his employer in Britain, the three following hogsheads of sugar, their neat weight in pounds, that being the usual form, is computed as under.

N ^o	C.	Q.	lb.	Tare in pounds.
1.	16	1	14	114
2.	14	3	7	110
3.	15	2	21	116
	46	3	14	340
	46			
	46			
	4698			
	5250			gross.
	340			tare.
	4910			neat.

EXAMPLE II.

A grocer buys 54 chests of spice, each chest weighing 4 C. 2 Q. 14 lb. gross; tare on the whole, 21 C. 3 Q. 18 lb.: Required the neat weight.

C.	Q.	lb.	
4	2	14	54 = 9 × 6.
	9		
41	2	14	
249	3	0	total gross.
21	3	18	tare.
Ans. 227	3	10	neat.

2. When

2. When the tare is so much *per* frail, *per* firkin, *per* chest, *per* bag, &c. multiply the allowance for tare by the number of frails, firkins, chests, or bags; subtract the product from the gross; and the remainder is the neat weight.

Or, from the gross weight of 1 frail, firkin, hogf-head, &c. subtract its tare; the remainder is the neat weight of 1 frail, firkin, &c.; which, multiplied, by the number of frails, &c. gives the total neat weight.

EXAMPLE I.

What is the neat weight of 7 frails of raisins, each weighing 3 C. 3 Q. 10 lb. gross, tare at 24 lb. *per* frail?

C.	Q.	lb.	lb.
30	3	10	24
7 frails.			Frails 7
26	3	14	gross.
1	2	0	tare.
Ans. 25			1 14 neat.

Or,

C.	Q.	lb.
3	3	10
gross of 1 frail.		
24 tare.		
3	2	14
neat of 1 frail.		
25	1	14
neat of 7 frails.		

EXAMPLE II.

What is the neat weight of 14 bags of cotton, each weighing 2 C. 2 Q. 7 lb. tare at 9 lb. *per* bag?

C. Q. lb.			C. Q. lb.		
2 2 7			2 2 7		
14 2 7 3 2			9 tare.		
Bags. 17 3 21			2 1 26		
14			7		
9			17 1 14		
126			2		
35			34 3 09		
35			34		
3598			34		
4018			3484		
126			3892 lb. neat.		
35			3892 lb. neat.		
3598					
4018					
126					

Note. The tare for several sorts of goods is ascertained in the book of rates. Thus, for every bale of silk from Smyrna or Cyprus, weighing 300 lb. or upwards, the tare is fixed at 16 lb.; for every bale weighing 200 lb. or upwards to 300, the tare is 14 lb.; and for every bale under 200 lb. the tare is 12 lb.

Accordingly the neat weight of the five following bales of silk from Smyrna, will be made out from the book of rates, as under.

N ^o	lb.	Tare.
1.	346	16
2.	300	16
3.	284	14
4.	200	14
5.	158	12
Gross 1288		72
Tare 72		
Neat 1216		

3. When

Chap. III. TARE and TRET.

23

3. When the tare is so much per C. work by one or other of the two rules following.

R U L E I.

If the tare be 14 or 16 per C. take accordingly $\frac{1}{2}$ or $\frac{1}{3}$ of the gross for the tare; which, subtracted from the gross, leaves the neat weight.

E X A M P L E I.

		C.	Q.	lb.		lb.
lb.	14	$\frac{1}{2}$	493	2	24	gross.
			61	2	24	tare.
			432	0	0	neat

Tare 14 per C.

E X A M P L E II.

		C.	Q.	lb.		lb.
lb.	16	$\frac{1}{3}$	119	3	0	gross.
			17	0	12	tare.
			102	2	16	neat.

Tare 16 per C.

R U L E II.

If the tare be any other than 14 or 16 per C. first work as above for 14 or 16, and then take aliquot parts of the quot.

E X A M P L E I.

		C.	Q.	lb.		lb.
lb.	14	$\frac{1}{2}$	57	2	24	gross.
			7	0	24	
			3	2	12	tare.
7		$\frac{1}{2}$ of $\frac{1}{2}$	54	0	12	neat.

Tare 7 per C.

EX.

540 yd work

EXAMPLE II.

		C.	Q.	lb.	lb.
		573	2	13	gross.
		Tare 17 per C.			
lb.		71	2	22½	
14	$\frac{1}{8}$	10	0	27	
2	$\frac{1}{7}$ of $\frac{1}{8}$	5	0	13½	
1	$\frac{1}{2}$ of $\frac{1}{7}$ of $\frac{1}{8}$	87	0	7	tare.
17		486	2	6	neat.

EXAMPLE III.

		C.	Q.	lb.	lb.
		44	2	12	gross.
		Tare 21 per C.			
lb.		5	2	8½	
14	$\frac{1}{8}$	2	3	4½	
7	$\frac{1}{2}$ of $\frac{1}{8}$	8	1	12¾	tare.
21		36	0	27¼	neat.

In the examples following, the first part of the compound fraction only is expressed, the other part or parts, for brevity's sake, being omitted.

EXAMPLE IV.

		C.	Q.	lb.	lb.
		170	2	14	gross.
		Tare 6 per C.			
lb.		24	1	14	
16	$\frac{1}{7}$	12	0	21	
8	$\frac{1}{2}$	3	0	5½	
2	$\frac{1}{4}$	9	0	15¾	tare.
6		161	1	26¼	neat.

EX.

EXAMPLE V.

		C.	Q.	lb.		lb.
		550	1	0	gross.	Tare 10 per C.
lb.						
16	$\frac{1}{7}$	78	2	12		
8	$\frac{1}{2}$	39	1	6		
2	$\frac{1}{4}$	9	3	$8\frac{1}{2}$		
10		49	0	$14\frac{1}{2}$	tare.	
		501	0	$13\frac{1}{2}$	neat.	

EXAMPLE VI.

		C.	Q.	lb.		lb.
		237	1	14	gross.	Tare 12 per C.
lb.						
16	$\frac{1}{7}$	33	3	18		
8	$\frac{1}{2}$	16	3	23		
4	$\frac{1}{2}$	8	1	$25\frac{1}{2}$		
12		25	1	$20\frac{1}{2}$	tare.	
		211	3	$21\frac{1}{2}$	neat.	

Note 1. That 14 and 16 pounds per C. may be considered as the standards of tare, in regard tare at any other rate may easily be computed from them, as is done in the above examples.

Note 2. It is not usual in computing tare to take notice of any thing lower, or less, than $\frac{1}{4}$ lb.; and accordingly, in Ex. 2. some small fractions are neglected or thrown away.

Note 3. Though the method of computing tare, explained above, be the more usual way; yet some chuse to multiply the pounds gross by the rate, or allowance of tare, and dividing the product by 112, the quot gives the tare in pounds. Ex. 1. wrought in this manner follows,

C. Q. lb.

57 2 24 gross. Tare 7 per C.

57

57

5780

6464

7

lb. C. Q. lb.

112)45248(404 = 3 2 12 as before.

448

448

448

(o)

Others multiply the C.'s by the rate of tare, and for the quarter and pounds take proportional parts, thus.

C. Q. lb.

57 2 24 gross. Tare 7 per C.

7

Tare of 57 C. is 399

— of 2 Q. is $3\frac{1}{2}$

— of 16 lb. is 1

— of 8 lb. is $0\frac{1}{2}$

C. Q. lb.

Total tare 404 = 3 2 12 as above.

Note 4. The tare for oil in uncertain casks is 18 lb. per C. but pays duty by the gallon, and the number of gallons is computed from the neat weight, by allowing $7\frac{1}{2}$ lb. to a gallon; that is, multiply the neat pounds by 2, the product divided by 15, quotes gallons; or, divide the product by 3, and that quot by 5.

EX.

EXAMPLE

		G.	Q.	lb.	
		128	2	0	oil gros. Tare 18 lb. per C.
		1536			
		56			
		14392			lb. gros.
lb.		16	$\frac{1}{7}$		
		2	$\frac{1}{8}$		
		2056			
		257			
		2313			lb. tare.
		12079			lb. neat.
		2			
		24158			half-pounds.
$\frac{1}{15}$		1610	$\frac{1}{2}$		gallons.

In working decimally the tare may be found by multiplying the gross weight by the number of pounds tare in one C. or by the decimal of the tare; and this product subtracted from the gross weight leaves the neat. But it is easier and more usual to work by one or other of the three methods following.

1. Multiply the gross weight by the neat pounds in one C. and the product is the pounds neat.

D :

Ex.

C. Q. lb.
Ex. 573 2 13 gross. Tare 17 lb. *per C.*

C.
 573.616 gross.
 95 lb. neat in 1 C.
 2868080
5162544 *C. Q. lb.*
 112)54493.520(486 2 5½ neat.
 448
 969
 896
 733
 672
 (61)

2. Multiply the gross weight by the decimal of the neat pounds in 1 C. and the product is neat C.

C.
lb. C. **Ex. 573.616** gross.
 95 = .8482 2848. multiplier inverted.

4588928
229446
45889
1147

486.5410 neat weight nearly.

3. Work by aliquot parts, as in Practice.

C.
Ex. 573.616 gross. Tare 17 lb. *p. C.*
lb.
 14 = $\frac{1}{8}$ of the gross = 71 702
 2 = $\frac{1}{7}$ of the former = 10.243
 1 = $\frac{1}{2}$ of the former = 5.121
17
87.066 tare.
 486.550 neat weight nearly.

In working by aliquot parts it will be convenient to have at hand a table of divisors, such as the following; which is constructed in the same manner as the table of rates and divisors in decimal practice.

lb. Tare.	Divisors.	lb. Tare.	Divisors.	lb. T.	Divisors.
1	2, 7, 8	10	2, 7, + 4	19	7, + 4, - 20
2	7, 8	11	8, 7, - 12	20	7, + 4
3	7, 8, + 2	12	8, - 7	21	8, + 2
4	4, 7	13	8, - 7, + 12	22	7, + 4, + 2
5	4, 7, + 4	14	8	23	7, + 2, - 8
6	4, 7, + 2	15	8, + 7, - 2	24	7, + 2
7	2, 8	16	7	25	7, + 2, + 8
8	2, 7	17	8, + 7, + 2	26	7, + 2, + 4
9	2, 7, + 8	18	7, + 8	27	7, + 2, + 4, + 2

II. Tret.

Tret, being always 4 lb. per 104 lb. futtle, is found by taking $\frac{1}{26}$ of the futtle-weight.

EXAMPLE I.

$\frac{1}{26}$	G.	Q. lb.		
	428	1	19	futtle. Tret 4 lb. per 104 lb.
	16	1	25 $\frac{1}{2}$	tret.
	411	3	21 $\frac{1}{2}$	neat.

In working have at hand a table of the following which is contained in the same as the table of rates and divisors in decimal practice.

Divisor	Divisor	Divisor	Divisor
12 rem.	12 rem.	12 rem.	12 rem.
4	4	4	4
26)428	26)49(1	26)49(1	26)49(1
26	26	26	26
23 rem.	23 rem.	23 rem.	23 rem.
28	28	28	28
193	193	193	193
47	47	47	47
26)663(25½	26)663(25½	26)663(25½	26)663(25½
52	52	52	52
143	143	143	143
130	130	130	130
(13)	(13)	(13)	(13)

EXAMPLE II.

lb.	C.	Q.	lb.	gross.	Tare 22 lb. per C.	Tret 4 lb. per 104 lb.
16	836	2	17			
4	119	2	2½			
2	29	3	14½			
22	14	3	21¼			
	164	1	10	tare.		
	672	1	7	futtle.		
	25	3	12	tret.		
	646	1	23	neat.		

Note. The neat weight may also be had by annexing two ciphers to the futtle pounds, and then dividing by 104. Ex. 1. wrought in this manner follows.

C. Q. lb.

428 1 19 futtle. Tret 4 lb. per 104 lb.

5136

47

416

638

624

143

104

390

312

780

728

(52)

In working decimally, multiply the futtle by .0385 = $\frac{4}{104}$, and the product is the tret; which deduced from the futtle, leaves the neat.

But it is shorter to multiply the futtle by .9615 = $\frac{100}{104}$, and the product is the neat. Ex. 2. done in this manner follows.

C. Q. lb.

836 2 17 = 836.6517 gross.

Tare 21 lb. per C.

Tret 4 lb. per 104 lb.

7 + 4 + 2 { 119.5216
29.8804
14.9402

164.3422 tare.

672.3095 futtle.

5169 multiplier inverted.

60507855

4033857

67230

33615

646.42557 neat weight nearly.

III. Cloff.

III. Cloff.

When tret is allowed, the remainder, after deducing the tare from the gross, is called *futtle*; and when cloff is allowed, the remainder of the *futtle*-weight, after deducing the tret, is again called *futtle*, or rather *subfuttle*.

Cloff, being always 2 lb. on every 3 C. *subfuttle*, is found by taking $\frac{1}{15}$ of the *subfuttle* weight.

Or, divide the *subfuttle* C.'s by 3, the quot is cloff in double pounds; these divided by 14 quote quarters; and the remainder is double pounds.

EXAMPLE I.

C.	Q.	lb.	
4033	2	14	<i>subfuttle</i> . Cloff 2 lb. per 3 C.
24	0	1	<i>cloff</i> .
4009	2	13	<i>neat</i> .

Method 1.

168)4033(24 C.
 336

673

672

1

4

168)60 Q.

28

168)182(1 lb.

168

(14)

Method 2.

3)4033(1

4) C. Q. lb.

14)1344(96(24

126

84

84

(0)

Note, The 1 C. remaining in the first division, and the 2 Q. in the *subfuttle*, afford the 1 lb. *cloff*.

E X.

EXAMPLE II.

C. Q. lb.
32 3 20 gross.

4 0 13 tare

28 3 7 futtle.

1 0 12 tret.

27 2 23 subfuttle.

18 $\frac{1}{4}$ cloff.

27 2 4 $\frac{3}{4}$ neat.

lb.

Tare 14 per C.

Tret 4 per 104

Cloff 2 per 3 C.

lb.

Note, The 27 C. affords 18 cloff.

And the 2 Q. afford - 0 $\frac{1}{4}$

18 $\frac{1}{4}$

The cloff may be found decimally by multiplying the subfuttle by .00595 = $\frac{1}{168}$.

Any operation in tare and tret may be proved by working the same question by the rule of three.

PRACTICAL QUESTIONS.

1. What is the neat weight of 346 C. 3 Q. 12 lb. gross, tare 12 lb. per C.? *Ans.* 309 C. 2 Q. 21 $\frac{1}{2}$ lb. neat.

2. What is the neat weight of 139 C. 1 Q. 22 lb. gross, tare 8 lb. per C. tret 4 lb. per 104, and cloff 2 lb. per 3 C.? *Ans.* 123 C. 3 Q. 1 $\frac{3}{4}$ lb. neat.

3. The neat weight being 216 C. tare 14 per C. what is the tare and gross? *Ans.* The tare is 30 C. 3 Q. 12 lb. and the gross 246 C. 3 Q. 12 lb.

4. The neat weight being 97 C. 3 Q. 2 lb. tret 4 lb. per 104, and tare 16 lb. per C. what is the tret, futtle, tare, and gross? *Ans.* The tret is 3 C. 3 Q. 18 lb. the futtle 101 C. 2 Q. 20 lb. the tare 16 C. 3 Q. 22 lb. and the gross 118 C. 2 Q. 14 lb.

5. The gross weight being 44 C. 3 Q. 18 lb. the tare on the whole 1 C. 1 Q. 2 lb. and the tret 4 lb. per

104, what is the neat weight, and what the price, at 2 l. 16 s. *per C.* neat? *Ans.* The neat weight is 41 C. 3 Q. 24 lb. and the price is 117 l. 10 s.

6. Six hogheads contain in all 55 C. 0 Q. 26 lb. gross; now tare being allowed at 30 lb. *per hoghead*, tret at 4 lb. *per 104*, and cloff at 2 lb. *per 3 C.* what will be the neat weight, and what the price thereof, at 9 d. *per lb.* neat? *Ans.* Neat weight 51 C. 1 Q. 0 $\frac{3}{4}$ lb. price 215 l. 5 s. 6 $\frac{3}{4}$ d.

C H A P. IV.

B A R T E R.

Barter, or truck, is the exchanging of one commodity for another; in doing of which the price of one of the commodities, and an equivalent quantity of the other, must be found either by practice, or by the rule of three.

Quest 1. How many pounds of cotton, at 9 d. *per lb.* must be given in barter for 13 C. 3 Q. 14 lb. of pepper, at 2 l. 16 s. *per C*?

First, Find the price or value of the commodity whose quantity is given, as follows.

	C.	Q.	lb.	L.	s.
	13	3	14	at	2 16
2 l.			26		
16 s.			10	8	
2 Q.			1	8	
1 Q.				14	
14 lb.				7	
	L.	38	17		

Secondly,

Secondly, Find how much cotton, at 9 d. per lb. 38 l. 17 s. will purchase, as under.

$$\begin{array}{r}
 \text{d. lb.} \quad \text{L. s.} \\
 \text{If } 9 : 1 :: 38 \ 17 \\
 \quad \quad \quad 20 \\
 \quad \quad \quad \hline
 \quad \quad \quad 777 \\
 \quad \quad \quad 12 \\
 \quad \quad \quad \hline
 \quad \quad 9)9324(\quad \text{C. Q.} \\
 \text{Ans. } 1036 \text{ lb.} = 9 \ 1
 \end{array}$$

If the above question be wrought decimally, the operation may stand as follows.

$$\begin{array}{r}
 \text{C.} \quad \text{L.} \quad \text{C.} \\
 \text{If } 1 : 2.8 :: 13.875 \\
 \quad \quad \quad 2.8 \\
 \quad \quad \quad \hline
 \quad \quad \quad 111000 \\
 \quad \quad \quad 27750 \quad \text{lb.} \quad \text{C. Q.} \\
 .0375)38.8500(1036 = 9 \ 1 \text{ Ans.} \\
 \quad \quad \quad 375 \dots \\
 \quad \quad \quad \hline
 \quad \quad \quad 1350 \\
 \quad \quad \quad 1125 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2250 \\
 \quad \quad \quad 2250 \\
 \quad \quad \quad \hline
 \end{array}$$

Quest. 2. A merchant receives 126 yards of cloth in barter for 189 gallons brandy, at 6 s. 8 d. per gallon, what was the cloth valued at per yard?

$$\begin{array}{r}
 \text{Gall.} \quad \text{s. d.} \quad \text{yds.} \quad \text{L.} \quad \text{yd.} \quad \text{s.} \\
 189 \text{ at } 6 \ 8 \quad \text{If } 126 : 63 :: 1 : 10 \text{ Ans.} \\
 \hline
 \frac{1}{3} \mid \text{L. } 63
 \end{array}$$

The decimal operation of the same question follows.

$$\begin{array}{r}
 G. \quad L. \quad 3) G. \\
 \text{If } 1 : 3 :: 189 \quad L. \quad s. \\
 126 \overline{) 63.0(.5} = 10 \text{ Ans.} \\
 \underline{63 \quad 0}
 \end{array}$$

Quest. 3. A and B barter ; A gets 20 C. of cheese, at 21 s. 6 d. *per* C. ; B gets eight pieces of cloth, at 3 l. 14 s. *per* piece : What is the balance, and to whom due ?

	L.	s.
B's 8 pieces cloth, at 3 l. 14 s. come to	29	12
A's 20 C. cheese, at 21 s. 6 d. amount to	21	10
Balance due to B.	8	2

Quest. 4. A and B barter ; A hath ginger worth 1 l. 17 s. 4 d. *per* C. ready money, but in barter he will have 2 l. 16 s. *per* C. ; B hath nutmegs worth L. 5 12 s. *per* C. ready money : How must B rate his nutmegs in barter, to be on an equal footing with A. Say,

$$\begin{array}{ccccccc}
 L. & s. & d. & L. & s. & L. & s. & L. & s. \\
 \text{If } 1 & 17 & 4 : 2 & 16 :: 5 & 12 : 8 & 8 \text{ Ans.}
 \end{array}$$

The value or price of the goods received and delivered in barter being always equal, it is obvious, that the product of the quantities received and delivered, multiplied into their respective rates, will be equal.

Hence arises a rule which may be used with advantage in working several questions ; namely, Multiply the given quantity and rate of the one commodity, and the product divided by the rate of the other commodity quotes the quantity sought ; or divided by the quantity quotes the rate.

Quest. 5. How many yards of linen, at 4 s. *per* yard, should I have in barter for 120 yards of velvet, at 15 s. 6 d ?

$$\begin{array}{rcl}
 \text{Yards.} & \text{Sixpences.} & \text{Sixp.} & \text{Yards.} \\
 120 \times 31 & = & 3720, \text{ and } 8)3720(465 \text{ Ans.}
 \end{array}$$

Quest.

Quest. 6. If I receive 140 yards of broad cloth in barter, for 28 yards of velvet, at 16 s. 8 d. *per* yard, what was the broad cloth valued at *per* yard?

Yds. Groats.

s. d.

$28 \times 50 = 1400$, and $140 \div 1400 (10 \text{ groats, or } 3 \text{ } 4)$ *Ans.*

Quest. 7. How many hats at 7 s. napkins at 4 s. and caps at 1 s. of each an equal number, may I have in barter for 280 yards of linen, at 3 s *per* yard?

Ans. [fort.

$280 \times 3 = 840$, and $7 + 4 + 1 = 12$ $840 \div 12 = 70$ of each

Quest. 8. A has 41 C. hops, at 30 s. *per* C. for which B gives him 20 l. in money, and the rest in prunes, at 5 d. *per* lb.: How many prunes did B give A beside the 20 l.? *Ans.* 17 C. 3 Q. 4 lb.

Quest. 9. A has 608 yards broad cloth, at 14 s. *per* yard; for which B gives him 125 l. 12 s. and 85 C. 2 Q. 24 lb. bees wax: How was the wax rated *per* C.? *Ans.* At 3 l. 10 s. *per* C.

Quest. 10. A hath 120 yards druggets, worth 6 s. *per* yard, but in barter he will have 8 s. *per* yard; B hath shalloon worth 4 s. *per* yard: How ought B to rate his shalloon in barter? and how many yards ought he to give A for his druggets? *Ans.* The shalloon is to be rated at 5 s. 4 d. *per* yard; and 180 yards to be given for the druggets.

Quest. 11. A hath 324 yards linen, at 4 s. 6 d. *per* yard; for which B gives him hats, at 7 s. *per* piece, and stockings, at 6 s. 6 d. *per* pair: How many hats and pairs of stockings, of each an equal number, must B give A for his linen? *Ans.* 108 hats, and as many pairs of stockings.

Quest. 12. A has 100 pieces silk, worth 3 l. *per* piece; but barterers them at 4 l. *per* piece, and takes their value from B in wool, at 7 l. 10 s. *per* C. which was only worth 6 l. *per* C.: How much wool must B give A for the silk? Whether was A or B the gainer? and how much? *Ans.* B gives A $53\frac{1}{3}$ C. wool, and A gains 20 l.

CHAP. V.

LOSS and GAIN.

Questions relating to loss and gain, are likewise solved by practice, or the rule of three, as in the following examples.

Quest. 1. A merchant bought 436 yards of broad cloth, at 8 s. 6 d. *per* yard, which he sold at 10 s. 4 d. *per* yard: How much did he gain on the whole?

Find both the buying price and selling price by practice or the rule of three, and their difference is the gain.

	<i>Yds.</i>	<i>s.</i>	<i>d.</i>
	436	at	8 6
6 s.	130	16	
6 d.	54	10	
	185	6	buying price.

	<i>Yds.</i>	<i>s.</i>	<i>d.</i>
	436	at	10 4
6 s.	130	16	0
1 s.	21	16	0
4 d.	72	13	4
	225	5	4 selling price.
	185	6	0 buying price.
	39	19	4 gain.

Or, Find the gain *per* yard, by subtracting the buying rate from the selling rate; and then find the gain on 436 yards, by practice or by the rule of three.

s.	d.		Yds.
10	4	buying rate.	1 s. 436 at 1 s. 10 d.
8	6	selling rate.	6 d. 218
1	10	gain per yard.	4 d. 145 4
			$\frac{1}{20}$ 79 9 4
L. 39 19 4 gain.			

Yd. d. Yds.
Or, If 1 : 22 :: 436

$$\begin{array}{r} 22 \\ \times 436 \\ \hline 872 \\ 22 \times 436 \\ \hline 12992 \end{array}$$

Gain L. 39 19 4

Yd. L. Yds. L. s. d.
Or, decimally, If 1 : .0916 :: 436 : 39 19 4

Quest. 2. A draper bought 124 yards of holland for 31 l. : How must he sell it *per* yard to gain 10 l. 6 s. 8 d. on the whole ?

To the buying price add the intended gain, and then find the rate *per* yard, by the rule of three.

L. s. d.	Yds.	L. s. d.	Yd. s. d.
31 0 0	buying price.	If 124 : 41 6 8 :: 1 : 6 8	<i>Ans.</i>
10 6 8	gain.		
41 6 8	selling price.		

Yds. L. Yd. s. d.
Or, decimally, If 124 : 41.7 :: 1 : 6 8

Quest. 3. A grocer bought 3 C. 1 Q. 14 lb. of cloves, at 2 s. 4 d. *per* lb. which he sold for 52 l. 14 s. : How much did he gain on the whole ?

Find

Find the buying price, subtract it from the selling price, and the remainder is the gain, *viz.* 8 l. 12 s.

Quest. 4. A parcel of goods was bought for 60 l. and sold for 75 l.: What was gained *per cent.*? that is, How many pounds, shillings, or pence, would have been gained in the sale upon 100 l. 100 s. or 100 d. laid out in the purchase? *Ans.* 25 *per cent.* For,

L. L.

As 60 : 75 :: 100 : 125. Or, As 60 : 15 :: 100 : 25

Quest. 5. If a halfpenny in the shilling be a merchant's profit, what does he gain *per cent.*? *Ans.* 4 l. 3 s. 4 d. *per* 100 l. or $4\frac{1}{2}$ *per cent.*

Because $\frac{1}{2}$ d. *per* shilling is 10 d. *per* l. you may work thus.

Or, Because $\frac{1}{2}$ d. is $\frac{1}{24}$ s. and $24 = 4 \times 6$, you may work also thus.

L. 100
10
 12)1000 pence.

2|0)8|3 4
 L. 4 3 4

4)100 l.

6)25

L. 4 3 4 *per* 100 l.

Hence a penny in the shilling is L. 8 : 6 : 8 *per* 100 l. or $8\frac{1}{3}$ *per cent.* And a penny halfpenny in the shilling is 12 l. 10 s. *per* 100 l. or $12\frac{1}{2}$ *per cent.*

Quest. 6. If 2 d. in the shilling be a dealer's profit, what does he gain *per cent.*? *Ans.* L. 16 : 13 : 4. *per* 100 l. or $16\frac{2}{3}$ *per cent.*

Because 2 d. is $\frac{1}{6}$ s. take $\frac{1}{6}$ of 100 l. as follows.

6)100 l.

L. 16 13 4

Hence it is obvious, that whatever part of a shilling the profit or loss is, the same part of 100 will be the gain

gain or loss *per cent.* Thus, 3 d. of profit or loss on the shilling is 25 *per cent.*.. And in like manner, from the gain or loss *per cent.* given, may the profit or loss *per shilling* be found.

And whatever part of a pound the given gain or loss is, the like part of 100 will be the gain or loss *per cent.*; which, therefore, may be found as directed above.

Quest. 7. A chapman has goods to the value of £. 415 : 12 : 6; but coming to a bad market, was obliged to sell them at 12 *per cent.* loss: What were they sold for? *Ans.* 365 l. 15 s. See the work.

[illegible]

Quest. 8. A merchant buys goods to the value of 425 l. and offers to sell them for 15 per cent. profit: What come they to? *Ans.* 488 l. 15 s. Say,

L.
 If 100 : 115 :: 425
115
 2125
 425
425
 L. 488 75
20
 s. 15 00

L. s.
 Ans. 488 15

Quest. 9. The prime cost of a parcel of goods sent to Jamaica is L. 37 : 8 : $1\frac{1}{2}$; and being sold at 75 per cent. advance on the invoice, what does that amount to? Ans. L. 65 : 9 : $2\frac{1}{2}$. Say,

L. s. d.
 If 100 : 75 :: 37 8 $1\frac{1}{2}$
20
 748
12
 8977
4
 35910
75
 17955
25137
 4)26932.50
12)6733
20)5611

Advance L. 28 1 1
 Prime cost 37 8 $1\frac{1}{2}$
 Amount 65 9 $2\frac{1}{2}$

The 50 cut off is $\frac{50}{100}$ f. = $\frac{1}{2}$ f. more.

Or thus :
 425 at 15 per cent.
4
 400 60
 25 3 15
63 15 profit.
 425 prime cost.
 L. 488 15 price.

L. s. d.
 Or, If 100 : 175 :: 37 8 $1\frac{1}{2}$
20
 748
12
 8977
4
 35910
175
 17955
25137
 3591
4)62842.50
12)15710 2
20)1309 2
 Amount L. 65 9 $2\frac{1}{2}$

Quest.

Quest. 10. If 86 pieces of cloth cost in all 129 l. how must they be sold per piece to gain 15 per cent.?

Ans. L. 1 : 14 : 6. Say,

L. L. s.

If 100 : 115 :: 129 : 148 7.

Pc. L. s. Pc. L. s. d.

And, If 86 : 148 7 :: 1 : 1 14 6.

Quest. 11. A grocer bought 4 C. 1 Q. pepper for L. 15 : 17 : 4; which happening to be damnified, he is willing to lose 12½ per cent.: How must he sell it per lb. *Ans.* At 7 d. per lb.

From 100 l. subtract 12 l. 10 s. and there remains 87 l. 10 s. Then say,

L. L. s. L. s. d. L. s. d.

If 100 : 87 10 :: 15 17 4 : 13 17 8 the selling price of the whole.

C. Q. L. s. d. lb. d.

And, If 4 1 : 13 17 8 :: 1 : 7 the rate per lb.

Quest. 12. If a merchant, in selling goods at 8 d. per lb. gain 12 per cent. what will he gain per cent. in selling the same goods at 9 d. per lb.? *Ans.* 26 per cent.

If 8 : 112 :: 9 : 126

Quest. 13. Sold a puncheon of rum for 30 l. and gained 20 per cent.: What was the prime cost? *Ans.* 25 l.

L. L.

If 120 : 100 :: 30 : 25

Quest. 14. A plumber sold 10 fodder of lead, each fodder containing 19½ C. for 204 l. 15 s. and gained 12½ per cent.: How much did it cost him in all? and how much per C. *Ans.* 182 l. in all; and 18 s. 8 d. per C.

Quest. 15. A merchant sells 8 tuns of wines for 440 l. and loses 12 *per cent.* : How much did it cost *per tun*? and how does he sell it *per gallon*? *Ans.* It cost 62 l. 10 s. *per tun*; and he sells it at 4 s. 4 d. $1\frac{1}{2}$ f. *per gallon*.

Quest. 16. If a merchant sell cloth at 11 s. 6 d. *per yard*, and thereby gain 15 *per cent.* what would he gain *per cent.* by selling the same cloth at 12 s. *per yard*? *Ans.* 20 *per cent.*

Quest. 17. A merchant selling corn at 8 s. *per bushel*, gained 10 *per cent.*; but the market falling, he was obliged to sell the rest of the same corn at 7 s. *per bushel*: What did he gain or lose *per cent.* on this last sale? *Ans.* He lost $3\frac{3}{4}$ *per cent.*

Quest. 18. A draper bought 28 pieces of stuffs; at 4 l. *per piece*, and sells 10 of them at 4 l. 6 s. *per piece*, and 8 of them at 4 l. 8 s. *per piece*; At what rate must he sell the rest, to gain 10 *per cent.* on the whole? *Ans.* At 4 l. 10 s. *per piece*.

CHAP. VI.

MISCELLANIES.

I. Brokage.

Brokers are agents employed by merchants in transacting mercantile affairs; such as, buying, selling, negotiating bills, &c. They are generally men that have been bred merchants, understand trade and exchange, with the proper seasons of doing business. They must too be men of character; and no decayed merchant is allowed to officiate as broker without a special licence. The payment or fee for their service is called *brokage*, being a scanty allowance of a few shillings *per cent.* on the 100 l.; and is readily cast up by practice, as in the following examples.

RULE.

Find the brokage at 1 *per cent.* by dividing the given sum by 100, and then take aliquot parts of the quot.

1. What

1. What is the brokage of L. 985 : 15 : 6, at 6 s. per cent.?

$$\begin{array}{r|l} \text{L. } 9 & 85 \ 15 \ 6 \\ & \underline{20} \\ \text{s. } 17 & 15 \\ & \underline{12} \\ \text{d. } 1 & 86 \\ & \underline{4} \\ \text{f. } 3 & 44 \end{array}$$

Or,

$$\begin{array}{r|l} \text{L. s. d.} & \\ \hline & 9 \ 17 \ 13\frac{3}{4} \\ & \underline{2 \ 9 \ 3\frac{1}{4}} \\ & 9 \ 10\frac{1}{4} \\ & \underline{2 \ 19 \ 1\frac{1}{2}} \text{ Ans.} \end{array}$$

$$\begin{array}{r} \text{L. s. d.} \\ 6 \text{ s.} = \frac{3}{10} \text{ l.} \quad 9 \ 17 \ 13\frac{3}{4} \\ \quad \quad \quad \underline{3} \\ (10) \quad 29 \ 11 \ 5\frac{1}{4} \\ \text{Ans. } 2 \ 19 \ 1\frac{1}{2} \end{array}$$

2. What is the brokage of 439 l. 19 s. at 7 s. 6 d. per cent.?

$$\begin{array}{r|l} \text{L. } 4 & 39 \ 19 \\ & \underline{20} \\ \text{s. } 7 & 99 \\ & \underline{12} \\ \text{d. } 11 & 88 \\ & \underline{4} \\ \text{f. } 3 & 52 \end{array}$$

7 s. 6 d. = $\frac{3}{4}$ l.

$$\begin{array}{r|l} \text{L. s. d.} & \\ \hline & 4 \ 7 \ 11\frac{3}{4} \\ & \underline{3} \\ & 8) 13 \ 3 \ 11\frac{1}{4} \\ & \underline{11 \ 12 \ 11\frac{3}{4}} \text{ Ans.} \end{array}$$

3. What is the brokage of 345 l. 16 s. at 4 s. 9 d. per cent.?

L.

L. 3	45 16	s. d.	L. s. d.
	20		3 9 1 $\frac{3}{4}$
s. 9	16	4 0 =	13 9 $\frac{3}{4}$
	12	6 =	1 8 $\frac{1}{2}$
d. 1	92	3 =	10 $\frac{1}{4}$
	4		16 4 $\frac{1}{2}$ Ans.
f. 3	68		

MORE EXAMPLES.

4. What is the brokage of L. 834 : 18 : 4, at 5 s. 6 d. per cent.? *Ans.* L. 2 : 5 : 10 $\frac{3}{4}$.

5. What is the brokage of L. 476 : 12 : 9, at 6 s. 8 d. per cent.? *Ans.* L. 1 : 11 : 9 $\frac{1}{4}$.

6. What is the brokage of L. 954, 17 s. at 4 s. 9 d. per cent.? *Ans.* L. 2 : 5 : 4.

II. Bankruptcy.

When a person in trade fails, stops payment, or turns insolvent, he is called *bankrupt*; and the computations used in settling affairs among the creditors are such as occur in the following questions.

1. A bankrupt's effects amount to 811 l. 10 s. and he owes

	L.	s.	d.
To A	220	16	6
To B	312	0	0
To C	117	12	6
To D	106	12	6
To E	200	6	0
To F	124	12	6
In all	1082	0	0

How much can he afford to pay *per pound*? and what must each creditor have?

Ans. 15 s. *per pound*: and the creditors get as follows.

L,

	L.	s.	d.
A	165	12	4½
B	234	0	0
C	88	4	4½
D	79	19	4½
E	150	4	6
F	93	9	4½

Proof 811 10

First find the dividend for 1 l. by saying,

$$\begin{array}{r}
 \text{L.} \quad \text{L.} \quad \text{s.} \quad \text{L.} \\
 \text{If } 1082 : 811 \text{ } 10 :: 1 \\
 \quad \quad \quad 20 \\
 \hline
 1082 \overline{) 16230} (15 \text{ s.} \\
 \underline{1082 \cdot} \\
 5410 \\
 \underline{5410} \\
 0
 \end{array}$$

Then find the dividend to each creditor by practice, as follows.

	L.	s.	d.		L.
	A	220	16 6, at 15s.		B
10 = ½	110	8	3	10 = ½	156
5 = ½	55	4	1½	5 = ½	78
	165	12	4½		234

Work in like manner for each of the other creditors.

Or you may work by decimal practice, viz. turn the dividend for 1 l. into a decimal, multiply it through the nine digits; and you have a table constructed, from which may be collected the dividend for each creditor, by moving the decimal point, and taking aliquot parts, as follows.

L.

L. s. d.
A 220 16 6

200 00 0 = 150.

20 00 0 = 15.

10 0 = .375

5 0 = .1875

1 0 = .0375

6 = .01875

165.61875

TABLE.

1	.75
2	1.50
3	2.25
4	3.00
5	3.75
6	4.50
7	5.25
8	6.00
9	6.75

L.

312

300 = 225.

10 = 7.5

2 = 1.5

234

Or, Multiply 312 by .75

312

1560

2184

234.00

2. A breaks, and agrees with his creditors at 17 s. 6 d. per pound: What will B receive, to whom he owes 400 l.?

L.

400, at 17 s. 6 d.

10 0 = 200

5 0 = 100

2 6 = 50

350 Ans.

3. A compounds with his creditors for 13 s. 4 d. per pound: What may B expect, to whom he owes 415 l. 9 s. 6 d.?

L.

$$\begin{array}{r}
 \text{L. } s. \text{ d.} \\
 415 \quad 9 \quad 6, \text{ at } 13 \text{ s. } 4 \text{ d.} \\
 \hline
 s. \text{ d.} \\
 6 \text{ } 8 = \frac{1}{3} = 138 \quad 9 \quad 10 \\
 6 \text{ } 8 = \frac{1}{3} = 138 \quad 9 \quad 10 \\
 \hline
 276 \quad 19 \quad 8 \text{ Ans.}
 \end{array}$$

MORE EXAMPLES.

4. A agrees with his creditors at 5 s. 5 d. *per* pound : What will B get, to whom he was owing 1000 l.? *Ans.* L. 270 : 16 : 8.

5. A bankrupt's effects afford 11 s. 7½ d. *per* pound : What will B get, to whom there is owing 300 l.? *Ans.* L. 174 : 7 : 6.

6. A bankrupt settles with his creditors at 6 s. 8 d. *per* pound, and clears with B, by paying him 317 l. 6 s. 8 d. : What was the original debt? *Ans.* 952 l.

III. Insurance.

Insurance is a security given by the insurers or underwriters, to indemnify the insured from such losses as are mentioned in the policy of insurance, in consideration of a sum of money called *premium*; which varies according to the risk, and is generally so much *per cent.*

It is an usual article in the policy, that in case of loss, the insurer is to be allowed a small discount or abatement, commonly 2 *per cent.*; that is, he is to pay but 98 l. for every 100 l. insured; and the 98 l. is called the *short recovery*.

A merchant sometimes insures, not only the full value of his cargo, but even the premium and discount; that, in case of loss, he may be intitled to a sum from the underwriters, or insurance office, exactly equal to the value of the said cargo; and this is called *covering his offset*.

In case of damage or loss, an estimate thereof is made out, and the underwriters pay in proportion to their several subscriptions; and this is called *an average*.

Most of the computations relative to insurance, fall under one or other of the cases following.

Case I. Given the rate *per cent.* to find the premium.
Work by practice or aliquot parts.

Ex. What premium must be paid for insuring 854 l. at $7\frac{1}{2}$ *per cent.*? and for insuring 2860 l. at $13\frac{1}{2}$ *per cent.*?

Ex. 1.

$$\begin{array}{rcl}
 \text{p.c.} & \left| \begin{array}{r} 854 \\ 5 = \frac{1}{20} 42.7 \\ 2\frac{1}{2} = \frac{1}{2} 21.35 \\ \hline 7\frac{1}{2} \end{array} \right. & \text{L. s.} \\
 & 64.05 = 64 \text{ l. } 1 \text{ Ans.} &
 \end{array}$$

Ex. 2.

$$\begin{array}{rcl}
 \text{p.c.} & \left| \begin{array}{r} 2860 \\ 10 = \frac{1}{10} 286 \\ 2 = \frac{1}{5} 57.2 \\ 1 = \frac{1}{2} 28.6 \\ \frac{1}{2} = \frac{1}{2} 14.3 \\ \hline 13\frac{1}{2} \end{array} \right. & \text{L. s.} \\
 & 386.1 = 386 \text{ l. } 2 \text{ Ans.} &
 \end{array}$$

Case II. Given the discount *per cent.* to find the short recovery.

Work by aliquot parts.

Ex. What is the short recovery of L. 2000 insured, when discount is allowed at 2 *per cent.*? and at $2\frac{1}{2}$ *per cent.*?

$$\begin{array}{rcl}
 \text{p.c.} & \left| \begin{array}{r} 2000 \\ 2 = \frac{1}{50} 40 \text{ discount.} \\ \hline 1960 \end{array} \right. & \text{short reco-} \\
 & & \text{[very.]}
 \end{array}$$

$$\begin{array}{rcl}
 \text{p.c.} & \left| \begin{array}{r} 2000 \\ 2\frac{1}{2} = \frac{1}{40} 50 \text{ discount.} \\ \hline 1950 \end{array} \right. & \text{short reco-} \\
 & & \text{[very.]}
 \end{array}$$

Case III. Given the rate *per cent.* of premium and discount, to find what sum must be insured to cover the outset in a single voyage.

Subtract the sum of the premium, and discount from 100, and then say, As the remainder to 100, so the outset to the answer.

Ex. 1. If the premium be 10 *per cent.* and the discount 2 *per cent.* what sum must be insured to cover L. 100 outset?

10 premium.	100				
2 discount.	12				
12 sum.	As 88	: 100 :: 100	: 113	12 8	<i>Anf.</i>

Proof.

	L.	s.	d.	
Sum insured	113	12	8	
Deduct 2 per cent.	2	5	5	discount.
Insurer pays	111	7	3	in case of loss.
Deduct 10 per cent.	11	7	3	premium.
Outset covered	100			or made good.

Ex. 2. If the premium be $13\frac{1}{2}$ per cent. and the discount 2 per cent. what sum must be insured in order to cover 800 l. outset?

$13\frac{1}{2}$ prem.	100				
2 disc.	15.5				
$15\frac{1}{2}$ sum.	As 84.5	: 100 :: 800	: 946	14 10 $\frac{3}{4}$	<i>Anf.</i>

Case IV. Given the rate per cent. of premium and discount, to find the premium out and home, and what sum must be insured to cover the inset.

First, Find the premium on the voyage out, as taught above, and for the voyage home work thus: From the short recovery deduct the premium; then say, As the remainder to the premium, so the sum of the outset and premium of the voyage out, to the premium of the voyage home; which, added to the premium of the voyage out, gives the total premium. Lastly, Find what sum insured will cover the inset by case 3.

Ex. If the premium be 10 per cent. and the discount 2 per cent. what premium must be paid for insuring 100 l. out and home? and what sum must be insured to cover the inset?

By case 3. ex. 1. the premium of the voyage out is L. 11 : 7 : 3.

	<i>L. s. d.</i>	
98	11 7 3	premium out,
10	100 0 0	outset.

	<i>L. s. d.</i>	
As 88 : 10 ::	111 7 3 : 12 13 1	prem. home.
	11 7 3	prem. out.
	24 00 4	total prem.

Lastly, by case 3. find what sum must be insured to cover L. 111 : 7 : 3, viz. say,

	<i>L. s. d.</i>	
As 88 : 100 ::	111 7 3	
	10	
	1113 12 6	
	10	
	<i>L. s. d.</i>	
88)11136	5 0	(126 10 11 <i>Ans.</i>

In former times the policies of insurance were generally conceived in such vague terms, that sometimes it was no easy matter to settle an average when a loss happened. The affair was usually brought before a court, critical questions were started, difficulties and clouds of dust were raised, that tended to inveigle the cause, and render it a subject of endless altercation : but the case is otherwise now ; for scarce any misfortune can happen but what is expressly specified in the policies used at present. I shall conclude insurance with one example of a general average on ship, goods, and freight.

A ship, by stress of weather, lost her masts, was forced ashore, and the damage estimated at 361.

	L.
A's goods, -	1200
B's goods, -	480
Insured by Mess. Adair, 1680	
A's own risk, -	60
B's own risk, -	40
Value of the goods, 1780	
Value of the ship, -	600
Amount of freight, 200	
	2580

Now, if 2580 l. pays 36l. then,

L.		L.	s.	d.
1680	pays	23	8	10
60	pays	0	16	9
40	pays	0	11	2
600	the ship, pays	8	7	5 $\frac{1}{4}$
200	freight, pays	2	15	9 $\frac{3}{4}$
	Proof	36	0	0

IV. Of purchasing stocks.

The British government, ever since the revolution in 1688, have been in use to raise supplies, for carrying on war, or answering other exigencies of the state, by borrowing money, and appropriating the public taxes for payment of the interest.

These national debts, contracted by borrowing from the bank of England, from trading companies, and from private persons, or by selling annuities, with the provisions made by parliament for paying the interest due thereon, or for cancelling the debts, are commonly called the *public funds* or *stocks*; and come under various designations; such as, *Bank Stock*, *India Stock*, *India Annuities*, *India Bonds*, *South-Sea Stock*, *South-Sea Annuities*, *Three per cent. Reduced Annuities*, *Three per cent. Consolidated Annuities*, *Three and a half*

half per cent. Annuities, Four per cent. Annuities, Long Annuities, Exchequer Bills, Navy and Victualling Bills, &c.

When the parliament has voted the supplies, a subscription is opened, and generally a few men of fortune subscribe for the whole; and as the subscriptions are all transferrable, they afterwards sell off these stocks in small shares, and at high rates, and the profits thence arising are so much clear gain.

The annuities and growing interest have hitherto been always punctually paid; and as the stocks now amount to many millions Sterling, not only monied corporations and men of fortune in Britain, but many foreigners have come to be concerned in them. The British stocks are now become a considerable branch of commerce, carried on in Exchange alley; and commissions are sent up from all the parts of Europe to make purchases in them.

Stocks are bought or sold at so much *per cent.*; and the price, or sum to be paid, may be cast up by practice, or decimally, as in the following examples.

1. What must be paid for 403 l. 18 s. bank annuities, at 94 *per cent.*?

By practice.

L. s.		403 18, at 94 <i>per cent.</i>		
20 = $\frac{1}{5}$		80	15	7
5 = $\frac{1}{4}$		20	3	$10\frac{3}{4}$
1 = $\frac{1}{5}$		4	0	$9\frac{1}{4}$
6		24	4	8 sub.
		379	13	4 <i>Ans.</i>

Decimally.

Decimally.

$$\begin{array}{r}
 L. \\
 \text{As } 100 : 94 :: 403.9 \\
 \underline{94} \\
 16156 \\
 36351 \\
 \hline
 \text{Ans. } 379.666
 \end{array}$$

2. What sum will purchase 875 l. 10 s. bank stock, at $131\frac{1}{4}$ per cent.?

By practice.

		L.	s.	
100 =		875	10	at $131\frac{1}{4}$ per cent.
25 = $\frac{1}{4}$		218	17	6
5 = $\frac{1}{5}$		43	15	6
$1\frac{1}{4}$ = $\frac{1}{4}$		10	18	$10\frac{1}{2}$
$131\frac{1}{4}$		1149	1	$10\frac{1}{2}$ Ans.

Decimally.

$$\begin{array}{r}
 L. \\
 \text{As } 100 : 131.25 :: 875.5 \\
 \underline{875.5} \\
 65625 \\
 65625 \\
 91875 \\
 \underline{105000} \\
 1149.09375 \text{ Ans.}
 \end{array}$$

3. What sum will purchase 432 l. 15 s. South-Sea Stock, at $113\frac{5}{8}$ per cent.?

By

By practice.

			L.	s.	d.
100	=		432	15	0
10	=	$\frac{1}{10}$	43	5	0
2	=	$\frac{1}{5}$	8	13	1
1	=	$\frac{1}{2}$	4	6	$6\frac{1}{2}$
$\frac{1}{2}$	=	$\frac{1}{4}$	2	3	$3\frac{1}{4}$
$\frac{1}{8}$	=	$\frac{1}{8}$	0	10	$9\frac{3}{4}$
113 $\frac{5}{8}$			491	14	$2\frac{1}{2}$ Ans.

Decimally. L.

As 100 : 113.625 :: 432.75

Mult. 57.234 inverted.

454500
 34087
 2272
 795
 56

491.710 Ans.

If the stock be any just number of hundreds, multiply the rate *per cent.* by the number of hundreds, and the product is the price, as in the following example.

4. What must I pay for 800 three *per cent.* annuities, at $124\frac{7}{8}$ *per cent.*?

L.	s.	d.
124	17	6
		8
999	0	0 Ans.

V. An easy way of finding the value of the short hundred, or five score.

R U L E I.

If the rate, or price of 1, be shillings, multiply the rate by 5, and the product is the answer in pounds.

The

The answer, multiplied by 2, 3, 4, &c. or by 10, gives the price of 200, 300, 400, &c. or of 1000.

EXAMPLES.

1. What cost 100 ells, at 16 s. *per ell*?

$$\begin{array}{r} 16 \text{ s.} \\ \underline{5} \\ \text{L. } 80 \text{ } \textit{Ans.} \\ \underline{3} \\ 240 \text{ price of 300 ells.} \end{array}$$

2. What cost 100 yards, at 35 s. *per yard*?

$$\begin{array}{r} 35 \text{ s.} \\ \underline{5} \\ \text{L. } 175 \text{ } \textit{Ans.} \\ \underline{10} \\ 1750 \text{ price of 1000 yds.} \end{array}$$

3. What cost 100 lb. at 3 l. 14 s. *per lb.*?

$$\begin{array}{r} \text{L. } \text{s.} \\ 3 \text{ } 14 \\ \underline{20} \\ 74 \\ \underline{5} \\ \text{L. } 370 \text{ } \textit{Ans.} \end{array}$$

4. What cost 100 gallons, at 2 l. 17 s. *per gallon*?

$$\begin{array}{r} \text{L. } \text{s.} \\ 2 \text{ } 17 \\ \underline{20} \\ 57 \\ \underline{5} \\ \text{L. } 285 \text{ } \textit{Ans.} \end{array}$$

R U L E II.

If the rate be pence, multiply the rate by 5, divide the product by 12, the quot will be pounds, and every unit of the remainder 1 s. 8 d.

EXAMPLES.

1. What will 100 yards cost, at 9 d. *per yard*?

$$\begin{array}{r} 9 \text{ d.} \\ \underline{5} \text{ L. } \text{s.} \\ 12)45(3 \text{ } 15 \text{ } \textit{Ans.} \\ 36 \\ \underline{} \\ (9) \end{array}$$

2. What will 100 ells cost, at 11 d. *per ell*?

$$\begin{array}{r} 11 \text{ d.} \\ \underline{5} \text{ L. } \text{s.} \text{ } \text{d.} \\ 12)55(4 \text{ } 11 \text{ } 8 \text{ } \textit{Ans.} \\ 48 \\ \underline{} \\ (7) \end{array}$$

R U L E III.

If the rate be shillings and pence, multiply the rate by 5; the product under shillings will be pounds, and every unit of the product under pence will be 1 s. 8 d.

E X A M P L E S.

1. What will 100 ells cost, at 4 s. 5 d. *per ell*?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 4 \quad 5 \\ \hline 5 \\ \hline 22 \quad 1 \end{array}$$

Ans. L. 22 1 8

2. What will 100 yards cost, at 7 s. 6 d. *per yd*?

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 7 \quad 6 \\ \hline 5 \\ \hline 37 \quad 6 \end{array}$$

Ans. L. 37 19

R U L E IV.

If the rate be farthings, multiply the rate by 5, and every unit of the product will be 5 d.

E X A M P L E S.

1. What will 100 apples cost, at 1 f. each?

$$\begin{array}{r} \text{1 f.} \\ 5 \\ \hline \text{d.} \quad \text{s.} \quad \text{d.} \\ \text{Ans. } 5 \times 5 = 25 = 2 \quad 1 \end{array}$$

2. What will 100 oranges cost, at 3 f. each?

$$\begin{array}{r} \text{3 f.} \\ 5 \\ \hline \text{d.} \quad \text{s.} \quad \text{d.} \\ \text{Ans. } 15 \times 5 = 75 = 6 \quad 3 \end{array}$$

If the value of 100 be given, and the rate, or price of 1, be required, reverse the operation.

- VI. *An easy way of finding the value of the long hundred, or six score.*

R U L E

Esteem the pence in the rate so many pounds; and for the farthings, take such a part of a pound as they

they are of a penny, and the half of this amount is the answer.

EXAMPLES.

1. What will 120 ells cost, at $7\frac{1}{2}$ d. per ell?

$$\begin{array}{r} \text{L. } s. \\ 2) 7 \quad 10 \\ \hline \text{L. } 3 \quad 15 \quad \text{Ans.} \end{array}$$

3. What will 120 gallons cost, at $16\frac{3}{4}$ d. per gallon?

$$\begin{array}{r} \text{L. } s. \\ 2) 16 \quad 15 \\ \hline \text{L. } 8 \quad 7 \quad 6 \quad \text{Ans.} \end{array}$$

2. What will 120 yards cost, at $13\frac{1}{4}$ d. per yard?

$$\begin{array}{r} \text{L. } s. \\ 2) 13 \quad 5 \\ \hline \text{L. } 6 \quad 12 \quad 6 \quad \text{Ans.} \end{array}$$

4. What will 120 lb. cost, at 17 s. $3\frac{1}{2}$ d. per lb.?

$$\begin{array}{r} 12 \\ 2) 207 \quad 10 \\ \hline \text{L. } 103 \quad 15 \quad \text{Ans.} \end{array}$$

If the price of 120 be given, and the rate be required, reverse the operation.

VII. A short way of finding the value of 1 C. or 112 lb.

R U L E.

Esteem every penny in the rate 9 s. 4 d.; esteem three farthings 7 s.; esteem a halfpenny 4 s. 8 d.; and a farthing 2 s. 4 d.

EXAMPLES.

1. What will 1 C. cost, at $7\frac{1}{2}$ d. per lb.?

$$\begin{array}{r} \text{L. } s. \quad d. \\ 1 \text{ d.} = 0 \quad 9 \quad 4 \\ \hline 7 \\ 7 \text{ d.} = 3 \quad 5 \quad 4 \\ \frac{1}{2} \text{ d.} = 0 \quad 4 \quad 8 \\ \hline \text{L. } 3 \quad 10 \quad 0 \quad \text{Ans.} \end{array}$$

2. What will 1 C. cost, at $9\frac{3}{4}$ d. per lb.?

$$\begin{array}{r} \text{L. } s. \quad d. \\ 1 \text{ d.} = 0 \quad 9 \quad 4 \\ \hline 9 \\ 9 \text{ d.} = 4 \quad 4 \quad 0 \\ \frac{3}{4} \text{ d.} = 0 \quad 7 \quad 0 \\ \hline \text{L. } 4 \quad 11 \quad 0 \quad \text{Ans.} \end{array}$$

H 2

VIII. A

VIII. *A short way of finding the value of the great gross.*

R U L E.

Multiply the rate or price of 1 dozen, in pence, by 3, divide the product by 5, and the quot will be the answer in pounds.

E X A M P L E S.

1. What will 1 great gross cost, at 7 d. *per dozen*?

$$\begin{array}{r} 7 \text{ d.} \\ 3 \\ \hline 5 \overline{)21} \\ \hline \text{L. } 4 \text{ } 4 \text{ } \textit{Ans.} \end{array}$$

2. What will 1 great gross cost, at 3 s. 6 d. *per doz.*?

$$\begin{array}{r} 42 \text{ d.} \\ 3 \\ \hline 5 \overline{)126} \\ \hline \text{L. } 25 \text{ } 4 \text{ } \textit{Ans.} \end{array}$$

In the same manner may the value of the small gross be found from the price of 1 given.

If the value of the gross be given, and the rate be required, reverse the operation; that is, multiply by 5, and divide the product by 3, and the quot is the answer in pence.

IX. *The price of goods cast up by inverting the question.*

In many cases, the value or price of goods may be found with great ease, by inverting the question, *viz.* esteem the given quantity to be the rate, and the rate to be the quantity.

E X .

EXAMPLES.

1. What cost 15 yards, at 7 d. per yard?

Invert the question, by saying, What cost 7 yards, at 15 d. or 1 s. 3 d.; and then work as follows.

s. d.

1 3
7

s. 8 9 *Ans.*

2. What cost 53 ells, at 9 d. per ell? that is, 9 ells at 53 d.?

s. d.

4 5
9

L. 1 19 9 *Ans.*

3. What cost $58\frac{1}{2}$ gallons, at 11 d. per gallon? or 11 gallons, at $58\frac{1}{2}$ d.?

s. d.

4 10 $\frac{1}{2}$
11

L. 2 13 7 $\frac{1}{2}$ *Ans.*

4. What cost $42\frac{1}{4}$ hogf-heads, at 28 s. per hogf-head? or 28 hogf-heads at $42\frac{1}{4}$ s.?

L. s. d.

2 2 3
7

28 = 7 × 4

14 15 9
4

59 3 0 *Ans.*

CHAP. VII.

EXCHANGE.

EXchange is the receiving or paying money in one country for the like sum in another.

The par in exchange is when the sum received and paid away are of the same intrinsic value, or contain an equal quantity of pure gold or silver.

The course of exchange is the current price betwixt two places, which is always fluctuating and unsettled, being sometimes above and sometimes below par, according to the circumstances of trade.

When the course of exchange rises above par, the country where it rises may conclude for certain, that the

the balance of trade runs against them. The truth of this will appear, if we suppose Britain to import from any foreign place goods to the value of 100,000 l. at par, and export only to the value of 80,000 l.; in this case bills on the said foreign place will be scarce in Britain, and consequently will rise in value; and after the 80,000 l. is paid, bills must be procured from other places at a high rate to pay the remainder, so that perhaps 120,000 l. may be paid for bills to discharge a debt of 100,000 l.

Though the course of exchange be in a perpetual flux, and rises or falls according to the circumstances of trade, yet the exchanges of London, Holland, Hamburgh, and Venice, in a great measure regulate those of all other places in Europe.

I. Exchange with Holland.

MONEY-TABLE.

	Par in Sterling.	s.	d.
8 Pennings, or 2 duytes,	1 groat or penny	==	0 0.54
2 Groats, or 16 pennings,	1 stiver	==	0 1.09
6 Stivers, or 12 pence,	1 schilling	==	0 6.56
20 Schillings,	1 pound Flemish	==	10 11.18
20 Stivers, or 40 pence,	1 guilder or florin	==	1 9.86
6 Guilders, or florins,	1 pound Flemish	==	10 11.18
2½ Guilders, or florins,	1 rixdollar	==	4 6.66

In Holland there are two sorts of money, bank and current. The bank is reckoned good security; demands on the bank are readily answered; and hence bank money is generally rated from 3 to 6 *per cent.* better than the current. The difference between the bank and current money is called the *agio*.

Bills on Holland are always drawn in bank money; and if accounts be sent over from Holland to Britain in current money, the British merchant pays these accounts by bills, and in this case has the benefit of the *agio*.

PROB.

PROB. I.

To reduce bank money to current money.

R U L E.

As 100 to 100 + agio, so the given guilders to the answer.

E X A M P L E.

What will 2210 guilders in bank money amount to in Holland currency, the agio being $3\frac{1}{8}$ per cent.?

		<i>Guild.</i>		
As 100 :	$103\frac{1}{8}$	::	2210	
8	8		825	
800	825		11050	
			4420	
			17680	
			<i>Guild. st. pen.</i>	
8 00)	18232 50	(	2279	1 4 cur.
	16...		20	
	22		10 00	(
	16		8	
	63		2	
	56		16	
	72		32	
	72		32	

Or, by practice,

50)2210	
44.2	= 2 per cent.
22.1	= 1 per cent.
2.7625	= $\frac{1}{8}$ per cent.
2279.0625	

If the agio only be required, make the agio the middle term, thus:

Guil.

Guild. Guild. st. pen.

As 100 : $3\frac{1}{8}$:: 2210 : 69 1 4 agio. Or,
work by practice, as above.

P R O B. II.

To reduce current money to bank money.

R U L E.

As 100 + agio to 100, so the given guilders to the answer.

E X A M P L E.

What will 2279 guilders 1 stiver 4 pennings, Holland currency, amount to in bank money, the agio being $3\frac{1}{8}$ per cent.?

<i>Guild.</i>	<i>Guild.</i>	<i>Guild. st. pen.</i>
As $103\frac{1}{8}$	: 100	:: 2279 1 4
8	8	20
825	800	45581
20		16
16500		273490
16		45581
990		729300
165		800
8)264000		8)583440000
3)33		3)72930 <i>Guild.</i>
11		11)24310(2210 bank.

In Amsterdam, Rotterdam, Middleburgh, &c. books and accounts are kept by some in guilders, stivers, and pennings, and by others in pounds, schillings, and pence, Flemish.

Britain gives 1 l. Sterling for an uncertain number of schillings and pence Flemish. The par is 1 l. Sterling for 36.59 s. Flemish; that is, 1 l. 16 s. 7.08 d. Flemish.

When

When the Flemish rate rises above par, Britain gains and Holland loses by the exchange, and the contrary.

Sterling money is changed into Flemish, by saying,

As 1 l. Sterling to the given rate,

So is the given Sterling to the Flemish sought.

Or, the Flemish money may be cast up by practice.

Dutch money, whether pounds, schillings, pence Flemish, or guilders, stivers, pennings, may be changed into Sterling, by saying,

As the given rate to 1 l. Sterling,

So the given Dutch to the Sterling sought.

EXAMPLES.

1. A merchant in Britain draws on Amsterdam for 782 l. Sterling: How many pounds Flemish, and how many guilders will that amount to, exchange at 34 s. 8 d. per pound Sterling?

L. s. d.			Decimally.		
L.	s.	d.	L.	s.	L.
If 1	:	34 8	::	782	
		12			782
		<u>416</u>			<u>698</u>
		782			27733
		<u>832</u>			<u>242666</u>
		3328			2 0)2710 9.8
		2912			L. 1355 9 4 Flem.
12)	325312	d.			
2 0)	2710 9 4				
	<u>L. 1355 9 4</u>	Flem.			

By practice.			Or thus:		
	L.	s. d.		L.	s. d.
		782			782
10 s. = $\frac{1}{2}$		391	14 s. = $\frac{7}{10}$		547 8
4 s. = $\frac{1}{5}$		156 8	8 d. = $\frac{1}{30}$		26 1 4
8 d. = $\frac{1}{6}$		26 1 4			<u>1355 9 4 Fl.</u>
		<u>1355 9 4 Fl.</u>			

Multiply the Flemish pounds and shillings by 6, and the product will be guilders and stivers; and if there be any pence, multiply them by 8 for pennings; or, divide the Flemish pence by 40, and the quot will be guilders, and the half of the remainder, if there be any, will be stivers, and 1 penny odd will be half a stiver, or 8 pennings, as follows.

L.	s. d.	Flem. pence.
1355	9 4	40)32531 2(32 rem.
	6	
<u>Guild. 8132 16 stiv.</u>		<u>Guild. 8132 16 stiv.</u>

2. Change 591 l. 5 s. Flemish into Sterling money, exchange at 37 s. 6 d. Flemish per l. Sterling?

Flem.	Ster.	Flem.
s. d.	L.	L. s.
If 37 6 : 1 :: 591 5		
2		20
<u>5)75</u>		<u>11825</u>
5)15		2
3		<u>5)23650</u>
		<u>5) 4730</u>
		<u>3) 946</u>
		315 $\frac{1}{3}$
L. s. d.		
Ans. 315 6 8 Ster.		

Decimally.

Decimally.

$$\begin{array}{r}
 \begin{array}{ccc}
 5) L. & L. & 5) L. \\
 \text{If } 1.875 : 1 :: 591.25 \\
 \hline
 5) .375 & & 5) 118.25 \\
 5) .075 & & 5) 23.65 \\
 .015 & .015) & 4.73(315.3 \\
 & & \underline{45} \\
 & & 23 \\
 & & \underline{15} \\
 & & 80 \\
 & & \underline{75} \\
 & & 50 \\
 & & \underline{45} \\
 & & 5
 \end{array}
 \end{array}$$

3. Change 1693 guilders 6 stivers into Sterling money, exchange at 34 s. 5 d. per pound Sterling.

Flem. Ster.

s. d. L. Guild. stiv.

If 34 5 : 1 :: 1693 6

$$\begin{array}{r}
 12 \\
 \hline
 413 \\
 \hline
 33866 \\
 2
 \end{array}$$

413)67732(164 L. Ster.

$$\begin{array}{r}
 413 \\
 \hline
 2643 \\
 2478 \\
 \hline
 1652 \\
 1652 \\
 \hline
 \end{array}$$

I 2

Decimally,

Decimally.

$$\begin{array}{r}
 34.41\text{g} \\
 .3 \\
 \hline
 10.3250 : 1 :: 1693.3 \\
 10.325 \overline{) 1693.300} (164\text{L. Ster. Ans.} \\
 \underline{10325} \\
 66080 \\
 \underline{61950} \\
 41300 \\
 \underline{41300} \\
 0
 \end{array}$$

In working decimally, because 10 s. Flemish is equal to 3 guilders, we might say, As 10 : 3 :: 34.41^g to the guilders in the rate; but it is the same in effect, and shorter, to multiply by .3.

When the rate is 33 s. 4 d. Sterling money is easily converted into guilders, or guilders into Sterling, the guilder in this case being equal to 2 s. Sterling.

4. A merchant in London remits to Rotterdam 1000 l. Sterling, exchange at 35 s. 4 d. and some time afterwards draws for 1000 l. Sterling, exchange at 34 s. 3 d. How much Flemish money has he still in the bank, and how much Sterling money ought he to have drawn for to bring home the whole remittance?

<i>s. d.</i>	$1000 = 10 \times 10 \times 10$	<i>s. d.</i>
35 4		1 1
34 3		10
<u>1 1</u>		<u>10 10</u>
		10
		<u>L. 5 8 4</u>
		10
		<u>54 3 4 Fl. in bank.</u>

To

To find how much Sterling must be drawn for to bring home the remittance, say,

$$\begin{array}{r}
 \begin{array}{ccc} s. & d. & \\ As & 34 & 3 \end{array} : \begin{array}{ccc} s. & d. & \\ & 35 & 4 \end{array} :: \begin{array}{ccc} L. St. & L. & s. d. \\ 1000 & : & 1031 \end{array} \begin{array}{ccc} & 12 & 7 \end{array} \begin{array}{c} St. \\ Ans. \end{array} \\
 \hline
 \begin{array}{ccc} 12 & & \\ 411 & & 424 \end{array}
 \end{array}$$

5. When the rate of exchange is 36 s. 8 d. what is the value of the guilder?

$$\begin{array}{r}
 \begin{array}{ccc} s. & d. & \\ & 36 & 8 \end{array} \\
 \hline
 \begin{array}{ccc} 12 & & \\ 11 & & 6 \end{array} \\
 \hline
 \begin{array}{ccc} 40 & & \\ 11 & 240 & (21 \frac{2}{11} d. Sterl. Ans. \end{array}
 \end{array}$$

6. When the value of the guilder is 21 $\frac{2}{11}$ d. Sterling, what is the rate of exchange?

$$\begin{array}{r}
 \begin{array}{ccc} d. St. & d. Fl. & d. St. \\ If & 21 \frac{2}{11} & : 40 :: 240 \end{array} \\
 \hline
 \begin{array}{ccc} 11 & & 11 \\ 6 & 240 & 2640 \\ 4 & 4 & 44 \\ 1 & 1 & 11 \\ & 40 & s. d. \end{array} \\
 \hline
 12)440(36 \ 8 \ Flem. Ans.
 \end{array}$$

7. When the current guilder is worth 22 d. Sterling, what is the value of the bank guilder, theagio being 4 per cent.?

If

If 100 : 104 :: 22

22

208

2 8

100)2288(22.88 d. Sterling. *Anf.*

MORE EXAMPLES.

8. Britain draws on Holland for 1814 l. Sterling : How many guilders will that come to, exchange at 34 s. 8 d. Flemish *per* l. Sterling? *Anf.* 1886½ guild. 12 stivers.

9. What may I draw for on London, if I pay in Amsterdam 3628 guilders, exchange at 35 s. 2½ d. *per* l. Sterling? *Anf.* L. 343 : 9 : 7.

10. What Sterling money is equivalent to 687 guilders 5 stivers, exchange at 33 s. 4 d. *per* l. Sterling? *Anf.* L. 68 : 14 : 6.

Holland exchanges with other nations as follows, *viz.* with

	<i>Flem. d.</i>		<i>Flem. d.</i>
Hamburgh, on the dollar,	= 66 $\frac{2}{3}$	Leghorn, on the piastre,	= 100
France, on the crown,	= 54	Florence, on the crown,	= 120
Spain, on the ducat,	= 109 $\frac{4}{5}$	Naples, on the ducat,	= 74 $\frac{2}{3}$
Portugal on the crusade,	= 50	Rome, on the crown,	= 136
Venice, on the ducat,	= 93	Milan, on the ducat,	= 102
Genoa, on the pezzo,	= 100	Bologna, on the dollar,	= 94 $\frac{4}{5}$

Exchange betwixt Britain and Antwerp, as also the Austrian Netherlands, is negotiated the same way as with Holland, only the *par* is somewhat different, as will be described in article 2. following.

II. Exchange with *Hamburgh*.

MONEY-TABLE.

		<i>Par in Sterling.</i>	s.	d.
12 Phennings	} make	1 schilling-lub	=	0 1 $\frac{1}{8}$
16 Schilling-lubs		1 mark	=	1 6
2 Marks		1 dollar	=	3 0
3 Marks		1 rixdollar	=	4 6
6 $\frac{1}{4}$ Marks		1 ducat	=	9 4 $\frac{1}{2}$

Books and accounts are kept at the bank, and by most people in the city, in marks, schilling-lubs, and phennings; but some keep them in pounds, schillings, and groots Flemish.

The *agio* at *Hamburgh* runs between 20 and 40 *per cent*. All bills are paid in bank-money.

Hamburgh exchanges with Britain by giving an uncertain number of schillings and groots Flemish for the pound Sterling. The groot or penny Flemish here, as also at *Antwerp*, is worth $\frac{5}{8}$ of a penny Sterling; and so something better than in *Holland*, where it is only $\frac{5}{16}$ d. Sterling.

		<i>Flemish.</i>
6 Phennings	} make	1 groot or penny
6 Schilling-lubs		1 schilling
1 Schilling-lub		2 pence or groots
1 Mark		32 pence or groots
7 $\frac{1}{2}$ Marks		1 pound

The *par* with *Hamburgh*, and also with *Antwerp*, is 35 s. 6 $\frac{2}{3}$ d. Flemish for 1 l. Sterling.

EXAMPLES.

1. How many marks must be received at *Hamburgh* for 300 l. Sterling, exchange at 35 s. 3 d. Flemish *per* l. Sterling?

L.

L. s. d. L.
If 1 : 35 3 :: 300

12

423

300

M. sch.

32) 126900 (3965 10

96...

309

288

210

192

180

160

(20)

16

) 320

32

(00)

Decimally.

Flem. s. Marks. Flem. s.

If 20 : 7.5 :: 35.25

4 : 1.5 :: 35.25

1.5

17625

3525

4) 52.875

Marks in 1 l. Sterling 13.21875

300

Marks in 300 l. Sterling 3965.62500

16

3750

625

Schilling-lubs 10.000

2. How

2. How much Sterling money will a bill of 3965 marks 10 schilling-lubs amount to, exchange at 35s. 3 d. Flemish *per* l. Sterling?

<i>Fl.s.</i>	<i>d.</i>	<i>L. St.</i>	<i>Mks.</i>	<i>sch.</i>
1f 35	3	1	:: 3965	10
12			32	2
423			7930	20 d.
11897				

423)126900(300 l. Sterling.

1269

Decimally.

4 : 1.5 :: 35.25

1.5

17625

3525

4)52.875(13.21875

13.21875)3965.62500(300 l. Sterling.

3965625

3. Hamburgh is indebted to Britain 2648 marks current money: For how many marks may Britain draw on the bank, the *agio* being 30 *per cent.*?

Marks.

As 130 : 100 :: 2648

13 : 10

10

Mks. sch. phen.

13)26480(2036 14 9. *Ans.*

4. What is the Sterling value of a mark-lubs, when the exchange between Britain and Hamburgh is 36 s. 8 d. Flemish *per* l. Sterling?

Flem. s. d. d. St. d. Flem.

If 36 8 : 240 :: 32

12

6

440

11) 192 (17 $\frac{5}{11}$ d. Ster. *Ans.*

44

: 24

11

11

: 6

82

77

(5)

Because 40 d. Flemish make a guilder of Holland, and 32 d. Flemish make a mark of Hamburg, you may reduce marks to guilders, by saying, As 40 to 32, so the given marks to the guilders sought. Or, from the given number of marks, subtract one fifth of itself, and the remainder is the answer.

E X A M P L E

How many guilders of Holland are equivalent to 1730 marks of Hamburg?

Marks.

If 40 : 32 :: 1730

5 : 4

4

Guild.

5) 6920 (1384 *Ans.*

Or thus :

1730 marks.

$\frac{1}{5} = 346$

1384 guilders.

Hamburg exchanges with other countries as follows, viz. with

Holland, on the dollar,	=	66 $\frac{2}{3}$ d. Flemish.
France, on the crown,	=	51 $\frac{1}{2}$ d. Flemish.
Spain, on the ducat,	=	52 $\frac{1}{2}$ schilling-lubs.
Portugal, on the crusade,	=	48 d. Flemish.
Venice, on the ducat,	=	89 $\frac{3}{10}$ d. Flemish.
Leghorn, on the piaſtre,	=	96 d. Flemish.

III. Exchange with France.

M O N E Y - T A B L E

		<i>Par in Ster.</i>	<i>s.</i>	<i>d.</i>
12 deniers	} make	1 sol	=	0 0 $\frac{2}{8}$
20 sols		1 livre	=	0 9 $\frac{3}{4}$
3 livres		1 crown	=	2 5 $\frac{1}{4}$

At Paris, Rouen, Lyons, &c. books and accounts are kept in livres, sols, and deniers; and the exchange with Britain is on the crown, or ecu, of 3 livres, or 60 sols Tournois. Britain gives for the crown an uncertain number of pence, commonly between 30 and 34, the par, as mentioned above, being $29\frac{1}{4}$ d.

EXAMPLES.

1. What Sterling money must be paid in London to receive in Paris 1978 crowns 25 sols, exchange at $31\frac{5}{8}$ d. per crown?

<i>Sols.</i>	<i>d.</i>	<i>Cr. sols.</i>
If 60 :	$31\frac{5}{8}$::	1978 25
	<u>253</u>	<u>60</u>
		118705
		<u>253</u>
		356115
		593525
		<u>237410</u>
		60)3003236 5 <i>Rem.</i>
		8)500539 3
		12)62567 11
		2)0)521 3 13
		L. 260 13 $11\frac{3}{8}$ <i>Anf.</i>

By practice:

<i>d.</i>	<i>Cr. sols.</i>
	1978 25, at $31\frac{5}{8}$ d.
30 = $\frac{1}{8}$	247 5 0
$1\frac{1}{2}$ = $\frac{1}{20}$	12 7 3
$\frac{1}{8}$ = $\frac{1}{12}$	1 0 $7\frac{1}{4}$
Sols 20 = $\frac{1}{3}$	0 0 $10\frac{1}{2}$
5 = $\frac{1}{4}$	0 0 $2\frac{1}{2}$
	<u>260 13 $11\frac{1}{4}$</u>

K 2

If

If you work decimally, say,

$$\begin{array}{rcl} \text{Cr. d. Ster.} & \text{Cr.} & \text{d. Ster.} \\ \text{As } 1 : 31.625 :: 1978.41\phi : 62567.42708\phi \end{array}$$

2. How many French livres will L. 121 : 18 : 6 Sterling amount to, exchange at $32\frac{7}{8}$ d. per crown?

$$\begin{array}{rcllcl} \text{d.} & \text{Liv.} & \text{L.} & \text{s.} & \text{d.} \\ \text{If } 32\frac{7}{8} : 3 :: 121 & 18 & 6 & & \\ \hline 263 & 8 & 20 & & \\ & 24 & 2438 & & \\ & & 12 & & \\ & & \hline & & 29262 & & \\ & & 24 & & \\ & & \hline & & 117048 & & \\ & & 58524 & & \text{Liv. sols den.} \end{array}$$

$$263)702288(2670 \quad 5 \quad 11 \quad \text{Ans.}$$

Rem. (78) = 5 sols 11 deniers.

3. What must I pay in Britain to receive in France 3965 livres, exchange at 31 d. Sterling per crown?

$$\begin{array}{rcllcl} \text{Liv. d.} & \text{Liv.} & \text{L.} & \text{s.} & \text{d.} \\ 3 : 31 :: 3956 : 170 & 6 & 6\frac{2}{3} \end{array}$$

Or thus:

$$\begin{array}{r} 3)3956 \\ \hline 8)1318 \quad 13 \quad 4 \\ \hline 30)164 \quad 16 \quad 8 \quad \text{at } 30 \text{ d.} \\ \quad 5 \quad 9 \quad 10\frac{2}{3} \quad \text{at } 1 \text{ d.} \\ \hline \text{L. } 170 \quad 6 \quad 6\frac{2}{3} \quad \text{Ans.} \end{array}$$

4. A banker in London, with a view to make some little gain, remits to France 6000 crowns, at a time when the rate of exchange was low, viz. at 30 d. Sterling per crown; and when the exchange rose to $31\frac{1}{2}$ d. he drew for the same number of crowns: What did he gain, and how much per cent.?

Cr.

Gr.

8)6000

20) 750 L. paid at 30 d.

37 10, at $1\frac{1}{2}$, equal to the gain.787 10 Received at $31\frac{1}{2}$ d.

Then say,

If 30 : 1.5 :: 100

100

30)150.0(5 per cent. *Ans.*

M O R E E X A M P L E S.

5. What is the value of 930 crowns 26 sols 3 deniers, at $32\frac{5}{8}$ d. per crown? *Ans.* 126 l. 9 s. $7\frac{1}{2}$ d.

6. How many French crowns shall I have for 102 l. 13 s. $11\frac{1}{2}$ d. at $31\frac{3}{8}$ d. per crown? *Ans.* 785 crowns 34 sols.

France exchanges with other places also on the crown, the par being as follows, viz.

1 French crown in	100 French crowns in
Spain = $184\frac{3}{8}$ mervadies.	Naples = $72\frac{1}{4}$ ducats.
Portugal = $431\frac{1}{2}$ rees.	Florence = 45 crowns.
Milan = 62 soldi.	Denmark = 54 rubles.
Sardinia = $92\frac{1}{2}$ soldi.	Sweden = 78 rixdollars.

IV. *Exchange with Portugal.*

M O N E Y - T A B L E.

Par in Ster. s. d. f.

1 ree = 0 0 0.27

400 rees } make { 1 crusade = 2 3

1000 rees } { 1 millree = 5 $7\frac{1}{2}$

In Lisbon, Oporto, &c. books and accounts are generally kept in rees and millrees; and the millrees are distinguished from the rees, by a mark set between them thus, 485 v 372; that is, 485 millrees and 372 rees.

Britain,

Britain, as well as other nations, exchanges with Portugal on the millree, the par, as in the table, being $67\frac{1}{2}$ d. Sterling. The course with Britain runs from 63 d. to 68 d. Sterling *per millree*.

E X A M P L E S.

1. How much Sterling money will pay a bill of 827 $\times$ 160 rees, exchange at $63\frac{3}{8}$ d. Sterling *per millree*?

<i>Rees.</i>	<i>d.</i>	<i>Rees.</i>	
If 1000	: $63\frac{3}{8}$	:: 827.160	
8		507	
8000	507	579012	
		413580	
	8000)	419370.120	<i>Rem.</i> 2
	12)	52421	— 5 d.
	20)	4368	— 8 s.
		L. 218 8 $\frac{1}{4}$	<i>Ans.</i>

By practice.

<i>d.</i>	<i>Rees.</i>
	827.160, at $63\frac{3}{8}$ d.
60 = $\frac{1}{4}$	206.790
3 = $\frac{1}{20}$	10.3395
$\frac{3}{8}$ = $\frac{1}{12}$	.861625
$\frac{1}{8}$ = $\frac{1}{2}$	.4308125
	218.4219375

The rees being thousandth parts of the millrees, are annexed to the integer, and the operation proceeds exactly as in decimals.

2. How many rees of Portugal will 500 l. Sterling amount to, exchange at 5 s. 4 $\frac{5}{8}$ d. *per millree*?

d.

$$\begin{array}{rcl}
 d. & Rees. & L. \\
 \text{If } 64\frac{1}{2} & : 1000 :: 500 & \\
 \hline
 517 & \frac{8}{8000} & \frac{20}{10000} \\
 & & \hline
 & & 12 \\
 & & \hline
 & & 120000 \\
 & & \hline
 & & 8000 \\
 & & \hline
 & & Rees. \\
 517 &) 9600000000 & (1856.866 \text{ Ans.}
 \end{array}$$

MORE EXAMPLES.

3. How much Sterling money will pay a bill of 181 millrees, exchange at $64\frac{1}{2}$ d. *per* millree? *Ans.* 48 l. 12 s. $10\frac{1}{2}$ d.

4. If I pay in London 933 l. 18 s. 8 d. how many rees shall I receive at Lisbon, exchange at 5 s. 4 d. *per* millree? *Ans.* 3502.250 rees.

Portugal exchanges with other places as follows, *viz.* with

	Rees.
Spain, on the piaſtre	= 637
Venice, on the ducat	= 744
Genoa, on the pezzo	= 800
Naples, on the ducat	= 597
Sardinia, on the dollar	= 637
Palermo, on the crown	= 889

V. Exchange with Spain.

MONEY-TABLE.

	Par in Ster.	s.	d.
34 mervadies	} make	1 rial	= 0 $5\frac{3}{8}$
8 rials		1 piaſtre	= 3 7
375 mervadies		1 ducat	= 4 $11\frac{1}{4}$

In Madrid, Bilboa, Cadiz, Malaga, Seville, and most of the principal places, books and accounts are kept in piaſtres, called also *dollars*, rials, and mervadies; and they exchange with Britain generally on the piaſtre, and sometimes

sometimes on the ducat. The course runs from 35 d. to 45 d. Sterling for a piastre or dollar of 8 rials.

E X A M P L E S.

1. London imports from Cadiz, goods to the value of 2163 piastres and 4 rials: How much Sterling will this amount to, exchange at $38\frac{3}{8}$ d. Sterling per piastre?

Piastr. Rials.

d.		2163	4.	at $38\frac{3}{8}$ d.
24 =	$\frac{1}{10}$	216	6	
12 =	$\frac{1}{20}$	108	3	
2 =	$\frac{1}{100}$	18	0 6	
2 =	$\frac{1}{500}$	2	5 0 $\frac{3}{4}$	
1 =	$\frac{1}{1000}$	1	2 6 $\frac{3}{8}$	
		345	17 1 $\frac{1}{8}$	
			1 7 $\frac{3}{8}$	
		L. 345	18 8 $\frac{5}{8}$	Ans.

d.		$38\frac{3}{8}$ each.
Rials		
4 =		19 $\frac{3}{8}$

2. London remits to Cadiz 345 l. 18 s. $8\frac{5}{8}$ d. How much Spanish money will this amount to, exchange at $38\frac{3}{8}$ d. Sterling per piastre?

d. Piastr. L. s. d.

If $38\frac{3}{8} : 1 :: 345 \text{ l. } 18 \text{ s. } 8\frac{5}{8} \text{ d.}$

307	20
2	6918
614	12
	83024
	16
	498149
	83024
	1328389

614)1328389(2163 piastres.

1228
1003
614
3898
3684
2149
1842
307
8

614)2456(4 rials.

2456

MORE

MORE EXAMPLES.

3. Spain remits to Britain 1768 dollars, or piaſtres; 7 rials, at $40\frac{7}{8}$ d. Sterling *per* piaſtre: How much Sterling will this amount to? *Ans.* L. 301 : 5 : $2\frac{5}{8}$.

4. If I pay in Britain L. 1000 : 16 : 10, how much Spaniſh money may I draw for, exchange at $41\frac{1}{2}$ d. Sterling *per* piaſtre? *Ans.* 5788 piaſtres.

The ſilver money in Spain is of two ſorts, *viz.* old and new plate; the old plate is 25 *per cent.* better than the new; and ſo the piaſtre of exchange conſiſts of 8 rials old plate, or of 10 rials new plate.

The copper money in Spain is called *vellon*; and though the piaſtre vellon be of the ſame value with the piaſtre of exchange old plate, yet the rial vellon is little more than half a rial old plate; for 32 rials vellon are equal in value to 17 rials old plate, or 100 rials vellon are equal to $53\frac{1}{8}$ rials old plate.

Spain exchanges with other places as follows, *viz.* with

Mervadies.

Venice, on the ducat,	=	318
Genoa, on the pezzo,	=	$341\frac{5}{8}$
Rome, on the crown,	=	464
Florence, on the crown,	=	409
Naples, on the ducat,	=	255
Milan, on the ducat,	=	348
Bologna, on the dollar,	=	$322\frac{5}{8}$
Mefſina, on the crown,	=	380

VI. *Exchange with Venice.*

MONEY-TABLE.

$5\frac{1}{8}$ Soldi } make { 1 gros
24 Gros } { 1 ducat = $50\frac{1}{4}$ d. Sterling.

The money of Venice is of three ſorts, *viz.* two of bank money, and the picoli money. One of the banks

deals in banco money, and the other in banco current. The bank money is 20 *per cent.* better than the banco current, and the banco current 20 *per cent.* better than the picoli money. Exchanges are always negotiated by the ducat banco, the par being 4 s. 2½ d. Sterling, as in the table

Though the ducat be commonly divided into 24 gros, yet bankers and negotiators, for facility of computation, usually divide it as follows; and keep their books and accounts accordingly.

12 Deniers d'or } make { 1 sol d'or
20 Sols d'or } { 1 ducat = 50½ d. Sterling.

The course of exchange is from 45 d. to 55 d. Sterling *per ducat.*

EXAMPLES.

1. How much Sterling money is equal to 1459 ducats 18 sols 1 denier, bank money of Venice, exchange at 52¾ d. Sterling *per ducat*?

Duc. d.	Duc. sol. den.		d.
If 1 : 52¾ :: 1459 18 1			52¾ rate.
	52¾	Sols.	10 = ½ 26¾
	2918		5 = ½ 13¾
	7295		2 = ½ 6¾
	75868		1 = ½ 3¾
d. 75868		den. 1 = ½	0¾
¼ = 7294			47½
¼ = 3648			
	76962¾		
	47½ Rem.		
	12)77010(6d.		
	2)06417(17 s.		
	L. 320 17 6 Sterling. <i>Ans.</i>		

2. How

2 How many ducats at Venice are equal to 385 l. 12 s. 6 d. Sterling, exchange at 4 s. 4 d. per ducat?

L. Duc. L.

If .216 : 1 :: 385.625

.216)385.625

211385625

Duc.

195)347062.5(1779.8 Ans.

195

1529

1365

1556

1365

1912

1755

1575

1560

(15)

Bank money is reduced to current money, by allowing for the agio, as was done in exchange with Holland; viz. say, As 100 to 120, or as 10 to 12, or as 5 to 6, so the given bank money to the current sought. And current money is reduced to bank money by reverfing the operation. And in like manner may picoli money be reduced to current or to bank money, and the contrary.

100 ducats banco of Venice.

In Leghorn = 93 pezzos | In Lucca = 77 crowns
In Rome = 68½ crowns | In Francfort = 139½ florins

VII. Exchange with Genoa.

MONEY-TABLE.

12 Denari } make { 1 soldi s. d.
20 Soldi } { 1 pezzo = 4 6 Sterling.

L 2

Books

Books and accounts are generally kept in pezzos, soldi, and denari; but some keep them in lires, soldi, and denari; and 12 such denari make 1 soldi, and 20 soldi make 1 lire.

The pezzo of exchange is equal to $5\frac{3}{4}$ lires; and, consequently, exchange money is $5\frac{3}{4}$ times better than the lire money. The course of exchange runs from 47 d. to 58 d. Sterling *per pezzo*.

E X A M P L E.

How much Sterling money is equivalent to 3390 pezzos 16 soldi, of Genoa, exchange at $51\frac{7}{8}$ d. Sterling *per pezzo*?

Soldi.	d.	Pez. soldi.
If 20 :	$51\frac{7}{8}$::	3390 16
8	<u>415</u>	20
160		67816
		<u>415</u>
		339080
		67816
		<u>271264</u>

$$160)28143640(175897\frac{3}{4} = 732\ 18\ 1\frac{3}{4}$$

If Sterling money be given, it may be reduced or changed into pezzos of Genoa, by reversing the former operation.

Exchange money is reduced to lire money, by being multiplied by $5\frac{3}{4}$, as follows.

Pez soldi.	Decimally.
3390 16	3390.8
<u>5$\frac{3}{4}$</u>	<u>5.75</u>
16954 0	169540
$\frac{1}{2} =$ 1695 8	237356
$\frac{1}{4} =$ 847 14	<u>169540</u>
Lires 19497 2	Lires 19497.100

And

And lire money is reduced to exchange money by dividing it by 53.

Soldi of Genoa.

In Milan, 1 crown	=	80
In Naples, 1 ducat	=	86
In Leghorn, 1 piastra	=	20
In Sicily, 1 crown	=	127½

VIII. Exchange with Leghorn.

MONEY-TABLE.

12 Denari	} make	1 soldi	s. d.
20 Soldi		1 piastra	= 4 6 Ster.

Books and accounts are kept in piastres, soldi, and denari. The piastra here consists of 6 lres, and the lire contains 20 soldi, and the soldi 12 denari, and consequently exchange money is 6 times better than lire money. The course of exchange is from 47 d. to 58 d. Sterling per piastra.

EXAMPLE.

What is the Sterling value of 731 piastres, at 55½ d. each?

s.	d.		731 piastres, at 55½ d.
4	or 48	= ½	146 4
	6	= ⅙	18 5 6
	1½	= ¼	4 11 4½
			L. 169 0 10½ Ans.

Sterling money is reduced to money of Leghorn, by reversing the former operation; and exchange money is reduced to lire money by multiplying by 6, and lire money to exchange money by dividing by 6.

In

100 piastres of Leghorn are
In Naples = 134 ducats. | In Geneva = 185½ crowns.

Soldi of Leghorn.

In Sicily, 1 crown = 133½

In Sardinia, 1 dollar = 95½

The above are the chief places in Europe with which Britain exchanges directly; the exchanges with other places are generally made by bills on Hamburgh, Holland, or Venice. I shall here however subjoin the par of exchange betwixt Britain and most of the other places in Europe, with which we have any commercial intercourse.

Par in Sterling.			L. s. d.
Rome,	1	crown	= 6 13½
Naples,	1	ducat	= 3 4½
Florence,	1	crown	= 5 4½
Milan,	1	ducat	= 4 7
Bologna,	1	dollar	= 4 3
Sicily,	1	crown	= 5 0
Vienna,	1	rixdollar	= 4 8
Ausburgh,	1	florin	= 3 1½
Francfort,	1	florin	= 3 0
Bremen,	1	rixdollar	= 3 6
Breslau,	1	rixdollar	= 3 3
Berlin,	1	rixdollar	= 4 0
Stetin,	1	mark	= 1 6
Emden,	1	rixdollar	= 3 6
Bolsenna,	1	rixdollar	= 3 8
Dantzic,	13½	florins	= 1 0 0
Stockholm,	34½	dollars	= 1 0 0
Russia,	1	ruble	= 4 5
Turkey,	1	asper	= 4 6

The following places, viz. Switzerland, Noremburgh, Leipzig, Dresden, Osnaburgh, Brunswick, Cologne, Liege, Strasburgh, Cracow, Denmark, Norway, Riga, Revel, Narva, exchange with Britain, when direct exchange is made, upon the rixdollar, the par being 4 s. 6 d. Sterling.

IX. *Exchange with America and the West Indies.*

In North America and the West Indies, accounts, as in Britain, are kept in pounds, shillings, and pence. In North America they have few coins circulating among them, and on that account have been obliged to substitute a paper-currency for a medium of their commerce; which having no intrinsic value, is subjected to many disadvantages, and generally suffers a great discount. In the West Indies coins are more frequent, owing to their commercial intercourse with the Spanish settlements.

Exchange betwixt Britain and America, or the West Indies, may be computed as in the following examples.

1. The neat proceeds of a cargo from Britain to Boston amount to 845 l. 17 s. 6 d. currency: How much is that in Sterling money, exchange at 80 per cent.?

If 180 : 100

18 : 10 L. s. d.

9 : 5 :: 845 17 6

5

9) 4229 7 6

L. 469 18 7½ Ster. *Ans.*

2. Boston remits to Britain a bill of 469 l. 18 s. 7½ d. Sterling: How much currency was paid for the bill at Boston, exchange at 80 per cent.?

If 100 : 180 L. s. d.

5 : 9 :: 469 18 7½

9

5) 4229 7 6

845 17 6 currency. *Ans.*

3. Philadelphia is indebted to Britain in 985 l. 12 s. 3 d. currency: How much Sterling will that amount to, exchange at 75 per cent.?

If

If 175 : 100
 35 : 20 L. s. d.
 7 : 4 :: 985 12 3
 4
 7) 3942 9
 563 4 1 $\frac{5}{7}$ Sterling, *Ans.*

4. Philadelphia grants a bill on Britain for 563 l. 4 s. 1 $\frac{5}{7}$ d. Sterling: What is the value of the bill in currency, exchange at 75 per cent.?

If 100 : 175 L. s. d.
 4 : 7 :: 563 4 1 $\frac{5}{7}$
 7
 4) 3942 9
 985 12 3 currency, *Ans.*

By practice thus:

	L.	s.	d.
100 =	563	4	1 $\frac{5}{7}$
50 = $\frac{1}{2}$	281	12	0 $\frac{6}{7}$
25 = $\frac{1}{4}$	140	16	0 $\frac{3}{7}$
175	985	12	3 currency.

5. A parcel of goods from Britain, amounting, by invoice, to 323 l. 5 s. Sterling, are sold by a factor in Virginia for L. 469 : 2 : 6 currency: What Sterling ought the factor to remit, deducting 5 per cent. for commission? and how much does the employer gain per cent. when exchange is at 30 per cent.?

Exch.

Exch. com.

If $130 + 5 = 135 : 100 \text{ L.}$

$27 : 20 :: 469.125$

20

$3)9382.500$

$9)3127.5$

347.5 Ster. to be remitted.

323.25 value consigned.

24.25 gain.

If $323.25 : 24.25 :: 100$

100

$323.25)2425.00(7.5 \text{ or } 7\frac{1}{2} \text{ per cent.}$

226275

162250

161625

625

6. How much currency in Virginia will purchase a bill of 345 l. 13 s. 4 d. Sterling, when exchange is $33\frac{1}{3}$ per cent.?

If $100 : 133\frac{1}{3}$

$\frac{3}{300} : \frac{400}{400}$

300 : 400

L. s. d.

$3 : 4 :: 345 \text{ } 13 \text{ } 4$

4

$3)1382 \text{ } 13 \text{ } 4$

Ans. currency 460 17 $9\frac{1}{3}$

By practice.

L. s. d.

$100 =$

345 13 4

$33\frac{1}{3} = \frac{1}{3}$

115 4 $5\frac{1}{3}$

$133\frac{1}{3}$

460 17 $9\frac{1}{3} \text{ cur.}$

7. If an ounce of gold worth 4 l. Sterling be rated in Carolina at 9 l. 15 s. 7 d. currency, how much currency will 100 l. Sterling purchase?

M

L.

$$\begin{array}{r}
 \text{L. currency.} \quad \text{L.} \\
 \text{If } 4 : 9 \ 15 \ 7 :: 100 \\
 \quad 1 \quad \quad \quad 12 \quad \quad 25 = 12 + 12 + 1 \\
 \hline
 \quad 117 \ 7 \\
 \quad 117 \ 7 \\
 \quad \quad 9 \ 15 \ 7
 \end{array}$$

L. 244 9 7 currency. *Ans.*

8. How much Sterling money will 1780 l. Jamaica currency amount to, exchange at 40 per cent.?

$$\begin{array}{r}
 \text{If } 140 : 100 \\
 \quad 14 : 10 \quad \text{L.} \\
 \quad 7 : 5 :: 1780 \\
 \quad \quad \quad 5 \\
 \quad \quad \quad \hline
 \quad \quad 7) 8900 \quad \text{s. d.} \\
 \quad \quad \quad \hline
 \quad \quad 1271 \ 8 \ 6\frac{3}{4} \text{ Ster. } \textit{Ans.}
 \end{array}$$

Bills of exchange from America, the rate being high, is an expensive way of remitting money to Britain; and therefore merchants in Britain generally chuse to have the debts due to them remitted home in sugar, rum, or other produce.

X. Exchange with Ireland.

At Dublin, and all over Ireland, books and accounts are kept in pounds, shillings, and pence, as in Britain; and they exchange on the 100 l. Sterling.

The par of one shilling Sterling is one shilling and one penny Irish; and so the par of 100 l. Sterling is 108 l. 6 s. 8 d. Irish. The course of exchange runs from 6 to 15 per cent.

E X A M P L E S.

1. London remits to Dublin 586 l. 10 s. Sterling: How much Irish money will that amount to, exchange at $9\frac{1}{2}$ per cent.?

If

$$\begin{array}{r}
 \text{L.} \\
 \text{If } 100 : 109\frac{5}{8} :: 586.5 \\
 \underline{8} \qquad \qquad \underline{877} \qquad \underline{41055} \\
 800 : 877 \qquad 41055 \\
 \qquad \qquad \underline{46920} \\
 800) 514360.5 \\
 \underline{642.950625} \\
 \text{Ans. } 642 \text{ l. } 19 \text{ s. } 11\frac{1}{2} \text{ d.}
 \end{array}$$

By practice:

p. cent.	586.5
10 = $\frac{1}{10}$	58.65
2 = $\frac{1}{5}$	11.73 sub.
8 =	46.92
1 = $\frac{1}{8}$	5.865
$\frac{4}{8}$ = $\frac{1}{2}$	2.9325
$\frac{1}{8}$ = $\frac{1}{4}$	.733125
9 $\frac{5}{8}$	56.450625 add.
	642.950625

2. How much Sterling will 625 l. Irish amount to, exchange at 10 $\frac{3}{8}$ per cent.?

$$\begin{array}{r}
 \text{If } 110\frac{3}{8} : 100 :: 625 \\
 \underline{8} \qquad \qquad \underline{800} \text{ L. s. d.} \\
 883 \quad 800 \quad 883) 500000 (566 \text{ } 5 \text{ } 0\frac{1}{4} \text{ Ster. } \text{Ans.}
 \end{array}$$

3. Britain remitted to Ireland 10000 l. Sterling when exchange was at 12 per cent. but, exchange falling to 8 per cent. Britain draws back the whole remittance: How much Sterling was drawn back, and how much did Britain gain?

M 2

As

As 108 : 112 :: 10000

$$\begin{array}{r} 10000 \\ \hline 108 \text{) } 1120000 \text{ (} 10370 \text{ } 7 \text{ } 4\frac{3}{4} \text{ Ster. drawn back.} \\ \hline 10000 \end{array}$$

370 7 4 $\frac{3}{4}$ gained.

Which is more than 3 $\frac{1}{2}$ per cent.

4. When exchange was up at 12 per cent. Dublin drew on London for 10000 l. Sterling; and when exchange fell to 8 per cent. Dublin remitted 10000 l. Sterling to London: How much Irish money did Dublin gain?

$$\begin{array}{r} \text{Irish.} \\ 112 \\ \hline 108 \\ \hline \text{Ster.} \end{array} \quad \begin{array}{r} \text{Ster.} \\ 10000 \\ \hline 4 \end{array}$$

If 100 : 4 :: 10000

10000
100) 40000 (400 l. Irish gained.

XI. Exchange betwixt London and other places in Britain.

The several towns in Britain exchange with London for a small premium in favour of London; such as, 1, 1 $\frac{1}{4}$, &c. per cent. The premium is more or less according to the demand for bills.

E X A M P L E.

Edinburgh draws on London for 860 l. exchange at 1 $\frac{3}{8}$ per cent.: How much money must be paid at Edinburgh for the bill?

L.

	L.	
	860	
per cent.		
1 = $\frac{1}{100}$	8 12	
$\frac{2}{8}$ = $\frac{1}{4}$	2 3	
$\frac{1}{8}$ = $\frac{1}{2}$	1 1 6	
	11 16 6 premium.	
	871 16 6 paid for the bill.	

To avoid paying the premium, it is an usual practice to take the bill payable at London a certain number of days after date; and in this way of doing, 73 days is equivalent to 1 per cent.

XII. Arbitration of Exchanges.

The course of exchange betwixt nation and nation naturally rises or falls according as the circumstances and balance of trade happen to vary. Now to draw upon, and remit to foreign places, in this fluctuating state of exchange, in the way that will turn out most profitable, is the design of arbitration. Which is either simple or compound.

I. Simple arbitration.

In simple arbitration the rates or prices of exchange from one place to other two are given; whereby is found the correspondent price between the said two places, called the *arbitrated price*, or *par of arbitration*; and hence is derived a method of drawing and remitting to the best advantage.

EXAMPLES.

1. If exchange from London to Amsterdam be 33 s. 9 d. *per* l. Sterling; and if exchange from London to Paris be 32 d. *per* crown; what must be the rate of exchange from Amsterdam to Paris, in order to be on a par with the other two?

Ster.

Ster.	Flem.	Ster.
s.	s. d.	d.
If 20 :	33 9 ::	32
<u>12</u>	<u>12</u>	
240	405	
	<u>32</u>	
	810	
	<u>1215</u>	

240)12960(54 d. Flem. *per* crown. *Ans.*

2. If exchange from Paris to London be 32 d. Sterling *per* crown; and if exchange from Paris to Amsterdam be 54 d. Flemish *per* crown; what must be the rate of exchange between London and Amsterdam, in order to be on a par with the other two?

Ster.	Flem.	Ster.
d.	d.	d.
If 32 :	54 ::	240
	<u>240</u>	
	216	

32)12960(405 (33 9 Flem. *per* l. Ster. *Ans.*

3. If exchange from Amsterdam to Paris be 54 d. Flem. *per* crown; and if exchange from Amsterdam to London be 33 s. 9 d. Flem. *per* l. Sterling; what must be the rate of exchange between Paris and London, in order to be on a par with the other two?

Flem.	Ster.	Flem.
s. d.	d.	d.
If 33 9 :	240 ::	54
<u>12</u>	<u>54</u>	
405	96	
	<u>120</u>	

405)12960(32 d. Ster. *per* crown. *Ans.*

From

From these three operations it appears, that if any sum of money be remitted, at the rates of exchange mentioned, from any one of the three places to the second, and from the second to the third, and again from the third to the first, the sum so remitted will come home entire, without increase or diminution.

From the par of arbitration thus found, and the course of exchange given, is deduced a method of drawing and remitting to advantage, as in the following examples.

4. If exchange from London to Paris be 32 d. Sterling *per* crown, and to Amsterdam 405 d. Flemish *per* l. Sterling; and if, by advice from Holland or France, the course of exchange between Paris and Amsterdam is fallen to 52 d. Flemish *per* crown; what may be gained *per cent.* by drawing on Paris, and remitting to Amsterdam?

The par of arbitration between Paris and Amsterdam in this case, by Ex. 1. is 54 d. Flemish *per* crown. Work as under.

d. St. Cr. L. St. Cr.

If 32 : 1 :: 100 : 750 debit at Paris.

Cr. d. Fl. Cr. d. Fl.

If 1 : 52 :: 750 : 39000 credit at Amsterdam.

d. Fl. L. St. d. Fl. L. s. d. Ster.

If 405 : 1 :: 39000 : 96 5 11 $\frac{1}{9}$ to be remitted.

100

3 14 0 $\frac{8}{9}$ gained *per cent.*

But if the course of exchange between Paris and Amsterdam, instead of falling below, rise above the par of arbitration, suppose to 56 d. Flemish *per* crown; in this case, if you propose to gain by the negotiation, you must draw on Amsterdam, and remit to Paris. The computation follows.

L.

L. St. d. Fl. L. St. d. Fl.

If 1 : 405 :: 100 : 40500 debit at Amsterdam.

d. Fl. Cr. d. Fl. Cr.

If 56 : 1 :: 40500 : $723\frac{3}{4}$ credit at Paris.

Cr. d. St. Cr. L. s. d. Ster.

If 1 : 32 :: $723\frac{3}{4}$: 96 8 $\frac{6}{7}$ to be remitted.

100

3 11 $5\frac{1}{7}$ gained per cent.

In negotiations of this sort, a fund for remittance is afforded out of the sum you receive for the draught; and your credit at the one foreign place pays your debit at the other.

5. London is indebted to Petersburg 5000 rubles, and Petersburg can draw for them directly on England at 50 d. Sterling *per* ruble, or on Holland at 90 d. Flemish *per* ruble: Which of these two ways will be most advantageous to London, supposing the course of exchange between London and Holland to be 36 s. 4 d. Flemish *per* l. Sterling?

Find the par of arbitration, by saying,

d. St. d. Fl. d. St. d. Fl.

If 50 : 90 :: 240 : 432 or 36 s. Flemish.

If the course of exchange between London and Holland had been the same with the par of arbitration, *viz.* 36 s. payment in either way would be the same to London; but since the course is 36 s. 4 d. it is plainly an advantage to make payment by the way of Holland; for it is better to receive 36 s. 4 d. than 36 s. for a pound Sterling. The computation follows.

Rub. d. St. Rub. L. s. d. Ster.

If 1 : 50 :: 5000 : 1041 13 4 Lond. to Petersburg.

Rub. d. Fl. Rub. d. Fl.

If 1 : 90 :: 5000 : 450000 Holland to Petersburg.

s. d. F. L. St. d. Fl. L. s. d. Ster.

If 36 4 : 1 :: 450000 : 1032 2 $2\frac{1}{4}$ Lond. to Holland.

9 11 $1\frac{3}{4}$ gain.

If

If the course of exchange to Holland had fallen below 36 s. the direct exchange would have been preferable.

The following, or like questions, are solved, without finding the par of arbitration.

6. London was ordered to remit 500 ducats to Venice, at 50 d. Sterling *per* ducat, and to draw for the value upon Spain, at 40 d. Sterling *per* piaſtre; but when the order came to hand, bills on Venice were at $52\frac{1}{2}$ d.: At what rate of exchange must London draw upon Spain, to compensate the advance upon the remittance?

$$\begin{array}{ccc} d. & d. & d. \\ \text{If } 50 & : & 52\frac{1}{2} :: 40 \end{array}$$

$$\text{Or, If } 100 : 105 :: 40$$

$$\begin{array}{r} 40 \\ \hline 100)4200(42 \text{ d. } \textit{Ans.} \end{array}$$

Proof,

$$\begin{array}{ccc} \textit{Duc. d. St.} & \textit{Duc.} & \textit{d. St.} \\ \text{If } 1 & : & 50 :: 500 : 25000 \end{array}$$

$$\begin{array}{ccc} d. \textit{ Piaſt.} & d. & \textit{Piaſt.} \\ \text{If } 40 & : & 1 :: 25000 : 625 \end{array}$$

Then, by the course,

$$\begin{array}{ccc} \textit{Duc. d.} & \textit{Duc.} & d. \\ \text{If } 1 & : & 52\frac{1}{2} :: 500 : 26250 \end{array}$$

$$\begin{array}{ccc} d. \textit{ Piaſt.} & d. & \textit{Piaſt.} \\ \text{If } 42 & : & 1 :: 26250 : 625 \text{ as before.} \end{array}$$

If the course of exchange with Spain be lower than 42 d. the order cannot be executed without loss; and if it be higher, there will be gain.

7. London was ordered to remit to Paris 800 crowns, at $31\frac{3}{8}$ d. Sterling *per* crown, and to draw for the

the value upon Amsterdam, at 36 s. 9 d. Flem. *per* pound Ster. ; but when the order came up, bills on Paris were at $31\frac{5}{8}$ d. Sterling *per* crown : What must be the rate of exchange with Amsterdam to compensate the advance on the remittance ?

$$\begin{array}{cccc} d. & s. & d. & s. & d. \\ \text{If } 31\frac{5}{8} & : & 36 & 9 & :: 31\frac{3}{8} & : & 36 & 5\frac{1}{4} \text{ nearly } \textit{Ans.} \end{array}$$

The proportion is inverse ; for the rate of exchange with Holland must be lessened to answer our purpose ; which may be proved as in the former example. The proportions for the remittance and draught follow.

$$\begin{array}{ccc} \textit{Gr.} & d. \textit{St.} & \textit{Gr.} & d. \textit{Ster.} \\ \text{If } 1 & : & 31\frac{5}{8} & :: 800 & : & 25300 \end{array}$$

$$\begin{array}{cccc} d. \textit{St.} & s. & d. \textit{Fl.} & d. \textit{St.} & d. \textit{Fl.} \\ \text{If } 240 & : & 36 & 5\frac{1}{2} & :: 25300 & : & 46119\frac{1}{2} \text{ nearly.} \end{array}$$

8. A merchant in London had 6000 guilders in the bank at Amsterdam, and was offered 22 d. Sterling for each guilder ; but as this offer did not please, he indorses a bill for the whole to his factor at Paris ; who soon brought the money to France, by exchanging at 55 d. Flemish *per* crown : he allows the factor $\frac{1}{2}$ *per cent.* commission for his trouble, and then draws upon him for the whole, exchange at 32 d. Sterling *per* crown : How much was this better than the offer of 22 d *per* guilder ?

$$\begin{array}{rcl} & 6000 \text{ guilders.} & \\ & 40 \text{ d. Flem. in 1 guilder.} & \\ d. \textit{Fl.} & \textit{Gr.} & \\ \text{If } 55 & : & 1 & :: 240000 & : & 4363.63, \text{ crowns.} \\ & & & 21.81, \text{ com.} = \frac{1}{20} & \\ & & & \hline & & & 4341.81, \text{ neat.} & \end{array}$$

Gr.

$$\begin{array}{rcl}
 \text{Gr. } d. & \text{Gr.} & \\
 \text{If } 1 : 32 :: 4341.81, & & \\
 & \underline{32} & \\
 & 8683.63, & \\
 & \underline{130254.54}, & \text{L. } s. \text{ } d. \text{ } \text{Ster.} \\
 138938.18, = & 578 \text{ } 18 \text{ } 2 & \text{by exch.} \\
 & \underline{550 \text{ } 0 \text{ } 0} & \text{by offer.} \\
 \text{Guild.} & & \\
 6000, \text{ at } 22 \text{ d. St. } & 28 \text{ } 18 \text{ } 2 & \text{gained.} \\
 d. & & \\
 20 = \frac{1}{12} & 500 & \\
 2 = \frac{1}{10} & 50 & \\
 \hline
 & 550 \text{ l. Ster.} &
 \end{array}$$

II. Compound arbitration.

In compound arbitration the rate or price of exchange between three, four, or more places, is given, in order to find how much a remittance passing through them all will amount to at the last place; or to find the arbitrated price, or par of arbitration, between the first place and the last. And this may be done by the following

R U L E S.

I. Distinguish the given rates or prices into antecedents and consequents; place the antecedents in one column, and the consequents in another on the right, fronting one another by way of equation.

II. The first antecedent, and the last consequent to which an antecedent is required, must always be of the same kind.

III. The second antecedent must be of the same kind with the first consequent, and the third antecedent of the same kind with the second consequent, &c.

IV. If to any of the numbers a fraction be annexed, both the antecedent and its consequent must be multiplied into the denominator.

V. To facilitate the operation, terms that happen to

be equal or the same in both columns, may be dropped or rejected, and other terms may be abridged.

VI. Multiply the antecedents continually for a divisor, and the consequents continually for a dividend, and the quot will be the answer, or antecedent required.

E X A M P L E S.

1. If London remit 1000 l. Sterling to Spain, by way of Holland, at 35 s. Flemish *per* l. Sterling; thence to France, at 58 d. Flemish *per* crown; thence to Venice, at 100 crowns *per* 60 ducats; and thence to Spain, at 360 mervadies *per* ducat; how many piaftres, of 272 mervadies, will the 1000 l. Sterling amount to in Spain?

<i>Antecedents.</i>		<i>Consequents.</i>	<i>Abridged.</i>
1 l. Sterling	=	35 s. or 420 d. Fl.	1 = 210
58 d. Flemish	=	1 crown France	29 = 1
100 crowns France	=	60 ducats Venice	1 = 30
1 ducat Venice	=	360 mervadies Spain	1 = 45
272 mervadies	=	1 piaftre	17 = 1
How many piaftres	=	1000 l. Sterling?	= 10

In order to abridge the terms, divide 58 and 420 by 2, and you have the new antecedent 29, and the new consequent 210; reject two ciphers in 100 and 1000; divide 272 and 360 by 8, and you have 34 and 45; divide 34 and 60 by 2, and you have 17 and 30; and the whole will stand abridged as above.

Then, $29 \times 17 = 493$ divisor; and, $210 \times 30 \times 45 \times 10 = 2835000$ dividend; and, $493 \overline{)2835000} (5750\frac{1}{2}$ piaftres. *Anf.*

Or, the consequents may be connected with the sign of multiplication, and placed over a line by way of numerator, and the antecedents, connected in the same manner, may be placed under the line, by way of denominator, and then abridged, as follows.

$$\frac{410 \times 60 \times 360 \times 1000}{58 \times 100 \times 272} = \frac{210 \times 60 \times 360 \times 10}{29 \times 1 \times 272} = \frac{210 \times 60 \times 45 \times 10}{29 \times 34}$$

$$= \frac{210 \times 30 \times 45 \times 10}{29 \times 17} = \frac{2835000}{493},$$

And, $493)2835000(5750\frac{1}{2}$ piaftres. *Ans.*

The placing the terms by way of antecedent and consequent, and working as the rules direct, faves so many statings of the rule of three, and greatly shortens the operation. The proportions at large for the above question would stand as under.

<i>L. St.</i>	<i>d. Fl.</i>		<i>L. St.</i>		<i>d. Fl.</i>
If 1	:	420 ::	1000	:	420000

<i>d. Fl.</i>	<i>Gr.</i>		<i>d. Fl.</i>		<i>Gr.</i>
If 58	:	1 ::	420000	:	$7241\frac{11}{29}$

<i>Gr.</i>	<i>Duc.</i>		<i>Gr.</i>		<i>Duc.</i>
If 100	:	60 ::	$7241\frac{11}{29}$	:	$4344\frac{24}{29}$

<i>Duc.</i>	<i>Mer.</i>		<i>Duc.</i>		<i>Mer.</i>
If 1	:	360 ::	$4344\frac{24}{29}$	:	$1564137\frac{27}{29}$

<i>Mer.</i>	<i>Piaft.</i>		<i>Mer.</i>		<i>Piaft.</i>
If 272	:	1 ::	$1564137\frac{27}{29}$	:	$5750\frac{250}{493}$

If we suppose the course of direct exchange to Spain to be $42\frac{1}{2}$ d. Sterling *per* piaftre, the 1000 l. remitted would only amount to $5647\frac{1}{2}$ piaftres; and, consequently, 103 piaftres are gained by the negotiation; that is, about 2 *per cent*.

2. A banker in Amsterdam remits to London 400 l. Flemish; first to France at 56 d. Flemish *per* crown; from France to Venice at 100 crowns *per* 60 ducats; from Venice to Hamburgh at 100 d. Flemish *per* ducat; from Hamburgh to Lisbon at 50 d. Flemish *per* crusade of 400 rees; and, lastly, from Lisbon to London at 64 d. Sterling *per* millree: How much Sterling money will the remittance amount to? and how much will be gained or saved, supposing the direct exchange from Holland

Holland to London at 36 s. 10 d. Flemish *per* l. Sterling?

Antecedents. Consequents.

56 d. Flem. = 1 crown.

100 crowns = 60 ducats.

1 ducat = 100 d. Flem.

50 d. Flem. = 400 rees.

1000 rees = 64 d. Sterling.

How many d. Ster. = 400 l. or 96000 d. Flemish?

This, in the fractional form, will stand as follows.

$$\frac{60 \times 100 \times 400 \times 64 \times 96000}{56 \times 100 \times 50 \times 1000} = \frac{368640}{7}, \text{ and}$$

7)368640(52662 $\frac{6}{7}$ d. Ster. = 219 l. 8 s. 6 $\frac{6}{7}$ d. St. *Ans.*

To find how much the exchange from Amsterdam directly to London, at 36 s. 10 d. Flemish *per* l. Sterling, will amount to, say,

s.	d.	d. Fl.	L. St.	d Fl.	L.	s.	d. St.
36	10	lf 442	: 1 :: 96000	: 217	3	10 $\frac{1}{2}$	
12				219	8	6 $\frac{3}{4}$	
442		Gained or saved,		2	4	8 $\frac{1}{4}$	

In the above example, the par of arbitration, or the arbitrated price, between London and Amsterdam, *viz.* the number of Flemish pence given for 1 l. Sterling, may be found thus,

Make 64 d. Sterling, the price of the millree, the first antecedent; then all the former consequents will become antecedents, and all the antecedents will become consequents. Place 240, the pence in 1 l. Sterling, as the last consequent, and then proceed as taught above, *viz.*

Antecedents.

Antecedents. Consequents.

64 d. Ster. = 1000 rees.

400 rees = 50 d. Flem.

100 d. Flem. = 1 ducat.

60 ducats = 100 crowns.

1 crown = 56 d. Flem.

How many d. Flem. = 240 d. Ster.?

$$\frac{1000 \times 50 \times 100 \times 56 \times 240}{64 \times 400 \times 100 \times 60} = \frac{875}{2}, \text{ and}$$

2)875(437½ d. = 36 s. 5½ d. Flem. *per* l. Ster. *Ans.*

Or the arbitrated price may be found from the answer to the question, by saying,

d. Ster. d. Flem. d. St.

If $\frac{368640}{7} : 96000 :: 240$

$$\begin{array}{r} 7 \\ \hline 672000 \\ 240 \\ \hline 2688 \end{array}$$

1344

d. s. d. Flem.

368640)161280000(437½ = 36 5½ as before.†

The work may be proved by the arbitrated price thus: As 1 l. Sterling to 36s. 5½ d. Flemish, so 219 l. 8 s. 6½ d. Sterling to 400 l. Flemish.

The arbitrated price compared with the direct course shows whether the direct or circular remittance will be most advantageous, and how much. Thus the banker at Amsterdam will think it better exchange to receive 1 l. Sterling for 36 s. 5½ d. Flemish, than for 36 s. 10 d. Flemish.

3. A merchant at London has credit for 680 piaſtres at Leghorn, for which he can draw directly at 50 d. Sterling *per* piaſtre; but chuſing to try the circular way, they are by his order remitted, firſt to Venice at 94 piaſtres *per* 100 ducats Banco; thence to Cadiz at 320 mervadies *per* ducat; thence to Liſbon at 630 rees *per* piaſtre

piastre of 272 mervadies; thence to Amsterdam at 50 d. Flemish *per* crusade of 400 rees; thence to Paris at 56 d. Flemish *per* crown; thence to London at $31\frac{1}{3}$ d. Sterling *per* crown: What is the arbitrated price between London and Leghorn *per* piastre? and how much is the circular remittance better than the direct draught, without reckoning charges?

Antecedents. Consequents.

94 piales = 100 ducats.

1 ducat = 320 mervadies,

272 mervad. = 630 rees.

400 rees = 50 d. Flem.

56 d. Flem. = 1 crown.

3 crowns = 94 d. St. = $3 \times 31\frac{1}{3}$

How many d. Ster. = 1 piastre?

$$\frac{100 \times 320 \times 630 \times 50 \times 94}{94 \times 272 \times 400 \times 56 \times 3} = \frac{1875}{34}, \text{ and}$$

34)1875($55\frac{5}{34}$ d. Sterling *per* piastre. *Ans.*

	L.	s.	d.
680 piales, at $55\frac{5}{34}$ d. =	156	5	0
at 50 d. =	141	13	4

Gained 14 11 8

If the arbitrated price of any intermediate antecedent be required, put the question at the void place, fill up the place of the first question with its answer; or, to avoid the fraction, make 680 piales the last consequent, and their value in pence Sterling the last antecedent, and then work as before.

Ex. How many rees are equal in value to 50 d. Flemish?

Antecedents.

<i>Antecedents.</i>	<i>Consequents.</i>
94 piastres	= 100 ducats.
1 ducat	= 320 mervadies.
272 mervad.	= 630 rees.
How many rees	= 50 d. Flem.
56 d. Flem.	= 1 crown.
3 crowns	= 94 d. Sterling.
37500 d. Ster.	= 680 piastres.

$$\frac{100 \times 320 \times 630 \times 50 \times 94 \times 680}{94 \times 272 \times 56 \times 3 \times 37500} = \frac{5 \times 2 \times 10 \times 4}{1} = 400$$

rees. *Ans.*

If the arbitrated price of a consequent be required, at the void place put the question, and work as before; only in this case the product of the antecedents is the dividend, and the product of the consequents the divisor.

Ex. How many rees are equal in value to a piastre of 272 mervadies?

<i>Antecedents.</i>	<i>Consequents.</i>
94 piastres	= 100 ducats.
1 ducat	= 320 mervadies.
272 mervad.	= how many rees?
400 rees	= 50 d. Flemish.
56 d. Flem.	= 1 crown.
3 crowns	= 94 d. Sterling.
37500 d. Ster.	= 680 piastres.

$$\frac{94 \times 272 \times 400 \times 56 \times 3 \times 37500}{100 \times 320 \times 50 \times 94 \times 680} = \frac{14 \times 3 \times 15}{1} = 630$$

rees. *Ans.*

Note. The value of an antecedent is always of the same denomination with the preceding consequent; and the value of a consequent is of the same denomination with the subsequent antecedent.

In this circular exchange, the agents or factors at the different places through which the money passes, retain so much in name of commission; and the regular accurate

curate method of computation is, to deduce the commission from the several consequents.

Thus, if we resume the former question, and suppose the commission to be $\frac{1}{2}$ per cent. the antecedents and consequents with commission deduced will stand as under.

<i>Antecedents.</i>	<i>Consequents.</i>
94 piaſtres =	99.5 ducats, com. deduced.
1 ducat =	318.4 mervadies, com. ded.
272 mervad. =	626.85 rees, com. ded.
400 rees =	49.75 d. Flem. com. ded.
56 d. Flem =	.995 crowns, com. ded.
3 crowns =	94 d. Ster.
How many d. Ster. =	1 piaſtre?

By working as formerly, the arbitrated price, or value of the piaſtre, will be found to be 53.78 d. Sterling nearly, which is ſomething leſs than the answer found, excluſive of charges. But ſtill there will be profit by the circular remittance: for

	<i>L.</i>	<i>s.</i>	<i>d.</i>
680 piaſtres at 53.78 d. Ster. =	152	7	6
———— at 50 d. Ster. =	141	13	4
	Gain	10	14 2

The value of the above piaſtre, with commission allowed, may be found eaſily, and pretty nearly, by deducting five commissions, equal to $2\frac{1}{2}$ per cent. from the arbitrated price, excluſive of charges, thus:

$$\begin{array}{rcl}
 55\frac{5}{34} & = & 55.147 \\
 2\frac{1}{2} \text{ per cent.} & = & \frac{1}{40} \mid 1.378 \\
 \hline
 & & 53.769 \text{ value nearly.}
 \end{array}$$

Thus a perſon who knows at what rate he can draw or remit directly, and at the ſame time has advice of the courſe of exchange in foreign places, may chalk out a path for circulating his money, ſo as to make a benefit of

of his skill and credit: and herein lies the great art of such negotiations.

Not only may different sorts of money be equated in the manner above described, but also weights and measures.

EXAMPLE I.

If 102 lb. of Hamburgh be equal to 100 lb. of Amsterdam, and 100 of Amsterdam to 98 at Francfort, and 98 at Francfort to 105 at Leipsic, and 105 of Leipsic to 145 at Leghorn, and 145 at Leghorn to 106 at Cadiz, and 100 at Cadiz to $103\frac{1}{3}$ at London; how many lbs. at London are equal to 3060 at Hamburgh?

Antecedents.

Consequents.

102 at Hamburgh = 100 at Amsterdam.

100 at Amsterdam = 98 at Francfort.

98 at Francfort = 105 at Leipsic.

105 at Leipsic = 145 at Leghorn.

145 at Leghorn = 106 at Cadiz.

300 at Cadiz = 310 at London.

How many at London = 3060 at Hamburgh?

$$\frac{100 \times 98 \times 105 \times 145 \times 106 \times 310 \times 3060}{102 \times 100 \times 98 \times 105 \times 145 \times 300} = \frac{106 \times 310 \times 3060}{102 \times 300} =$$

$\frac{55862}{17}$, and 17)55862(3286 lb. *Ans.*

EXAMPLE II.

If $1\frac{1}{2}$ ells, or aunes, of Hamburgh, make 1 ell in Holland, and 7 in Holland make 4 in France, and 7 in France make 5 yards in England; how many yards in England are equal to 588 ells or aunes at Hamburgh? and what their price, at the rate of 4 l. Sterling for 5 yards English?

Antecedents.

Consequents.

$1\frac{1}{2} \times 5 = 6$ of Hamburgh = 5 of Holland = 1×5

7 of Holland = 4 of France.

7 of France = 5 of England.

How many yards of England = 588 of Hamburgh?

$$\frac{5 \times 4 \times 5 \times 588}{6 \times 7 \times 7} = \frac{5 \times 4 \times 5 \times 2}{1} = 200 \text{ yards. } \textit{Ans.}$$

O 2

Then

Then for the price:

<i>Antecedents.</i>	<i>Consequents.</i>
6 of Hamburg	= 5 of Holland.
7 of Holland	= 4 of France.
7 of France	= 5 of England.
5 of England	= 4 l. Sterling.
How much Sterling	= 588 of Hamburg?

$$\frac{5 \times 4 \times 5 \times 4 \times 588}{6 \times 7 \times 7 \times 5} = \frac{4 \times 5 \times 4 \times 2}{1} = 160 \text{ l. Ster. } \textit{Ans.}$$

E X A M P L E III.

<i>Antecedents.</i>	<i>Consequents.</i>
If 82 bushels at London	= 27 muddles at Amster.
27 muddles at Amster.	= 19 setiers at Paris,
38 setiers at Paris	= 65 veertels at Antwerp,
65 veertels at Antwerp	= 104 hanegas at Cadiz,
52 hanegas at Cadiz	= 216 alquiers at Lisbon,
How many alquiers at Lisb.	= 410 bushels at London?

$$\frac{27 \times 19 \times 65 \times 104 \times 216 \times 410}{82 \times 27 \times 38 \times 65 \times 52} = 108 \times 10 = 1080 \text{ al-} \\ \text{quiers at Lisbon. } \textit{Ans.}$$

Though foreign weights and measures may be equated with those of Britain by arbitration, as in the above examples; yet this is done more easily and readily by the rule of three, from the following tables, which exhibit the proportion or conformity that the weights, long measures, and corn-measures, of the principal places in Europe, have with those of London and Amsterdam.

TABLE

TABLE I. OF WEIGHTS.

100 lb. of Eng- land are e- qual to		100 lb. of Am- sterdam are equal to		At
lb.	oz.	lb.	oz.	
100	0	109	8	London
91	8	100	0	Amsterdam
96	8	105	8	Antwerp
88	0	96	4	Rouen
106	0	116	0	Lyons
90	9	99	0	Rochelle
107	11	118	0	Toulouse
113	0	123	8	Marseilles
81	7	89	0	Geneva
93	5	102	0	Hamburgh
89	7	98	0	Francfort
96	1	105	0	Leipsick
137	4	150	0	Genoa
132	11	145	0	Leghorn
153	11	168	0	Milan
152	0	166	0	Venice
154	10	169	0	Naples
97	0	106	0	Seville
97	0	106	0	Cadiz
104	13	114	8	Portugal
96	5	105	4	Liege
112	0 $\frac{2}{3}$	123	0 $\frac{1}{12}$	Russia
107	0 $\frac{1}{24}$	117	0	Sweden
89	0 $\frac{1}{2}$	97	13	Denmark

As the weights of Amsterdam, Paris, Bourdeaux, Strasburgh, Besançon, and several other places, have but a very minute difference, they are all comprehend-
ed under that of Amsterdam, as that of Nuremberg is
under Francfort, and others in like manner.

TABLE

TABLE II. OF LONG MEASURE.

100 ells or aunes, or 125 yards of England, are e- qual to	100 aunes of Amster- dam are e- qual to	At
<i>Aunes.</i>	<i>Aunes.</i>	
100	60	London
166 $\frac{2}{3}$	100	Amsterdam
162 $\frac{4}{7}$	98 $\frac{3}{4}$	Brabant
97 $\frac{1}{12}$	58 $\frac{1}{2}$	France
200 $\frac{1}{24}$	120	Hamburgh
208 $\frac{1}{3}$	125	Breslau in Silesia
187 $\frac{1}{2}$	112 $\frac{1}{2}$	Dantzick
183 $\frac{1}{4}$	110	Bergue and Drontheim
194 $\frac{1}{2}$	116 $\frac{2}{3}$	Sweden
143 $\frac{1}{7}$	86	St Gall for linen
186 $\frac{2}{3}$	112	St Gall for cloth
100	60	Geneva
164 $\frac{1}{2}$	98 $\frac{3}{4}$	Brussels, Antwerp
<i>Canes.</i>	<i>Canes.</i>	
58 $\frac{1}{4}$	35	Marseilles
62 $\frac{3}{8}$	37 $\frac{1}{2}$	Toulouse
50 $\frac{1}{8}$	30 $\frac{1}{2}$	Genoa
55 $\frac{1}{3}$	33	Rome
<i>Varas.</i>	<i>Varas.</i>	
133 $\frac{1}{3}$	80	Spain
101 $\frac{2}{3}$	61	Portugal
<i>Cavidos.</i>	<i>Cavidos.</i>	
166 $\frac{2}{3}$	100	Portugal
<i>Brasses.</i>	<i>Brasses.</i>	
170 $\frac{1}{3}$	102	Venice
174 $\frac{1}{2}$	105 $\frac{1}{4}$	Bergamo
194 $\frac{1}{7}$	117	Florence, Leghorn, Lucca
214 $\frac{1}{6}$	128 $\frac{1}{4}$	Milan
<i>Arsheens.</i>	<i>Arsheens.</i>	
178 $\frac{4}{7}$	297 $\frac{1}{2}$	Petersburgh

The aunes or ells of Haerlem, Leyden, the Hague, Rotterdam, and other places of Holland, also that of Nuremberg, are all equal to that of Amsterdam, and comprehended under it; the aune of Osnaburg is equal to that of France; and the aunes of Bern, Basil, Francfort, and Leipfick, are equal to that of Hamburgh.

TABLE III. OF CORN-MEASURES.

$10\frac{1}{4}$ quarters, or 82 Winchester bushels of England, or 1 last of Amsterdam, make, at

Aiguillon, 41 sacks	Coningsberg, 1 last
Albi, 25 setiers	Copenhagen, 42 tuns
Alicant, 12 cahizes	Dantzick, 1 last
Alkmaar, 36 sacks	Delft, 29 sacks
Amersfort, 16 muddes	Deventer, 36 muddes
Amsterdam, 27 muddles, or 1 last	Doesbourg, 22 mouwers
Antwerp, $32\frac{1}{2}$ veertels	Dort, 24 sacks
Arles, 49 setiers	Dunkirk, 18 razieres
Bayonne, 36 sacks	Edam, 27 muddes
Beaucaire, 28 setiers	Elbing, 1 last
Beaumont, 38 sacks	Emden, $15\frac{1}{4}$ tuns
Bergen-op-zoom, 63 sisters	Erfelsteyn, 21 muddes
Bois-le-Duc, $20\frac{1}{2}$ mouwers	Francfort, 27 malders
Bommel, 18 muddes	Ghent, 56 halsters
Bourdeaux, 38 boisseux	Genoa, 25 mines
Breda, 33 veertels	Gimond, 20 sacks
Bruges, $17\frac{1}{2}$ hoedts	Graveline, 22 razieres
Brussels, 25 sacks	Haerlem, 38 sacks
Bueren, 21 muddes	Hamburgh, $\frac{1}{3}$ of a last
Cadillac, $33\frac{1}{3}$ sacks	Heulden, $17\frac{1}{4}$ muddes
Cadiz, 52 hanegas	Hoorn, 44 sacks
Cahors, 100 cartes	Ireland, 38 bushels
Campan, $24\frac{1}{2}$ muddes	La Brille, 40 sacks
Carcassone, 35 setiers	La Reole, 30 sacks
Clairac, $34\frac{1}{2}$ sacks	Lavour, 21 setiers
Cleves, $16\frac{1}{4}$ mouwers	Leyden, 44 sacks
Condom, 41 sacks	Libourne, 35 sacks
	Liege, 96 setiers

10 $\frac{1}{4}$ quarters, or 82 Winchester bushels of England,
or 1 last of Amsterdam, make, at

Lisle, 38 razieres	Ruremonde, 68 schepels
Lisbon, 216 alquiers	Riga, 46 loopens
Leghorn, 40 sacks	Rotterdam, 29 sacks
Louvain, 27 muddes	St Giles, 40 charges
Lubeck, 95 schepels	St Omer, 22 $\frac{1}{2}$ razieres
Lyons, 14 $\frac{1}{4}$ anees	St Valery, 19 setiers
Middleburgh, 41 $\frac{1}{2}$ sacks	Saumur, 19 setiers
Montfort, 21 muddes	Steenbergen, 35 veertels
Muyden, 44 sacks	Stockholm, 23 tuns
Naerden, 44 sacks	Terveer, 39 sacks
Nerac, 33 $\frac{1}{3}$ sacks	Thiel, 21 muddes
Nieuport, 17 $\frac{1}{2}$ razieres	Toulouse, 26 setiers
Oudewater, 21 muddes	Tongres, 15 muddes
Paris 19 setiers	Tonningen, 24 tuns
Porto Port, 180 alquiers	Venloo, 21 $\frac{3}{4}$ mouwers
Purmerent, 27 muddes	Vianen, 20 muddes
Rabastens, 17 setiers	Utrecht, 25 muddes
Rhenen, 20 muddes	Zurickzee, 40 sacks.

The use of the tables.

E X A M P L E I.

What are 800 lb. of Lyons equal to in England?

Say, from Table I. As 106 lb. of Lyons : 100 lb.
of England :: 800 lb. of Lyons : 754 $\frac{1}{2}$ of England.

E X A M P L E II.

How many aunes of Amsterdam are equal to 500
aunes of Dantzick?

Say, from Table II. As 112 $\frac{1}{2}$ aunes of Dantzick : 100
aunes of Amsterdam :: 500 aunes of Dantzick : 444 $\frac{1}{4}$
aunes of Amsterdam.

E X A M P L E III.

How many setiers will 400 bushels of England make
at Paris?

Say,

Say, from Table III. As 82 bushels of England : 19 setiers of Paris :: 400 bushels of England : $92\frac{1}{2}$ setiers of Paris.

C H A P. VIII.

A L L I G A T I O N.

Alligation serves to solve questions that relate to the mixing of simples; and is either *medial* or *alternate*.

I. *Alligation Medial.*

Alligation medial, from the rates and quantities of the simples given, discovers the rate of the mixture.

R U L E.

As the total quantity of the simples,
To their price or value;
So any quantity of the mixture,
To the rate.

E X A M P L E S.

1. A grocer mixeth 30 lb. of currants, at 4 d. *per* lb. with 10 lb. of other currants, at 6 d. *per* lb.: What is the value of 1 lb. of the mixture? *Ans.* $4\frac{1}{2}$ d.

lb.	d.	d.
30,	at 4	amounts to 120
10,	at 6	———— 60
<u>40</u>		<u>180</u>

lb. d. lb. d.

If 40 : 180 :: 1 : $4\frac{1}{2}$

2. A farmer mingleth several sorts of wheat as follows, viz. 20 bushels, at 5 s. *per* bushel; 36 bushels,

at 3 s.; and 40 bushels, at 2 s.: What is a bushel of the mixture worth? *Ans.* 3 s.

Bush.	s.	s.
20, at 5, comes to		100
36, at 3	—	108
40, at 2	—	80
<u>96</u>		<u>288</u>

Bush. s. *Bush.* s.
If 96 : 288 :: 1 : 3

3. A tobacco-nist mixeth 30 lb. of tobacco, at 9d. per lb. with 60 lb. at 14d. and 24 $\frac{3}{4}$ lb. at 2 s. per lb.: What is a pound of the mixture worth?

lb.	Rates.	L.
30	$\times .0375 =$	1.125
60	$\times .0583 =$	3.5
24.75	$\times .1 =$	2.475
<u>114.75</u>		<u>7.1</u>

lb. L. lb. L.
If 114.75 : 7.1 :: 1 : .0618

s. d.
114.75) 7.1000(.0618 = 1 2 $\frac{3}{4}$

68850

21500

11475

100250

91800

8450

Note 1. When the quantity of each simple is the same, the rate of the mixture is readily found by adding the rates of the simples, and dividing their sum by the number of simples, thus.

Suppose

Suppose a grocer mixes several sorts of sugar, and of each an equal quantity, viz. at 50 s. at 54 s. and at 60 s. per C. the rate of the mixture will be 54 s. 8 d. per C.; for,

$$\begin{array}{r} \text{s.} \quad \text{s.} \quad \text{d.} \\ 50 + 54 + 60 = 164, \text{ and } 3)164(54 \quad 8 \end{array}$$

Note 2. If it be required to increase or diminish the quantity of the mixture, say, As the sum of the given quantities of the simples, to the several quantities given; so the quantity of the mixture proposed, to the quantities of the simples sought. Thus,

If it be required to increase the quantity of the mixture in Ex. 2. to 120 bushels, say,

$$\begin{array}{r} \text{Bush.} \quad \text{s.} \\ 96 : 20 :: 120 : 25, \text{ at } 5 \\ 96 : 36 :: 120 : 45, \text{ at } 3 \\ 96 : 40 :: 120 : 50, \text{ at } 2 \end{array}$$

Proof 120

Note 3. If it be required to know how much of each simple is in an assigned portion of the mixture, say, As the quantity of the mixture, to the several quantities of the simples given; so the quantity of the assigned portion, to the quantities of the simples sought. Thus,

Suppose a grocer mixes 10 lb. of raisins, with 30 lb. of almonds, and 40 lb. of currans, and it be demanded, how many ounces of each sort are found in every pound, or in every sixteen ounces of the mixture, say,

$$\begin{array}{r} \text{oz.} \\ 80 : 10 :: 16 : 2 \text{ raisins.} \\ 80 : 30 :: 16 : 6 \text{ almonds.} \\ 80 : 40 :: 16 : 8 \text{ currants.} \end{array}$$

Proof 16

Note 4. If the rates of two simples, with the total value and total quantity of the mixture be given, the

P 2

quantity

quantity of each simple may be found as follows, *viz.* multiply the lesser rate into the total quantity, subtract the product from the total value, and the remainder will be equal to the product of the excess of the higher rate above the lower, multiplied into the quantity of the higher-priced simple; and, consequently, the said remainder, divided by the difference of the rates, will quote the said quantity. Thus,

Suppose a grocer has a mixture of 400 lb. weight, that cost him 7 l. 10 s. consisting of raisins, at 4 d. *per* lb. and almonds at 6 d. how many pounds of almonds were in the mixture?

		lb.	Rates.	
L. s.	d.	400	6 d.	
		4	4 d.	
7 10 =	1800			
	1600	1600 d.	2 d.	L. s.
				2 10
				5 0
				<u>7 10</u>
		Total 400	Proof	

MORE EXAMPLES.

4. A merchant mingled three different sorts of sugar, *viz.* 3 C. at 2 l. 16 s. *per* C. and 3 C. at 3 l. 14 s. 8 d. with 6 C. at 1 l. 17 s. 4 d.: What is a C. of the mixture worth? *Ans.* 2 l. 11 s. 4 d.

5. A vintner mingles several sorts of wine, *viz.* 36 gallons at 8 s. *per* gallon, 2 gallons at 7 s. *per* gallon, 13 gallons at 1 s. *per* gallon, with 12 gallons of water: What is a gallon of the mixture worth? *Ans.* 5 s.

6. If, with 40 bushels of corn at 4 s. *per* bushel, there are mixed 10 bushels at 6 s. *per* bushel, 30 bushels at 5 s. *per* bushel, and 20 bushels at 3 s. *per* bushel, what will 10 bushels of the mixture be worth? *Ans.* 2 l. 3 s.

7. A goldsmith mixes 30 ounces of gold 22 caracts fine, 10 ounces 20 caracts fine, and 20 ounces 18 caracts fine: How many caracts fine is the mixture? *Ans.* 20 $\frac{1}{3}$; that is, of the 24 parts or caracts into which the ounce

ounce is supposed to be divided, $20\frac{1}{2}$ are pure gold, and the remaining $3\frac{1}{2}$ are alloy.

Medicines are sometimes mixed or compounded by this rule; for understanding the manner whereof it will be necessary to observe,

That medicines, drugs, or simples, with respect to their qualities, are divided into five sorts, viz. hot, cold, dry, moist, temperate; and all, except the last, admit of four degrees, represented by indices, as in the following table.

Ind.	1	2	3	4	5	6	7	8	9	The index 3 denotes cold or moist in the 2d degree, and 9 denotes hot or dry in the 4th degree.
Deg.	4	3	2	1	0	1	2	3	4	
	of cold and moisture.					of heat and dryness.				

EXAMPLE.

If I have four simples, A, B, C, D, and mix them as follows, viz. 4 oz. of A hot in the 4th degree, 1 oz. of B hot in the 2d, 1 oz. of C temperate, and 3 oz. of D cold in the 3d degree; what will be the quality of the mixture or compound? *Ans.* hot in the 1st degree.

$$A \ 4 \times 9 = 36$$

$$B \ 1 \times 7 = 7$$

$$C \ 1 \times 5 = 5$$

$$D \ 3 \times 2 = 6$$

$$\begin{array}{r} 36 \\ 7 \\ 5 \\ 6 \\ \hline 9 \end{array} \quad 9) 54(6 \text{ hot in the 1st degree.}$$

II. Alligation Alternate.

Alligation alternate, being the converse of alligation medial, from the rates of the simples, and rate of the mixture given, finds the quantities of the simples.

RULES.

R U L E S.

I. Place the rate of the mixture on the left side of a brace, as the root; and on the right side of the brace set the rates of the several simples, under one another, as the branches.

II. Link or alligate the branches, so as one greater and another less than the root may be linked or yoked together.

III. Set the difference betwixt the root and the several branches, right against their respective yoke-fellows. These alternate differences are the quantities required.

Note 1. If any branch happen to have two or more yoke-fellows, the difference betwixt the root and these yoke-fellows must be placed right against the said branch, one after another, and added into one sum.

Note 2. In some questions, the branches may be alligated more ways than one; and a question will always admit of so many answers, as there are different ways of linking the branches.

Alligation alternate admits of three varieties, viz.
 1. The question may be unlimited, with respect both to the quantity of the simples, and that of the mixture.
 2. The question may be limited to a certain quantity of one or more of the simples.
 3. The question may be limited to a certain quantity of the mixture.

Variety I.

When the question is unlimited, with respect both to the quantity of the simples, and that of the mixture. This is called *Alligation Simple*.

E X A M P L E S.

1. A grocer would mix sugars, at 5 d. 7 d. and 10 d. per lb. so as to sell the mixture or compound at 8 d. per lb.: What quantity of each must he take?

lb.

$$\begin{array}{c}
 \text{lb.} \\
 8 \left\{ \begin{array}{l} 5 \\ 7 \\ 10 \end{array} \right\} \begin{array}{l} 2 \\ 2 \\ 3, 1 \end{array} \left| \begin{array}{l} 2 \\ 2 \\ 4 \end{array} \right.
 \end{array}$$

Here the rate of the mixture 8 is placed on the left side of the brace, as the root; and on the right side of the same brace are set the rates of the several simples, viz. 5, 7, 10, under one another, as the branches; according to Rule I.

The branch 10 being greater than the root, is alligated or linked with 7 and 5, both these being less than the root; as directed in Rule II.

The difference between the root 8 and the branch 5, viz. 3, is set right against this branch's yoke fellow 10. The difference between 8 and 7 is likewise set right against the yoke-fellow 10. And the difference betwixt 8 and 10, viz. 2, is set right against the two yoke-fellows 7 and 5; as prescribed by Rule III.

As the branch 10 has two differences on the right, viz. 3 and 1, they are added; and the answer to the question is, that 2 lb. at 5 d. 2 lb. at 7 d. and 4 lb. at 10 d. will make the mixture required.

The truth and reason of the rules will appear by considering, that whatever is lost upon any one branch is gained upon its yoke-fellow. Thus, in the above example, by selling 4 lb. of 10 d. sugar at 8 d. *per lb.* there is 8 d. lost: but the like sum is gained upon its two yoke-fellows; for by selling 2 lb. of 5 d. sugar at 8 d. *per lb.* there is 6 d. gained, and by selling 2 lb. of 7 d. sugar at 8 d. there is 2 d. gained; and 6 d. and 2 d. make 8 d.

Hence it follows, that the rate of the mixture must always be mean or middle with respect to the rates of the simples; that is, it must be less than the greatest, and greater than the least; otherwise a solution would be impossible. And the price of the total quantity mixed, computed at the rate of the mixture, will always be equal to the sum of the prices of the several quantities

cast

cast up at the respective rates of the simples. This may be used by way of proof; and the above example proved in this manner follows.

$$\begin{array}{r|l}
 8 \left\{ \begin{array}{l} 5 \\ 7 \\ 10 \end{array} \right\} \begin{array}{l} 2 \\ 2 \\ 3, 1 \end{array} & \begin{array}{l} 2 \times 5 = 10 \\ 2 \times 7 = 14 \\ 4 \times 10 = 40 \\ \hline 8 \times 8 = 64 \end{array} & \begin{array}{l} 10 \\ 14 \\ 40 \\ \hline 64 \text{ proof.} \end{array}
 \end{array}$$

Or say,

$$\text{If } 8 : 64 :: 1 : 8$$

That is, the sum of the prices or price of the mixture, divided by the sum of the quantities, quotes the rate of the mixture.

From the above proof it is obvious, that the rates of the simples, their total quantity, and the price of the mixture, being given; the rate of the mixture, and consequently the quantities of the several simples, may be found: for the price of the mixture divided by the total quantity of the simples, quotes the rate of the mixture; the alternate differences of which, and the several rates, are the quantities sought.

The answer in the above example comes out to be 2, 2, 4; and as the branches can be linked but one way, we can, by the operation, have only this single answer: but then any other three numbers that have the same proportion to one another, found by division or multiplication, will also answer the question. By this means the answer found may be increased or diminished, till it suit the stock of simples on hand for making the mixture; as in the following specimen.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. }

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. } &c.

2. 4. 6. 8. 10. 12. 14. 16. 18. 20. 22. 24. 26. }

But we may also find innumerable other answers, by increasing or diminishing the alternate differences of any pair

pair of simples, without varying the rest, by one or other of the two rules following.

I. When the rates of the variable pair of simples are both greater, or both less, than the rate of the mixture; increase the alternate difference of the one, by adding to it the difference betwixt the rate of the mixture and the rate of the other variable simple; and diminish the alternate difference of the other, by subtracting from it the difference betwixt the rate of the mixture and the rate of the former variable simple.

The alternate difference to be increased is optional, or which of them you please; and instead of the alternate differences, you may use any other numbers that are in the same proportion; and the higher these numbers are, the more answers they will afford. We shall exemplify the rule by the proportional numbers, 13, 13, 26, found above, making 13, 13, the variable pair, their rates, 5 and 7, being both less than 8, the rate of the mixture, and shall continue 26 unvaried.

By increasing 13 in the upper line, we have,

| | | | | | | | | | |
|-----|-----|-----|-----|-----|--|--|--|--|--|
| 13. | 14. | 15. | 16. | 17. | | | | | |
| 13. | 10. | 7. | 4. | 1. | | | | | |
| 26. | 26. | 26. | 26. | 26. | | | | | |

By increasing 13 in the second line, we have,

| | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 13. | 12. | 11. | 10. | 9. | 8. | 7. | 6. | 5. | 4. | 3. | 2. | 1. |
| 13. | 16. | 19. | 22. | 25. | 28. | 31. | 34. | 37. | 40. | 43. | 46. | 49. |
| 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. | 26. |

II. When the rate of one of the variable simples is greater than the rate of the mixture, and the rate of the other less, their alternate differences, or, rather, the numbers proportional to them, must, as directed in the former rule, be both increased, or both diminished, and either or each of these ways may be used.

We shall exemplify the rule, as formerly, by the proportional numbers 13, 13, 26, making the first 13 and 26 the variable pair, whose rates are 5 and 10, the

one below and the other above 8, the rate of the mixture, and shall continue the other 13 unvaried;

By increasing both, we have,

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|----------------|
| 13. | 15. | 17. | 19. | 21. | 23. | 25. | 27. | 29. | } to infinity, |
| 13. | 13. | 13. | 13. | 13. | 13. | 13. | 13. | 13. | |
| 26. | 29. | 32. | 35. | 38. | 41. | 44. | 47. | 50. | |

By diminishing both, we have,

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 13. | 11. | 9. | 7. | 5. | 3. | 1. |
| 13. | 13. | 13. | 13. | 13. | 13. | 13. |
| 26. | 23. | 20. | 17. | 14. | 11. | 8. |

Again, we may make the second 13 and 26 the variable pair, their rates being 7 and 10, the one below and the other above 8, the rate of the mixture, and continue the first 13 unvaried, as under,

By increasing both, we have,

| | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|----------------|
| 13. | 13. | 13. | 13. | 13. | 13. | 13. | 13. | 13. | } to infinity. |
| 13. | 15. | 17. | 19. | 21. | 23. | 25. | 27. | 29. | |
| 26. | 27. | 28. | 29. | 30. | 31. | 32. | 33. | 34. | |

By diminishing both, we have,

| | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|
| 13. | 13. | 13. | 13. | 13. | 13. | 13. |
| 13. | 11. | 9. | 7. | 5. | 3. | 1. |
| 26. | 25. | 24. | 23. | 22. | 21. | 20. |

The reason of the rules is obvious: for by increasing or diminishing the alternate differences, or other numbers proportional to them, in the manner prescribed, the value of every pound of increase or diminution is the rate of the mixture.

2. A vintner would mix wines at 14s. 15s. 19s. and 22 s. *per* gallon, so as the mixture may be worth 18 s. *per* gallon: What quantity of each must he take?

First,

First, thus. Gal. Sec. thus. Gal. Thirdly, thus. Gal.

$$\begin{array}{l}
 18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 4 \\ 1 \\ 3 \\ 4 \end{array} \quad \left| \quad 18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 1 \\ 4 \\ 4 \\ 3 \end{array} \quad \left| \quad 18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 1, 4 \\ 1 \\ 4, 3 \\ 4 \end{array} \quad \left| \quad \begin{array}{l} 5 \\ 1 \\ 7 \\ 4 \end{array}
 \end{array}$$

Fourthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 4 \\ 1, 4 \\ 3 \\ 4, 3 \end{array} \quad \left| \quad \begin{array}{l} 4 \\ 5 \\ 3 \\ 7 \end{array}$$

Fifthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 1, 4 \\ 4 \\ 4 \\ 4, 3 \end{array} \quad \left| \quad \begin{array}{l} 5 \\ 4 \\ 4 \\ 7 \end{array}$$

Sixthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 1 \\ 1, 4 \\ 4, 3 \\ 3 \end{array} \quad \left| \quad \begin{array}{l} 1 \\ 5 \\ 7 \\ 3 \end{array}$$

Seventhly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right. \begin{array}{l} 1, 4 \\ 1, 4 \\ 4, 3 \\ 4, 3 \end{array} \quad \left| \quad \begin{array}{l} 5 \\ 5 \\ 7 \\ 7 \end{array}$$

Here the simples are linked seven different ways; and accordingly we have seven different answers to the question; and, by multiplication or division, each of these will produce other answers to infinity. Again, by taking each pair of simples successively, and increasing or diminishing their alternate differences, or other numbers proportional to them, in the manner taught above, we shall have other answers innumerable.

3. A grocer mixes teas, at 7 s. at 8 s. at 10 s. and at 13 s. per lb.: How much of each sort must he take, that a pound of the mixture may be worth 9 s. 6 d.?

First, lb.

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right. \begin{array}{l} 3.5 \\ .5 \\ 1.5 \\ 2.5 \end{array}$$

Secondly, lb.

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right. \begin{array}{l} .5 \\ 3.5 \\ 2.5 \\ 1.5 \end{array}$$

Q 2

Thirdly,

Thirdly, *lb.*

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ .5 \\ 2.5, 1.5 \\ 2.5 \end{array} \left| \begin{array}{l} 4 \\ .5 \\ 4 \\ 2.5 \end{array} \right.$$

Fourthly, *lb.*

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} 3.5 \\ .5, 3.5 \\ 1.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 3.5 \\ 4 \\ 1.5 \\ 4 \end{array} \right.$$

Fifthly, *lb.*

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ 3.5 \\ 2.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 4 \\ 3.5 \\ 2.5 \\ 4 \end{array} \right.$$

Sixthly, *lb.*

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5 \\ .5, 3.5 \\ 2.5, 1.5 \\ 1.5 \end{array} \left| \begin{array}{l} .5 \\ 4 \\ 4 \\ 1.5 \end{array} \right.$$

Seventhly, *lb.*

$$9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ .5, 3.5 \\ 2.5, 1.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 4 \\ 4 \\ 4 \\ 4 \end{array} \right.$$

Other answers innumerable may be found by the methods explained above.

4. An apothecary would mix four simples, *viz.* A hot in the fourth degree, B hot in the second, C temperate, and D cold in the third, so as the mixture may be hot in the first degree: What quantity of each simple must he take? See the table on composition of medicines in alligation medial.

$$\begin{array}{r|l} 9 & 1 \text{ A} \\ 7 & 4 \text{ B} \\ 5 & 3 \text{ C} \\ 2 & 1 \text{ D} \\ \hline 6 \times 9 = 54 & \text{Proof } 54 \end{array} \begin{array}{l} 1 \times 9 = 9 \\ 4 \times 7 = 28 \\ 3 \times 5 = 15 \\ 1 \times 2 = 2 \end{array}$$

The branches may be alligated other six ways, which will afford the like number of other answers.

Variety II.

When the question is limited to a certain quantity of one or more of the simples. This is called *Alligation Partial*.

If the quantity of one of the simples only be limited, alligate the branches, and take their differences, as if there

there had been no such limitation; and then work by the following proportion.

As the difference right against the rate of the simple
whose quantity is given,
To the other differences respectively;
So the quantity given,
To the several quantities sought.

EXAMPLES.

1. A distiller would, with 40 gallons of brandy at 12 s. *per* gallon, mix rum at 7 s. *per* gallon, and gin at 4 s. *per* gallon: How much of the rum and gin must he take, to sell the mixture at 8 s. *per* gallon?

$$8 \left\{ \begin{array}{l} 12 \\ 7 \\ 4 \end{array} \right) \begin{array}{l} 1,4 \\ 4 \\ 4 \end{array} \left| \begin{array}{l} 5 \\ 4 \\ 4 \end{array} \right| \begin{array}{l} 40 \text{ of brandy.} \\ 32 \text{ of rum.} \\ 32 \text{ of gin.} \end{array} \right\} \text{Ans.}$$

The operation gives for answer 5 gallons of brandy, 4 of rum, and 4 of gin. But the question limits the quantity of brandy to 40 gallons; therefore say,

$$\text{If } 5 : 4 :: 40 : 32$$

The quantity of gin, by the operation being also 4, the proportion needs not be repeated. The proof is the same as formerly.

As the branches can be linked only one way, we can have but one answer by the operation; and because the quantity of brandy is limited to 40 gallons, there can be no recourse to proportional numbers; but by proceeding with the remaining pair of simples in the manner formerly taught, we may have a great many other answers, as follows, *viz.*

By increasing the first 32, we have,

$$\left. \begin{array}{l} 40. 40. 40. 40. 40. 40. 40. 40. \\ 32. 36. 40. 44. 48. 52. 56. 60. \\ 32. 31. 30. 29. 28. 27. 26. 25. \end{array} \right\} \text{ \&c.}$$

By

By increasing the last 32, we have,

40. 40. 40. 40. 40. 40. 40. 40.
 32. 28. 24. 20. 16. 12. 8. 4.
 32. 33. 34. 35. 36. 37. 38. 39.

2. An innkeeper would, with 72 bushels of his best corn, at 60d. *per* bushel, mix other corns at 48 d. at 36d. at 24 d. *per* bushel: What quantity of the last three must he take, that the mixture may be worth 42 d. *per* bushel?

First, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 36 \\ 6 \\ 6 \\ 18 \end{array} \left| \begin{array}{l} 72 \\ 12 \\ 12 \\ 36 \end{array} \right.$$

Secondly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6 \\ 18 \\ 13 \\ 6 \end{array} \left| \begin{array}{l} 72 \\ 216 \\ 216 \\ 72 \end{array} \right.$$

Thirdly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 6 \\ 18, 6 \\ 18 \end{array} \left| \begin{array}{l} 24 \\ 6 \\ 24 \\ 18 \end{array} \right| \begin{array}{l} 72 \\ 18 \\ 72 \\ 54 \end{array}$$

Fourthly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 18 \\ 6, 18 \\ 6 \\ 18, 6 \end{array} \left| \begin{array}{l} 18 \\ 24 \\ 6 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 96 \\ 24 \\ 96 \end{array}$$

Fifthly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 18 \\ 18 \\ 18, 6 \end{array} \left| \begin{array}{l} 24 \\ 18 \\ 18 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 54 \\ 54 \\ 72 \end{array}$$

Sixthly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6 \\ 6, 18 \\ 18, 6 \\ 6 \end{array} \left| \begin{array}{l} 6 \\ 24 \\ 24 \\ 6 \end{array} \right| \begin{array}{l} 72 \\ 288 \\ 288 \\ 72 \end{array}$$

Seventhly, *Ans.*

$$42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 6, 18 \\ 18, 6 \\ 18, 6 \end{array} \left| \begin{array}{l} 24 \\ 24 \\ 24 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 72 \\ 72 \\ 72 \end{array}$$

In each of the seven answers here given, 72 is limited; but then any two of the three remaining simples may be esteemed a pair, affording in all three pairs, from which innumerable other answers may be found.

3. A goldsmith would, with 85 oz. of gold, worth 5 l. *per* oz. mix other sorts of gold at 4 l. 10 s. *per* oz. and at 4 l. *per* oz.: What quantity of the last two, and what quantity of alloy must he take, that the mass may be worth 4 l. 5 s. *per* oz.?

$$\begin{array}{r}
 \text{L.} \\
 4.25 \left\{ \begin{array}{l} 5 \\ 4.5 \\ 4 \\ 0 \end{array} \right\} \begin{array}{l} 4.25 \\ .25 \\ .25 \\ .75 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{oz.} \\
 \text{As } 4.25 : 85 :: \left\{ \begin{array}{l} 4.25 : 85 \\ .25 : 5 \\ .25 : 5 \\ .75 : 15 \end{array} \right\} \text{Ans.}
 \end{array}$$

By other ways of linking, and from the pairs that arise from the three variable simples, other answers innumerable may be found.

4. A vintner would, with 10 gallons of wine at 12 s. *per* gallon, and 20 gallons at 15 s. *per* gallon, mix other wines, at 18 s. and at 20 s. *per* gallon: What quantity of the last two must he take, that the mixture may be worth 16 s. *per* gallon?

This question is limited to a certain quantity of two of the simples, and, previous to the operation, the two given quantities must be supposed to be mixed by themselves; and the rate of the mixture found by alligation medial as under.

Gallons.

$$\begin{array}{r}
 \text{s.} \\
 10, \text{ at } 12 \text{ s. come to } 120 \\
 20, \text{ at } 15 \text{ s. amount to } 300 \\
 \hline
 30 \qquad 420 (14 \text{ s. per gallon.}
 \end{array}$$

The question now may be considered as limited only in the quantity of one of the simples, and may be proposed and solved as follows.

A

A vintner would, with 30 gallons of wine at 14 s. per gallon, mix other wines at 18 s. and at 20 s. per gallon: What quantity of the last two must he take, that the mixture may be worth 16 s. per gallon?

$$16 \left\{ \begin{array}{l} 14 \\ 18 \\ 20 \end{array} \right\} \begin{array}{l} 2, 4 \\ 2 \\ 2 \end{array} \left| \begin{array}{l} 6 \\ 2 \\ 2 \end{array} \right| \begin{array}{l} 30 \\ 10 \\ 10 \end{array} \quad \text{Ans.}$$

That is, 30 gallons of wine at 14 s. per gallon; or, resolving this compound into its simples, the answer is, that 10 gallons of wine at 12 s. per gallon, and 20 gallons at 15 s. together with 10 gallons of each of the other two sorts of wine, will make the mixture proposed.

$$10 \times 12 = 120$$

$$20 \times 15 = 300$$

$$10 \times 18 = 180$$

$$10 \times 20 = 200$$

$$\underline{50} \quad \underline{50} 800 (16 \text{ proof.})$$

The quantity 30, or rather 10 and 20, are limited; but from the remaining pair a great many other answers may be found.

The procedure is the same when the question is limited in the quantity of three or more of the simples.

Variety III.

When the question is limited to a certain quantity of the mixture, this is called *Alligation Total*.

After linking the branches, and taking the differences, work by the proportion following.

As the sum of the differences,

To each particular difference;

So the given total of the mixture,

To the respective quantities required.

EX-

E X A M P L E S.

1. A vintner hath wine at 3 s. *per* gallon, and would mix it with water, so as to make a composition of 144 gallons, worth 2 s. 6 d. *per* gallon: How much wine and how much water must he take?

$$\begin{array}{r|l} \text{Gal.} & \\ 30 \left\{ \begin{array}{l} 36 \\ 0 \end{array} \right\} 30 & \left. \begin{array}{l} 120 \text{ of wine.} \\ 24 \text{ of water.} \end{array} \right\} \text{Ans.} \\ \hline 36 & 144 \text{ total.} \end{array}$$

$$120 \times 36 = 4320$$

$$24 \times 0 = 0$$

$$\text{Proof } 144)4320(30$$

$$\text{As } 36 : 30 :: 144 : 120$$

$$\text{As } 36 : 6 :: 144 : 24$$

There being here only two simples, and the total of the mixture limited, the question admits but of one answer.

2. A distiller would mix brandy at 8 s. wine at 7 s. and cyder at 1 s. *per* gallon, so that the mixture may contain 26 gallons, and be worth 5 s. *per* gallon: What quantity of each must he take?

$$\begin{array}{r|l} \text{Ans.} & \\ 5 \left\{ \begin{array}{l} 8 \\ 7 \\ 1 \end{array} \right\} \begin{array}{l} 4 \\ 4 \\ 3,2 \end{array} & \left| \begin{array}{l} 4 \\ 4 \\ 5 \end{array} \right| \begin{array}{l} 8 \\ 8 \\ 10 \end{array} \\ \hline & \left| \begin{array}{l} 13 \\ 26 \end{array} \right| \end{array} \quad \begin{array}{l} \text{If } 13 : 4 :: 26 : 8 \\ \text{If } 13 : 5 :: 26 : 10 \end{array}$$

From the alternate differences, an immense variety of other unlimited answers may be obtained by the methods explained above; and any one of these unlimited answers, that suits the purpose best, may be reduced to the limits of the question by the proportion assigned above. And even from the limited answer other answers may be found, by observing the directions following, *viz.*

When three simples, as here, are mixed; though the total be limited, yet from the answer found by the operation, other answers may be deduced in the manner following.

First, Increase or diminish the quantity of that simple whose rate alone is greater or less than the rate of the mixture, by the difference of the differences between the rate of the mixture and the rates of the other two simples. Thus, in the above example, the rate of the cyder alone is less than the rate of the mixture, and the differences betwixt the rate of the mixture and the rates of the other two simples, are respectively 3 and 2, whose difference is 1; and accordingly the quantity of the cyder found by the operation, *viz.* 10, may be increased or diminished by 1, that is, it may be made 11 or 9.

Secondly, Of the two remaining simples, increase or diminish the quantity of that one whose rate is farthest from the rate of the mixture, by the sum of the differences betwixt the rate of the mixture and the rates of the other two simples. Thus, in the above example of the two remaining simples, the rate of the brandy is farthest from the rate of the mixture, and the rate of the mixture differs from the rates of the other two simples, *viz.* the wine and cyder respectively, by 2 and 4, whose sum is 6; and accordingly the quantity of the brandy, *viz.* 8, may be increased or diminished by 6, that is, it may be made 14 or 2.

Thirdly, Diminish or increase the quantity of the remaining simple, by the sum of the differences betwixt the rate of the mixture and the rates of the other two simples. But here observe, that the quantity of this simple is to be diminished when those of the former two are increased; and the contrary. Thus the quantity of the wine, *viz.* 8, is to be diminished or increased by 7, the sum of 3 and 4, the respective differences betwixt the rate of the mixture and the rates of the brandy and cyder; that is, it may be made 1 or 15.

By this means other two answers to the above question are obtained, which, with the answer found by the operation, are here subjoined.

Brandy

| | | | |
|--------|-----------|-----------|-----------|
| Brandy | 14. | 8. | 2. |
| Wine | 1. | 8. | 15. |
| Cyder | 11. | 10. | 9. |
| | <u>26</u> | <u>26</u> | <u>26</u> |

3. A goldsmith hath several sorts of gold, viz. N^o 1. of 24 caracts fine, N^o 2. of 24 caracts fine, N^o 3. of 18 caracts fine, and N^o 4. of 16 caracts fine: How much of each sort must he take, to make a mass of 60 ounces, of 21 caracts fine?

| | | | | | |
|----|------|-------------|--|-----------|----------------------|
| | | <i>Ans.</i> | | | |
| 21 | { 24 | 5 | | 25 | |
| | { 22 | 3 | | 15 | As 12 : 5 :: 60 : 25 |
| | { 18 | 1 | | 5 | As 12 : 3 :: 60 : 15 |
| | { 16 | 3 | | 15 | As 12 : 1 :: 60 : 5 |
| | | <u>12</u> | | <u>60</u> | |

The branches in the above example may be linked other six ways, which will accordingly afford so many other answers.

When four simples, as here, are mixed, and it is required to deduce other answers from that found by the operation, in this case one of the simples must be considered as invariable, and the three variable simples must be such, that the rate of one of them may, with respect to the rate of the mixture, be opposite to the rates of the other two; that is, if the rate of one of the variable simples be greater than the rate of the mixture, then the rates of the other two must both be less; and the contrary.

Thus, in the above example, making N^o 1. invariable, the rate of N^o 2. is greater than the rate of the mixture, and the rates of the other two are both less; and accordingly by the three directions subjoined to Ex. 2. we may, from the answer in Ex. 3. deduce other answers, as follows.

R 2

N^o 1.

| | | | | |
|-------------------|-----------|-----------|-----------|-----------|
| N ^o 1. | 25. | 25. | 25. | 25. |
| N ^o 2. | 15. | 13. | 11. | 9. |
| N ^o 3. | 5. | 11. | 17. | 23. |
| N ^o 4. | 15. | 11. | 7. | 3. |
| | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> |

Next, making N^o 2. invariable, we shall have,

| | | | |
|-------------------|-----------|-----------|-----------|
| N ^o 1. | 25. | 23. | 21. |
| N ^o 2. | 15. | 15. | 15. |
| N ^o 3. | 5. | 13. | 21. |
| N ^o 4. | 15. | 9. | 3. |
| | <u>60</u> | <u>60</u> | <u>60</u> |

Again, making N^o 3. invariable, we shall have,

| | | | | | | |
|-------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| N ^o 1. | 31. | 25. | 19. | 13. | 7. | 1. |
| N ^o 2. | 7. | 15. | 23. | 31. | 39. | 47. |
| N ^o 3. | 5. | 5. | 5. | 5. | 5. | 5. |
| N ^o 4. | 17. | 15. | 13. | 11. | 9. | 7. |
| | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> |

Lastly, making N^o 4. invariable, we shall have,

| | | | | | |
|-------------------|-----------|-----------|-----------|-----------|-----------|
| N ^o 1. | 17. | 21. | 25. | 29. | 33. |
| N ^o 2. | 27. | 21. | 15. | 9. | 3. |
| N ^o 3. | 1. | 3. | 5. | 7. | 9. |
| N ^o 4. | 15. | 15. | 15. | 15. | 15. |
| | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> | <u>60</u> |

By taking the six answers that will arise from the other six ways of linking, and proceeding in like manner with each of them, a great many other answers may be found.

4. A merchant having wines at 12 d. 14 d. 15 d. 18 d. and 22 d. *per* pint, would make a mixture of 1240 pints, to sell at 17 d. *per* pint: What quantity of each must he take?

| | | |
|------|----------|-------------|
| | | <i>Ans.</i> |
| 17 { | 12 1,5 | 6 240 |
| | 14 5 | 5 200 |
| | 15 5 | 5 200 |
| | 18 5 | 5 200 |
| | 22 5,3,2 | 10 400 |
| | | <hr/> |
| | | 31 1240 |

As 31 : 6 :: 1240 : 240

As 31 : 5 :: 1240 : 200

As 31 : 10 :: 1240 : 400

The branches in this example may be alligated a great many other ways; each of which will afford a different answer.

When five simples, as here, are mixed; in order to deduce other answers from that found by the operation, two of the simples, taken by turns, must always be made invariable; and the three remaining simples must be such, that the rate of one of them being greater or less than the rate of the mixture, the rates of the other two must respectively be both less, or both greater, than the rate of the mixture; and then the procedure is the same as formerly. In like manner, when six, seven, eight, &c. simples, are mixed, all the simples must by turns be made invariable, except three; and these three must be qualified as above.

5. Suppose 9 lb. of pure gold immersed in a vessel full of water to expel 3 lb. of water, 9 lb. of pure silver to expel 6 lb. of water, and 9 lb. of a mass made up of gold and silver to expel 4 lb. of water; required the quantity of gold and silver in the said mass.

In order to solve this question, the three several quantities of water expelled, which denote the specific bulks or gravities of the simples, must be considered as the rates; that of the mass being esteemed the root, and the other two the branches, as follows.

Ans.

Ans.

$$4 \left\{ \begin{array}{l} 3 \\ 6 \end{array} \right\}_1^2 \left| \begin{array}{l} 6 \text{ lb. of pure gold} \\ 3 \text{ lb. of pure silver} \end{array} \right\} \text{ in the mass.}$$

3 9

Hence is solved the famous question with respect to the crown of Hiero King of Syracuse. He had ordered a crown to be made of pure gold; but suspecting that the founder had mixed silver with the gold, he desired Archimedes to examine the affair, and tell him, how much gold and how much silver was in the crown. Archimedes, by immersing the crown, and also the like quantities of pure gold and silver, in water, severally, and observing the respective quantities of water expelled, found an answer to the question.

CHAP. IX.

RULE of FALSE.

THE Rule of False is so called, because, by the help of false supposed numbers, the true ones are discovered; and is divided into Single and Double, or into Single Position and Double Position.

Before the knowledge of algebra came to be common, this rule was in high esteem, and found to be of great service; and though not so universally practised now as formerly, yet still it continues to be of considerable use.

I. *Single Position.*

Single position is, when, by one supposition, or one position, the number sought is discovered: in the finding of which we work first with the position, as if it was the true number; and, having brought out the result, we then proceed by the following proportion.

As

As the result by operation, or false result,
To the false position;
So the true result,
To the true position, or number sought.

E X A M P L E S.

1. A, B, and C, buy a quantity of wine for 340 l.; of which A pays three times more than B, and B four times more than C: What paid each?

L.

Suppose A paid 36
Then B paid 12
And C paid 3

Result 51

As 51 : 36 :: 340 : 240

L.

So A paid 240
B paid 80
C paid 20

340 proof.

2. A person having about him a certain number of crowns, said, If a third, a fourth, and a sixth of them were added together, the sum would be 45: How many crowns had he?

Suppose 48, whereof a third is 16
a fourth is 12
a sixth is 8

Result 36

As 36 : 48 :: 45 : 60. A third of 60 is 20

a fourth is 15

a sixth is 10

Proof 45

3. Suppose the sum of 20 l. was to be paid by A, B, and C, in the following proportion, viz. A $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{4}$: What must each pay?

L.

$$\begin{array}{r}
 \frac{1}{2} \text{ of } 20 \text{ l. is } 10 \quad 0 \quad 0 \\
 \frac{1}{3} \text{ is } 6 \quad 13 \quad 4 \\
 \frac{1}{4} \text{ is } 5 \quad 0 \quad 0 \\
 \hline
 \text{Result } 21 \quad 13 \quad 4
 \end{array}$$

$$\begin{array}{r}
 \text{L. s. d.} \\
 \text{As } 21 \quad 13 \quad 4 : \left\{ \begin{array}{l} 10 \quad 0 \quad 0 \\ 6 \quad 13 \quad 4 \\ 5 \quad 0 \quad 0 \end{array} \right\} :: 20 : \left\{ \begin{array}{l} 9 \quad 4 \quad 7\frac{5}{13} \\ 6 \quad 3 \quad 0\frac{12}{13} \\ 4 \quad 12 \quad 3\frac{9}{13} \end{array} \right\}
 \end{array}$$

$$\text{Proof } 20 \quad 0 \quad 0$$

4. A schoolmaster being asked how many scholars he had, answered, If I had as many, half as many, and one fourth as many, I should have 99: How many had he? *Ans.* 36.

5. A, B, and C, discoursing of their age, B says to A, I am as old, and half as old again, as you; says C to B, I am twice as old as you; and A affirms, that the sum of their three ages amounted to 165: What was each man's age? *Ans.* A 30, B 45, and C 90 years of age.

6. A gentleman bought a chaise, horse, and harness, for 55 l.; the horse came to twice the price of the chaise, and the harness cost one third the price of the horse: What did he pay for each? *Ans.* For the chaise 15 l. for the horse 30 l. and for the harness 10 l.

II. Double Position.

Double position is when two suppositions, or two positions, must be used, in order to discover the number sought: in finding whereof, work with each position, as if it was the true number, till by this means you bring out the results, and consequently the errors; and then proceed by the following

R U L E.

1 Multiply the two positions into their altern errors; then,

then, errors alike, that is, both excesses, or both defects, divide the difference of the products by the difference of the errors. But if the errors are unlike, that is, the one an excess and the other a defect, divide the sum of the products by the sum of the errors.

Or, instead of the above rule, you may use the following proportion.

As the difference of the errors, if alike, or their sum, if unlike,

To the difference of the positions ;

So either error,

To a fourth number.

Which fourth number added to, or subtracted from, the position producing the error, gives the number sought.

E X A M P L E S.

1. A person buys 30 pints of liquor for 50 shillings; part beer, at 6d. *per* pint, and part wine, at 3s. *per* pint: How much was there of each?

| | | |
|---------------------|-------------|-----------------------------|
| | <i>Pts.</i> | <i>s.</i> |
| Position 1. Beer 6, | at 6d. is, | 3 |
| Then wine 24, | at 3s. is, | 72 |
| | | <hr/> 75 result. |
| | | <hr/> 50 |
| | | <hr/> + 25 error of excess. |

| | | |
|----------------------|-------------|-----------------------------|
| | <i>Pts.</i> | <i>s.</i> |
| Position 2. Beer 10, | at 6d. is, | 5 |
| Then wine 20, | at 3s. is, | 60 |
| | | <hr/> 65 result. |
| | | <hr/> 50 |
| | | <hr/> + 15 error of excess. |

Pos. Er.
 $6 \times 15 = 90$
 $10 \times 25 = 250$ *Pts.*

 $10 \overline{) 160} (16 \text{ beer.}$
 VOL. III.

Pts. *s.*
 Beer 16, at 6d. is, 8
 Wine 14, at 3s. is, 42

 Proof 50
 Or,

Or, work by the proportion, as follows.

$$\text{As } 10 : 4 :: 25 : 10$$

$$\text{Or, As } 10 : 4 :: 15 : 6$$

$$\text{And } 10 + 6 = 16 \text{ pints of beer.}$$

$$\text{And } 6 + 10 = 16 \text{ pints of beer, as before.}$$

It is easy to perceive, that the fourth number must here be added to the position, because, by increasing the quantity of beer, the quantity of wine is lessened; and this of course lessens the result, which is too great.

The positions might have been so chosen, that the errors thence arising would have been both defects, or the one an excess, and the other a defect; but the doing of this is left as a proper exercise to the learner. These varieties shall be illustrated in other examples.

It is worth while here also to observe, that double position takes place, or becomes necessary, when there is no partition of prices or quantities given, whereby to found or apply a proportion; for the question says, A person bought 30 pints of liquor, part beer, part wine, without partitioning the quantities; whereas, had the question said, That for every 8 pints of beer, he had 7 pints of wine, the answer might have been found by single position.

The reason of the assigned proportion will appear by considering, that the result varies with the position; and that the error of a result is ever proportional to the error of its position; so that the error of any one result is to the error of its own position, as the error of any other result is to the error of its position; and the difference of any two results has the same proportion to the difference of the positions from which they flow, as the difference of any other two results has to the difference of their positions. But the difference of the errors is equal to the difference of the results; therefore, As the difference of the errors, if alike, to the difference of the positions, so either error to a fourth number, viz. the error of the position.

When

When the errors are unlike, one of the false results is greater and the other less than the true result given in the question; and consequently the difference of the errors is found the same way as the difference of an affirmative and negative quantity, namely, by taking their sum; or, in the same manner as is found the difference of latitude of two places, whereof one lies to the north and the other to the south of the equator; that is, by adding the latitudes of the two places into one sum.

The truth of the rule will appear, by observing, that the difference of the two products arising from the multiplication of the positions into their altern errors, may be conceived as made up of two small parts or products, *viz.* first, the product of half the sum of the positions into the difference of the errors; secondly, the product of half the sum of the errors into the difference of the positions. Now, the former of these small products, divided by the difference of the errors, quotes half the sum of the positions, the divisor being one of the factors; the latter of these small products, divided also by the difference of the errors, quotes half the sum of the errors of the positions, by the proportion; and consequently the sum of these two small products, (equal to the difference of the alternate products), divided by the difference of the errors, quotes the true position, or answer sought, when the errors are alike.

When the errors are unlike, their difference, as also the difference of the alternate products, is found by taking their sum, as has been already shown.

2. A, B, and C, built a house, which cost 76*l.*; whereof A paid a sum unknown, B paid 10*l.* more than A, and C paid as much as A and B: What did each partner pay?

| | <i>L.</i> | | <i>L.</i> |
|--------------------|---------------|--------------------|---------------|
| <i>Position 1.</i> | | <i>Position 2.</i> | |
| A | 6 | A | 9 |
| B | 16 | B | 19 |
| C | 22 | C | 28 |
| | <u> </u> | | <u> </u> |
| Result | 44 | Result | 56 |
| | 76 | | 76 |
| | <u> </u> | | <u> </u> |
| Error of defect | —32 | Error of defect | —20 |

Pos. Er.
 $6 \times 20 = 120$
 $9 \times 32 = 288$
 L.
 12)168(14 A

L.
 A 14
 B 24
 C 38

 76 proof.

By the proportion,

As 12 : 3 :: 32 : 8. And $8 + 6 = 14$ } A's share,
 Or, As 12 : 3 :: 20 : 5. And $5 + 9 = 14$ } as before.

It is obvious, that the fourth numbers here must be added to the positions, because the results were both too small.

3. A labourer having threshed out 40 quarters of grain, part wheat, part barley, received, in name of wages, 28 s. or 336 d. being paid at the rate of 12 d. for every quarter of wheat, and 6 d. for every quarter of barley: How many of the 40 quarters were wheat, and how many were barley?

| | <i>Q.</i> | <i>d.</i> |
|------------------------|-----------|---------------|
| <i>Position 1.</i> | | |
| Wheat 10, at 12 d. is, | 10 | 120 |
| Barley 30, at 6 d. is, | 30 | 180 |
| | | <u> </u> |
| Result | | 300 |
| | | 336 |
| | | <u> </u> |
| Error of defect | | —36 |

Position

Position 2. $\begin{array}{r} Q. \\ \text{Wheat } 26, \text{ at } 12 \text{ d. is, } 312 \\ \text{Barley } 14, \text{ at } 6 \text{ d. is, } 84 \end{array}$

$\begin{array}{r} \text{Result } 396 \\ 336 \end{array}$

Error of excess $+ 60$

Pos. Er.

$$10 \times 60 = 600$$

$$26 \times 36 = 936$$

$96 \mid 1536$ (16 quarters of wheat.

$\begin{array}{r} 96 \cdot \\ 576 \end{array}$

$\begin{array}{r} 576 \end{array}$

$\begin{array}{r} 576 \end{array}$

(0)

$\begin{array}{r} Q. \end{array}$

Wheat 16, at 12 d. is, 192

Barley 24, at 6 d. is, 144

Proof 336

By the proportion.

$$\text{As } 96 : 16 :: 36 : 6$$

$$\text{Or, As } 96 : 16 :: 60 : 10$$

$$\text{And } 6 + 10 = 16 \text{ } \} \text{ quarters of wheat,}$$

$$\text{And } 26 - 10 = 16 \text{ } \} \text{ as before.}$$

The fourth number of the first proportion is added to the position, because the result was too little. But the fourth number of the second proportion is subtracted from the position, the result being too great.

4. What number is that whereof $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, make 19?

1 pos.

1 pos. 30.

$$\begin{array}{r} \frac{1}{2} = 10 \\ \frac{1}{4} = 7.5 \\ \frac{1}{3} = 6 \\ \frac{1}{6} = 5 \\ \hline \end{array}$$

Result 28.5

19

Error + 9.5

2 pos. 10.

$$\begin{array}{r} \frac{1}{2} = 3.33 \\ \frac{1}{4} = 2.5 \\ \frac{1}{3} = 2. \\ \frac{1}{6} = 1.66 \\ \hline \end{array}$$

Result 9.50

19.

Error - 9.5

Pos. Er.

$$30 \times 9.5 = 285$$

$$10 \times 9.5 = 95$$

19) 380 (20 *Ans.*

$$\frac{1}{2} \text{ of } 20 = 6.6$$

$$= 5$$

$$= 4$$

$$= 3.3$$

Proof 19

Hence observe, that any question, such as this, that properly belongs to single position, may also be solved by double position.

Again, observe, that when the errors are equal and unlike, half the sum of the positions is the answer, or number sought.

5. A, B, and C, discoursing of their age, A affirmed, that he was 18 years old; B said, his age was equal to that of A, and half the age of C; and C affirmed, he was as old as both A and B: What was the age of each person? *Ans.* A 18, B 54, and C 72.

6. A father dying, left to his three sons, A, B, C, his estate in money, dividing it as follows, *viz.* To A he gave the half, wanting 44 l.; to B he gave a third, and 14 l. over; and to C he gave the remainder, which was 82 l. less than the share of B: What was the sum left, and what was each son's share? *Ans.* The sum left was 588 l. whereof A had 250 l. B 210 l. and C 128 l.

7. A and B had each a certain number of crowns, and said A to B, If you give me one of your crowns, I shall then have five times as many as you have behind; but said B to A, If you give me one of your crowns, then

then shall each of us have an equal number: How many crowns had each person? *Ans.* A had 4, and B 2.

8. A gentleman had two horses, Chestnut and Swift, and a saddle worth 50 l. which set on the back of Chestnut, makes his value double that of Swift; but the saddle set on the back of Swift, makes his value triple that of Chestnut: What was the value of each horse? *Ans.* Chestnut 30 l. and Swift 40 l.

9. The money staked by A, B, and C, playing at Hazard, was 324 crowns; but they happening to disagree, each seized as many of the crowns as he could: A got a number unknown; B as many as A, and 15 over; C got a fifth part of both their sums added together: How many crowns got each? *Ans.* A 127½, B 142½, and C 54.

10. A gentleman caught a fish, whose head was 6 inches long, the tail as long as the head and half the body, the body was just the length of the head and tail: What was the length of the fish, what the length of the body, and what the length of the tail? *Ans.* Length of the fish 48 inches, length of the body 24 inches, and length of the tail 18 inches.

11. When first the marriage-knot was tied

Betwixt my wife and me,

My age did hers as far exceed,

As three times three does three;

But after ten and half ten years,

We man and wife had been,

Her age came up as near to mine,

As eight is to sixteen,

Now, Tyro skill'd in numbers, say,

What were our ages on the wedding-day?

Answer.

Sir, Forty-five years you had been,
Your bride no more than just fifteen.

C H A P. X.

EXTRACTION of ROOTS.

IF unity be multiplied continually by any given number, the products thence arising are called *powers of that number*; and the given number is called *the root, or first power*.

Thus, if 2 be the given number, then $1 \times 2 = 2$ is the root or first power; and $2 \times 2 = 4$ is the square or second power; and $4 \times 2 = 8$ is the cube or third power; and $8 \times 2 = 16$ is the biquadrate or fourth power; and $16 \times 2 = 32$ is the sursolid or fifth power; and $32 \times 2 = 64$ is the sixth power, or cube squared, &c.

The natural numbers, 1, 2, 3, &c. are sometimes placed over these powers, denoting the number of multiplications used in producing them, or showing what powers they are; and are called *indices or exponents*, as in the following scheme.

| | | | | | | | | | |
|----------|----|----|----|----|-----|-----|-----|------|-----|
| Indices, | 0, | 1, | 2, | 3, | 4, | 5, | 6, | 7, | &c. |
| Powers, | 1, | 2, | 4, | 8, | 16, | 32, | 64, | 128, | &c. |

The raising any root or number given to any power required, is called *involution*; and is performed by multiplying the given root into unity continually, as taught above. But the finding the root of a given power is called *evolution*, or *extraction of roots*.

If the root of any power not exceeding the seventh power, be a single digit, it may be obtained by inspection, from the following table of powers.

TABLE.

T A B L E

| 1st power or
root. | 2d power or
square. | 3d power or
cube. | 4th power or
biquadrate. | 5th power or
surfold. | 6th power or
cube squa
red. | 7th power. |
|-----------------------|------------------------|----------------------|-----------------------------|--------------------------|-----------------------------------|------------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 |
| 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 |
| 6 | 36 | 216 | 1296 | 7776 | 46656 | 279936 |
| 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 |
| 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 |
| 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 |

I. *Extraction of the square root.*

R U L E S.

1. Divide the given number into periods of two figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of hundreds, and point toward the right. Every period will give one figure in the root.

2. Find by the table of powers, or by trial, the nearest lesser root of the left-hand period, place the figure so found in the quot, subtract its square from the said period, and to the remainder bring down the next period for a dividuall or resolvend.

3. Double the quot for the first part of the divisor; inquire how often this first part is contained in the whole resolvend, excluding the units place; and place the figure denoting the answer both in the quot and on the right of the first part; and you have the divisor complete.

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T

4. Multiply

4. Multiply the divisor thus completed by the figure put in the quot, subtract the product from the resolvend, and to the remainder bring down the following period for a new resolvend, and then proceed as before.

Note 1. If the first part of the divisor, with unity supposed to be annexed to it, happen to be greater than the resolvend, in this case place 0 in the quot, and also on the right of the partial divisor; to the resolvend bring down another period; and proceed to divide as before.

Note 2. If the product of the quotient-figure into the divisor happen to be greater than the resolvend, you must go back, and give a lesser figure to the quot.

Note 3. If, after every period of the given number is brought down, there happen at last to be a remainder, you may continue the operation, by annexing periods or pairs of ciphers till there be no remainder, or till the decimal part of the quot repeat or circulate, or till you think proper to limit it.

E X A M P L E I.

Required the square root of 133225.

| | | | |
|---------------|--------|------------|---------------|
| Square number | 133225 | (365 root | 365 |
| | 9 | | 365 |
| 1 div. 66) | 432 | resolvend. | 1825 |
| | 396 | product. | 2190 |
| 2 div. 725) | 3625 | resolvend. | 1095 |
| | 3625 | product. | 133225 proof. |

E X A M P L E II.

Required the square root of 54990 $\frac{1}{4}$.

Square

Square number 54990.25 (234.5 root.)

$$\begin{array}{r}
 4 \\
 \hline
 1 \text{ div. } 43) 149 \text{ resolvend.} \\
 \quad 129 \text{ product.} \\
 \hline
 2 \text{ div. } 464) 2090 \text{ resol.} \\
 \quad 1856 \text{ prod.} \\
 \hline
 3 \text{ div. } 4685) 23425 \text{ resol.} \\
 \quad 23425 \text{ prod.} \\
 \hline
 \end{array}$$

EXAMPLE III.

Required the square root of 72, to eight decimal places.

$$\begin{array}{r}
 72.00000000 (8.48528137 \text{ root.}) \\
 64 \\
 \hline
 164) 800 \\
 \quad 656 \\
 \hline
 1688) 14400 \\
 \quad 13504 \\
 \hline
 16965) 89600 \\
 \quad 84825 \\
 \hline
 169702) 477500 \\
 \quad 339404 \\
 \hline
 169704) 138096 \\
 \quad 135763 \\
 \hline
 \quad \quad 2333 \\
 \quad \quad 1697 \\
 \hline
 \quad \quad \quad 636 \\
 \quad \quad \quad 509 \\
 \hline
 \quad \quad \quad 127 \\
 \quad \quad \quad 118 \\
 \hline
 \quad \quad \quad (9)
 \end{array}$$

After getting half of the decimal places, I work by contracted division for the other half; and obtain them with the same accuracy as if the work had been at large.

EXAMPLE IV.

Required the square root of .2916

$$\begin{array}{r}
 \dot{.}291\dot{6}(.54 \text{ root.}) \\
 \underline{25} \\
 104) 416 \\
 \underline{416}
 \end{array}$$

EXAMPLE V.

Required the square root of .001225

$$\begin{array}{r}
 \dot{.}00122\dot{5}(.035 \text{ root}) \\
 \underline{9} \\
 65) \dots 325 \\
 \underline{325}
 \end{array}$$

EXAMPLE VI.

Required the square root of 5.875

$$\begin{array}{r}
 \dot{.}587\dot{5}(2.423 \text{ root.}) \\
 \underline{4} \\
 44) 187 \\
 \underline{176} \\
 482) 1150 \\
 \underline{964} \\
 4843) 18600 \\
 \underline{14529} \\
 (4071)
 \end{array}$$

The

The operation may be continued further by annexing to the remainder more periods of ciphers.

$$\begin{array}{r}
 2.423 \\
 2.423 \\
 \hline
 7269 \\
 4846 \\
 9692 \\
 4846 \\
 \hline
 5.870929 \\
 4071 \text{ rem.} \\
 \hline
 5.875000 \text{ proof.}
 \end{array}$$

EXAMPLE VII.

Required the square root of the repetend .x

.x1(.3333 &c. root.

$$\begin{array}{r}
 9 \\
 \hline
 63) 211 \\
 189 \\
 \hline
 663) 2211 \\
 1989 \\
 \hline
 6663) 22211 \\
 19989 \\
 \hline
 \end{array}$$

Here, instead of annexing pairs of ciphers to the remainders, I annex periods of the repetend.

222211 and so on to infinity.

EXAMPLE VIII.

Required the square root of the repetend .4

$\sqrt{44} (6.6 \text{ &c. root.})$

$$\begin{array}{r} 36 \\ \hline 126) 844 \\ \quad 756 \\ \hline 1326) 8844 \\ \quad 7956 \\ \hline \quad \quad 88844 \end{array}$$

Note. No repeating digit, except . x and . y have for their roots a pure single repetend.

E X A M P L E IX.

Required the square root of $1320.x$

$\sqrt{1320.x} (36.33 \text{ &c. root.})$

$$\begin{array}{r} 9 \\ \hline 66) 420 \\ \quad 396 \\ \hline 723) 2411 \\ \quad 2169 \\ \hline 7263) 24211 \\ \quad 21789 \\ \hline \quad \quad 242211 \end{array}$$

E X A M P L E X.

Required the square root of the circulate $138.518;$;
or, which is the same thing, of $13,8.51,$

$\sqrt{13,8.51}$

$\dot{1}3.\dot{8}.\dot{5}\dot{1},(11.769389 \text{ root.}$

$$\begin{array}{r}
 \text{I} \\
 \hline
 21) \quad 38 \\
 \quad 21 \\
 \hline
 227) 1751 \\
 \quad 1589 \\
 \hline
 2346) 16285 \\
 \quad 14076 \\
 \hline
 23529) 220918 \\
 \quad 211761 \\
 \hline
 23538) 9157 \\
 \quad \dots 7061 \\
 \hline
 \quad 2096 \\
 \quad 1883 \\
 \hline
 \quad 213 \\
 \quad 211 \\
 \hline
 \quad (2)
 \end{array}$$

Here I annex to the remainders periods of the circulating figures.

I continue the work till I have three decimal places in the root ; and then, by contracted division, I obtain three more.

If the square root of a vulgar fraction be required, find the root of the given numerator for a new numerator, and find the root of the given denominator for a new denominator. Thus, the square-root of $\frac{4}{9}$ is $\frac{2}{3}$, and the root of $\frac{16}{25}$ is $\frac{4}{5}$; and thus the root of $\frac{25}{4}$ ($= 6\frac{1}{4}$) is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Example 4. and 5.

The extraction of roots may be conducted in the way of vulgar fractions ; but then the operation becomes involved and troublesome. The following example done both ways will shew the superior excellence of decimals.

Required the square root of $20\frac{1}{4}$.

By

By vulgar fractions.

 $20\frac{1}{4}(4\frac{1}{2} \text{ root.})$

$$\begin{array}{r} 16 \\ \hline 8\frac{1}{2})4\frac{1}{2} \\ 4\frac{1}{2} \\ \hline (0 \end{array}$$

Decimally.

 $20.25(4.5$

$$\begin{array}{r} 16 \\ \hline 85)425 \\ 425 \\ \hline (0 \end{array}$$

The method of extracting the square root is founded on Euclid, book 2. prop. 4. where it is demonstrated, That if any line, and consequently any number, be divided into two parts, the square of the whole line or number will be equal to the squares of the two parts, together with twice a rectangle or product under the parts. Thus, if the number 10 be divided into two parts, suppose 6 and 4, the square of 10 will be equal to $6 \times 6 = 36 + 4 \times 4 = 16$, + twice $6 \times 4 = 48$; for $36 + 16 + 48 = 100 = 10 \times 10$.

In order to show the reason of the rules, we must have recourse to algebra, in which a letter, as a , is put to represent any number at pleasure; and aa denotes the square of a , and aaa its cube; ay denotes a product, and $2ay$ denotes a double product; and the factor into which any letter is multiplied, is called its *coefficient*.

Now, let the first two periods of the given square number be considered as a square number by themselves; and as every period gives a figure to the root, their root will consist of two figures, or two parts; call the first a , and the other y ; then $a + y$ is their root complete; and by Euclid ii. 4, or by algebraic multiplication, $aa + 2ay + yy$ will be equal to the first two periods of the square number given; and when aa , the square of the first part of the root, is subtracted from the first period, there will remain $2ay + yy$; and to find y , the other part of the root, I divide by its factor or coefficient, *viz.* by $2a + y$; that is, I find a divisor by doubling the quot, and annexing to it the next quotient-figure.

N. B. The figure of the quot denoted by a , in regard another figure is to follow, is considered as in the place of

of tens, having a cipher on the right, and consequently so has the first part of the divisor; and therefore the annexing the quotient-figure to the said partial divisor is the same thing in this case as adding it.

If there be more periods of the given square number, the two figures now in the quot are to be esteemed the first part of the root, and called a ; and by repeating the operation, as above directed, the other part y is found. And if there be still more periods, call the three figures now in the quot a , then repeat the operation, and find y as before. And thus proceed till every period is brought down.

M O R E E X A M P L E S.

1. There are two numbers, whereof the lesser is 3456, their difference is 293392: What is the greater, and what the square root of their sum? *Ans.* The greater is 296848, and the square root of their sum is 548.

2. There is a round floor, whose content is, 45.938271604 square feet, or 45 feet 11 inches 3 lines and 4 fourths: Required the side of a square floor equal to it in area? *Ans.* 6.7 feet, or 6 feet 9 inches 4 lines. See duodecimals in decimal practice, Ex. 3.

3. There are three fields, whereof the first contains 100 acres 3 roods and 36 poles; the second contains 118 acres 2 roods and 24 poles; and the third contains 122 acres 2 roods and 20 poles: Required the side of a square field that shall be equal to all the three in area? *Ans.* The side of the square field is 234 poles nearly, or 58 chains and 2 poles.

4. A round malt-kiln, whose diameter is 12 feet, is to be enlarged, so as to hold three times as much malt as formerly: Required the new diameter? *Ans.* 20.78 feet. The areas of circles are to one another as the squares of their diameters, by Euclid xii. 2.; multiply therefore the square of the given diameter by 3, and extract the square root of the product.

5. A tower is surrounded by a ditch 40 feet wide, and a scaling ladder 50 feet long will reach from the

outside of the ditch to the top of the tower: Required the height of the tower? *Ans.* 30 feet. The square of the hypotenuse, or longest side of a right-angled triangle, is equal to the sum of the squares of the other two sides, by Euclid i. 47.; and therefore, from the square of 50 subtract the square of 40, and the square root of the remainder is the height of the tower.

6. Three towns, A, B, C, are so situate, that B lies 80 miles south of A, and C 60 miles west of A: What is the distance between B and C? *Ans.* 100 miles.

II. *Extraction of the cube root.*

R U L E S.

1. Divide the given number into periods of three figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of thousands, and point toward the right. The number of periods shews the number of figures in the root.

2. Find by the table of powers, or by trial, the nearest lesser root of the left-hand period; place the figure so found in the quot; subtract its cube from the said period; and to the remainder bring down the next period for a dividend or resolvend.

The divisor consists of three parts, which may be found, as follows.

3. The first part of the divisor is found thus: Multiply the square of the quot by 3, and to the product annex two ciphers; then inquire how often this first part of the divisor is contained in the resolvend, and place the figure denoting the answer in the quot.

4. Multiply the former quot by 3, and the product by the figure now put in the quot; to this last product annex a cipher; and you have the second part of the divisor. Again, square the figure now put in the quot for the third part of the divisor; place these three parts under one another, as in addition; and their sum will be the divisor complete.

5. Multiply

5. Multiply the divisor, thus completed, by the figure last put in the quot, subtract the product from the resolvend, and to the remainder bring down the following period for a new resolvend, and then proceed as before.

Note 1. If the first part of the divisor happen to be equal to or greater than the resolvend, in this case, place 0 in the quot, annex two ciphers to the said first part of the divisor, to the resolvend bring down another period, and proceed to divide as before,

Note 2. If the product of the quotient-figure into the divisor happen to be greater than the resolvend, you must go back, and give a lesser figure to the quot.

Note 3. If after every period of the given number is brought down, there happen at last to be a remainder, you may continue the operation by annexing periods of three ciphers till there be no remainder, or till you have as many decimal places in the root as you judge necessary.

EXAMPLE I.

Required the cube root of 12812904.

Cube number 12812904 (234 root,
8

| | | |
|---------------|---|------------------|
| 1st part 1200 | } |)4812 resolvend. |
| 2d part 180 | | |
| 3d part 9 | | |

1 divisor 1389 $\times 3 = 4167$ product.

| | | |
|-----------------|---|--------------------|
| 1st part 158700 | } |)645904 resolvend. |
| 2d part 2760 | | |
| 3d part 16 | | |

2 divisor 161476 $\times 4 = 645904$ product.

U 2

PROOF.

P R O O F.

| | |
|--------------|---------------|
| 234 | Square 54756 |
| 234 | <u>234</u> |
| <u>936</u> | 219024 |
| 702 | 164268 |
| <u>468</u> | <u>109512</u> |
| Square 54756 | Cube 12812904 |

The reason of the rules will appear, if we take the first two periods of the given cube number, and consider them as a cube number by themselves, whose root will consist of two figures or parts, which call a and y ; and then, by algebraic multiplication, the first two periods of the cube number will be equal to $aaa + 3aay + 3ayy + yyy$. Now, when aaa , the cube of the first part of the root, is subtracted from the left-hand period, there remains $3aay + 3ayy + yyy$; and to find y , the other part of the root, I divide by its coefficient, *viz.* by $3aa + 3ay + yy$; and as the divisor consists of three parts, I begin with the first part, *viz.* $3aa$, and try how often it is contained in the resolvend; and then, by the help of the figure that goes to the quot, I make up the other two parts, as follows.

$$\begin{array}{lcl}
 \text{1st part } 1200 & = & 3aa = 3 \times 2 \times 2 \\
 \text{2d part } 180 & = & 3ay = 3 \times 2 \times 3 \\
 \text{3d part } 9 & = & yy = 3 \times 3 \\
 \hline
 \text{1 divisor } 1389
 \end{array}$$

The reason of annexing one cipher for every a , *viz.* two ciphers in the first part of the divisor, and one in the second, is because the first figure of the root 2, another figure being to follow on the right, is really and truly 20.

If, as in the above example, there be more periods of the given cube number, the two figures now in the quot are to be esteemed the first part of the root, and called

a ;

a ; and by repeating the operation for a new divisor, the other part y , may be found. Thus,

$$\begin{array}{rcl} \text{1st part } 158700 & = & 3aa = 3 \times 23 \times 23 \\ \text{2d part } 2760 & = & 3ay = 3 \times 23 \times 4 \\ \text{3d part } 16 & = & yy = 4 \times 4 \\ \hline \text{2 divisor } 161476 & & \end{array}$$

In the first and second part of the divisor, the ciphers are annexed for the reason assigned above.

If there be still more periods in the cube number, call the three figures now in the quot a , repeat the operation, and find y as before; and thus proceed till every period is brought down.

The reason of dividing the given number into periods of two figures in the square, of three in the cube, of five in the sursolid, &c. and of there being as many figures in the root as there are periods, arises from the nature of involution; and will appear by resolving any number into its constituent parts, and raising them separately to the second, third, fourth, fifth power, &c. Let the number be $7842 = 7000 + 800 + 40 + 2$. Now, 7 has three places after it; but in the square it will have six, and in the cube nine. Again, 8 has two places after it; but in the square it will have four, and in the cube six, &c.

$$\begin{array}{r} \text{Thus } 7\ 000 \\ \hline 7\ 000 \\ \hline \text{Square } 49,00,00,00 \\ \hline 7\ 000 \\ \hline \end{array}$$

$$\text{Cube } 343,000,000,000$$

$$\begin{array}{r} \text{And thus } 8\ 00 \\ \hline 8\ 00 \\ \hline \text{Square } 64,00,00 \\ \hline 8\ 00 \\ \hline \end{array}$$

$$\text{Cube } 512,000,000$$

Hence it is plain, that the square of 7842 will contain four periods of two figures, and its cube will contain four periods of three figures; and so the number of periods will be equal to the number of figures in the root. And the square and cube of 842 will in like manner contain three periods each.

E X.

EXAMPLE II.

Required the cube root of $28\frac{3}{4}$.

$$\begin{array}{r}
 28.750000(3.06 \text{ root,} \\
 27 \\
 \hline
) 1750000 \text{ resolv.} \\
 \begin{array}{l} 270000 \\ 5400 \\ 36 \end{array} \}
 \end{array}$$

$$\begin{array}{r}
 \text{Div. } 275436 \times 6 = 1652616 \text{ prod.} \\
 \hline
 97384 \text{ rem.}
 \end{array}$$

P R O O F.

$$\begin{array}{r}
 3.06 \\
 3.06 \\
 \hline
 1836 \\
 918 \\
 \hline
 \text{Sq. } 9.3636
 \end{array}$$

$$\begin{array}{r}
 \text{Sq. } 9.3636 \\
 3.06 \\
 \hline
 561816 \\
 280908 \\
 \hline
 28.652616 \\
 97384 \text{ rem.} \\
 \hline
 28.750000 \text{ cube,}
 \end{array}$$

EXAMPLE III.

Required the cube root of .000485613.

$$\begin{array}{r}
 .000485613(.078 \text{ root,} \\
 343 \\
 \hline
) \dots 142613 \text{ resolv.} \\
 \begin{array}{l} 14700 \\ 1680 \\ 64 \end{array} \}
 \end{array}$$

$$\begin{array}{r}
 \text{Div. } 16444 \times 8 = 131552 \text{ prod.} \\
 \hline
 11061 \text{ rem.}
 \end{array}$$

EX-

EXAMPLE IV.

Required the cube root of 7584.

$$\begin{array}{r}
 \begin{array}{l}
 300 \\
 270 \\
 81
 \end{array} \left. \vphantom{\begin{array}{l} 300 \\ 270 \\ 81 \end{array}} \right\} \begin{array}{l} 7584(19.64 \text{ root.} \\ \hline 1 \\ \hline \end{array} \\
 \hline
 1 \text{ div. } 651 \times 9 = 5859 \text{ prod.} \\
 \hline
 \begin{array}{l}
 108300 \\
 3420 \\
 36
 \end{array} \left. \vphantom{\begin{array}{l} 108300 \\ 3420 \\ 36 \end{array}} \right\} \begin{array}{l})6584 \text{ resolv.} \\ \\ \\ \hline \end{array} \\
 \hline
 2 \text{ div. } 111756 \times 6 = 670536 \text{ prod.} \\
 \hline
 \begin{array}{l}
 11524800 \\
 23520 \\
 16
 \end{array} \left. \vphantom{\begin{array}{l} 11524800 \\ 23520 \\ 16 \end{array}} \right\} \begin{array}{l})725000 \text{ resolv.} \\ \\ \\ \hline \end{array} \\
 \hline
 3 \text{ div. } 11548336 \times 4 = 46193344 \text{ prod.} \\
 \hline
 \text{Rem. } 8270656
 \end{array}$$

If the cube root of a vulgar fraction be required, find the cube root of the given numerator for a new numerator, and the cube root of the given denominator for a new denominator. Thus, the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, and the cube root of $\frac{27}{64}$ is $\frac{3}{4}$; and thus the cube root of $\frac{125}{8}$ ($= 15\frac{5}{8}$) is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Ex. 3.

MORE EXAMPLES.

1. There are two numbers, whereof the greater is 2579890752, their difference is 1152: What is the lesser, and what the cube root of their sum? *Ans.* The lesser

leffer is 2579889600, and the cube root of their sum is 1728.

2. The content of an oblong cellar is 1953.125 cubic feet: Required the side of a cubical cellar that shall contain just as much. *Ans.* 12.5 feet.

3. There are three boxes, the content of one is 10000 solid inches, of another 16656, and of the third 20000: Required the side of a cubical box that shall contain as much as all the three. *Ans.* 36 inches, or 3 feet.

4. Required the side of a cubical vessel that will contain 80 wine gallons. *Ans.* 26.43 inches.

231 cubic inches in a wine gallon.

80

18480 whose cube root is 26.43 inches.

5. A ship's length is 50 feet, breadth 20, and depth of the hold 10: Required the dimensions of a like vessel triple her burthen.

L. 50, its cube $125000 \times 3 = 375000$, its cube root 71.54

B. 20, its cube $8000 \times 3 = 24000$, its cube root 29.24

D. 10, its cube $1000 \times 3 = 3000$, its cube root 14.44

6. A brass bullet of 5 inches diameter weighs 20 lb.: What is the diameter of a like bullet that weighs 160 lb.?

Ans. 10 inches.

lb.

lb.

As 20 : 5 × 5 × 5 :: 160

4 : 5 × 5 :: 160

1 : 25 :: 40 : 1000, whose cube root is 10 inches.

III. Extraction of the biquadrate root.

R U L E.

Extract the square root of the given number; and again

gain extract the square root of the root so found, and the last of these roots is the root sought.

E X A M P L E.

Required the biquadrate root of 5308416

$$\begin{array}{r}
 \begin{array}{r}
 \overset{\cdot}{5}\overset{\cdot}{3}\overset{\cdot}{0}\overset{\cdot}{8}\overset{\cdot}{4}\overset{\cdot}{1}\overset{\cdot}{6} \\
 \underline{4} \\
 43)130 \\
 \underline{129} \\
 4604)18416 \\
 \underline{18416}
 \end{array}
 \quad
 \begin{array}{r}
 \overset{\cdot}{2}\overset{\cdot}{3}\overset{\cdot}{0}\overset{\cdot}{4} \text{ (48 root.)} \\
 \underline{16} \\
 88)704 \\
 \underline{704}
 \end{array}
 \end{array}$$

If, in the first extraction, there happen to be a remainder, continue the operation, by annexing pairs of ciphers, till you have twice as many decimal places in the square or first root, as you propose to have in the last root.

IV. Extraction of the root of the fifth power, or sur-solid.

R U L E S.

1. Divide the given number into periods of five figures, find the nearest lesser root of the left-hand period, put the figure so found in the quot, subtract its fifth power, and to the remainder bring down the next period for a resolvend.

2. Put $a + y$ for the root, and then the sur-solid or fifth power will be $aaaaa + 5aaaay + 10aaayy + 10aayyy + 5ayyyy + yyyyy$. Now, $aaaaa$ being already subtracted, there remains the other five parts; and to find y I divide by its coefficient, viz. by $5aaaa + 10aaay + 10aayy + 5ayyy + yyyy$; that is, I try how often $5aaaa$ is contained in the resolvend; and, by the help of the

quotient-figure, I make up the other four parts of the divisor.

E X A M P L E

Required the sursolid root of 33554432

$$\begin{array}{r}
 33554432 \text{ (32 root.} \\
 243 \\
 \hline
) 9254432 \text{ resolv.} \\
 4050000 = 5aaaa \\
 540000 = 10aaay \\
 36000 = 10aayy \\
 1200 = 5ayyy \\
 16 = yyyy \\
 \hline
 \end{array}$$

Divisor $4627216 \times 2 = 9254432$ prod.

(0)

V. Extraction of the root of the sixth power, or cube squared.

R U L E.

Extract the square root of the given number, and then extract the cube root of that root, the last is the root sought. Or, first extract the cube root, and then extract the square root of that root.

E X A M P L E.

Required the root of 191102976, being the sixth power.

191102976

$$\begin{array}{r}
 191102976 \quad (13824 \text{ root.}) \\
 \begin{array}{r}
 \overline{1} \qquad \qquad \overline{8} \\
 23) \cdot 91 \qquad 1200) 5824 \text{ resolv.} \\
 \underline{69} \qquad \quad 240 \\
 268) 2210 \qquad \underline{16} \\
 \underline{2144} \qquad 1456 \times 4 = 5824 \text{ prod.} \\
 2762) \quad 6629 \qquad \qquad \qquad (9) \\
 \underline{5524} \\
 27644) 110576 \\
 \underline{110576}
 \end{array}
 \end{array}$$

VI. Extraction of the root of the seventh power.

R U L E.

Put $a + y$ for the root, and the seventh power will be $aaaaaaa + 7aaaaaay + 21aaaaaayy + 35aaaaayyy + 35aaayyyyy + 21aayyyyyy + 7ayyyyyyy + yyyyyyy$, by the aid of which proceed as in extracting the root of the fifth power.

E X A M P L E.

Required the root of 3404825447, being the seventh power.

X 2

3404825447

3404825447 (23 root.
128

2124825447 resolv.

448000000 = 7aaaaaa

201600000 = 21aaaaay

50400000 = 35aaaayy

7560000 = 35aaayyy

680400 = 21aayyyy

34020 = 7ayyyyyy

729 = yyyyyy

Divisor 798275149 $\times 3 = 2124825447$ prod.

(o)

VII. Extraction of the root of the eighth power.

R U L E.

Extract the square root of the given number continually till you have three roots; the last of these is the root sought.

Thus, let 1785793904896 be the eighth power; by extracting the square root you get the biquadrate or fourth power, viz. 1336336; and by extracting the square root of the biquadrate, you get the square or second power, viz. 1156, whose square root is 34, the root sought.

VIII. Extraction of the root of the ninth power.

R U L E.

Extract the cube root of the given number, and you have the cube or third power, whose cube root is the root sought.

Thus, let 5159780352 be the ninth power; by extracting the cube root you get the cube or third power, viz. 1728, whose cube root 12 is the root sought.

Universally, whatever the given power be, put $a + y$ for

for the root, and by involution raise $a + y$ to the power of the given number; then, with this as your guide or canon, extract the root in the manner prescribed and exemplified in the extraction of the root of the fifth and seventh powers.

But if the index of the given power be a multiple of 2, the work may be rendered easier: for by extracting the square root of the given number, you obtain a power whose index is one half of the index of the given power. Thus, by extracting the square root of the tenth power, you have the fifth power; and the square root of the twelfth power is the sixth power, &c.

Again, if the index of the given power be a multiple of 3, by extracting the cube root you obtain a power whose index is one third of the index of the power given. Thus the cube root of the ninth power is the cube or third power; and the cube root of the twelfth power is the biquadrate or fourth power, &c.

Involution is directly contrary to extraction or evolution; and therefore, if a square number be squared, it will give the biquadrate or fourth power; and if a biquadrate be squared, it will give the eighth power. Again, if a cube number be cubed, it will give the ninth power; and if the biquadrate be cubed, it will give the twelfth power.

CHAP. XL

PROGRESSION.

THE comparing of numbers with one another is called *Comparative Arithmetic*; and in comparing any two numbers together, the first number, or the number compared, is called the *antecedent*, and the other number with which it is compared, is called the *consequent*.

In comparing two numbers, we may, by subtracting the lesser from the greater, find their difference; and the difference thus found is called the *arithmetical ratio*

ratio of the two numbers; and if two or more pairs of numbers have all the same ratio or difference, such numbers are said to be in *arithmetical proportion*; and the ratio in reference to the several proportional numbers, is usually called the *common difference*; the first and last term are called *extremes*, and the intermediate ones are denominated *means*.

If a rank of numbers, without interruption, have all the same difference, the proportion is said to be *continued* or *continual*; and such a rank is called an *arithmetical progression*, or an *arithmetical series*; such as, 2, 4, 6, 8, 10, 12.

But if, in the rank of proportional numbers, there be a breach or chasm, the proportion is said to be *discontinued*, *disjunct*, or *interrupted*; such as, 3, 5, 7, — 19, 21, 23.

Again, in comparing two numbers, we may find how often the one contains the other, by dividing the antecedent by the consequent; and the quotient thence arising is called the *geometrical ratio* of the two numbers; and if two or more pairs of numbers have all the same ratio, such numbers are said to be in *geometrical proportion*. The first and last of the proportional numbers are called *extremes*, and the other terms are denominated *means*.

If a rank of numbers, without breach or chasm, have all the same ratio, the proportion is said to be *continued* or *continual*; and such a rank is called a *geometrical progression*, or a *geometrical series*; such as, 2 : 4 : 8 : 16 : 32.

But if, in the rank of proportional numbers, there be a chasm, the proportion is said to be *discontinued*, *disjunct*, or *interrupted*; such as, 12 : 6 :: 30 : 15.

I. Arithmetical Progression.

An arithmetical progression, or series, is formed either from 0, or from some number assumed as the first term, by continual addition or subtraction; and is either increasing or decreasing.

Thus, 0, 3, 6, 9, 12, 15, 18, is an arithmetical progression

gression or series, formed from 0, assumed as the first term, and increasing by the continual addition of 3; and such a series may be continued upward to infinity.

And 14, 12, 10, 8, 6, 4, 2, is an arithmetical progression, or series, formed from 14, assumed as the first term, and decreasing by the continual subtraction of 2; but such a series can be continued downward only till the last term becomes equal to or less than the common difference.

In an arithmetical progression, or series, five things occur to be considered.

- I. The least term.
 - II. The greatest term.
 - III. The number of terms.
 - IV. The common difference.
 - V. The sum of all the terms.
- } Extremes.

The properties of numbers in arithmetical progression are various and numerous; but the chief, or most useful, which alone we propose to take notice of in this place, are explained in the following theorems.

THEOREM I.

Any term of an arithmetical series is equal to the sum of the least term, and the product of the common difference into the number of terms before or after it.

Thus, in the increasing series, 2, 5, 8, 11, 14, 17, the term $17 = 2 + 3 \times 5 = 2 + 15$.

Again, in the decreasing series, 18, 16, 14, 12, 10, 8, 6, 4, 2, the term $18 = 2 + 8 \times 2 = 2 + 16$.

The reason is plain; because every term subsequent or prior to the least is made up by the continual addition or subtraction of the common difference.

Hence the product of the common difference into the number of terms minus unity, added to the least term gives the greatest, or subtracted from the greatest term leaves the least.

THEO.

THEOREM II.

In an arithmetical series, consisting of three, five, or any odd number of terms, the double of the middle term is equal to the sum of the two extremes, or to the sum of any two means equally distant from the said middle term.

Thus, in the series, 1, 2, 3, 4, (5), 6, 7, 8, 9, the middle term 5 doubled is $10 = 1 + 9$, the two extremes, or $= 2 + 8$, or $3 + 7$, or $4 + 6$.

The reason is obvious: for of two terms equally distant from the middle term, the one exceeds the middle term just as much as the other is below it.

COROLLARIES.

1. Hence an arithmetical mean betwixt two given extremes is found by taking half the sum of the extremes. Thus, the mean betwixt 6 and 10 is 8; for $6 + 10 = 16$, and $2)16(8$. So 6, 8, 10, are in arithmetical proportion.

2. Hence also, to an extreme and mean given, a third proportional, or the other extreme, is found by subtracting the given extreme from the mean doubled. Thus, to 3, 6, given, the third proportional is 9; for $2 \times 6 = 12$, and $12 - 3 = 9$: so 3, 6, 9, are in arithmetical proportion. In like manner, to 10, 7, given, the third proportional is 4; for $2 \times 7 = 14$, and $14 - 10 = 4$: so 10, 7, 4, are arithmetical proportionals.

THEOREM III.

In an arithmetical series, consisting of four, six, or any even number of terms, the sum of the extremes is equal to the sum of the two middle terms, or to the sum of any two means equally distant from the extremes.

Thus, in the series, 3, 5, 7, 9, 11, 13, the sum of $3 + 13 = 7 + 9$, or $= 5 + 11$.

The reason is plain: for if, betwixt the two middle terms in the series, a middle term be inserted, the sum of the extremes, as also the sum of every pair of equidistant means,

means, will, by Theorem II. be equal to the double of the inserted middle term; and things equal to one and the same thing are equal to one another.

COROLLARIES.

1. Hence, to an extreme and two means given, the other extreme, or fourth proportional, is found, by subtracting the given extreme from the sum of the two given means. Thus, to 4, 6, 8, the fourth proportional is 10; for $6 + 8 = 14$, and $14 - 4 = 10$; so 4, 6, 8, 10, are in arithmetical proportion. And this takes place though the proportion be disjunct. Thus, to 18, 14, — 6, the fourth proportional is 2; for $14 + 6 = 20$, and $20 - 18 = 2$; so 18, 14, — 6, 2, are in arithmetical proportion disjunct.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by subtracting the given mean from the sum of the two extremes. Thus, to 13, 9, 7, the other mean is 11; for $13 + 7 = 20$, and $20 - 9 = 11$; so 13, 11, 9, 7, are arithmetical proportionals. This also takes place, though the proportion be disjunct. Thus, to 9, 15, — 36, the other mean is 30; for $9 + 36 = 45$, and $45 - 15 = 30$; so 9, 15, — 30, 36, are in arithmetical proportion disjunct.

THEOREM IV.

In an arithmetical series, the difference of the extremes, divided by the common difference, gives a quot, which, increased by unity, is the number of terms.

Thus, in the series, 2, 4, 6, 8, 10, 12, 14, the difference of the extremes 2 and 14 is 12, and 12, divided by the common difference 2, quotes 6, and $6 + 1 = 7$, the number of terms.

The reason is obvious: for the difference of the extremes is made up by a repetition of the common difference as often as there are terms save one; that is, the difference of the extremes is a product of the common difference into the number of terms minus unity.

THEOREM V.

In an arithmetical series, the difference of the extremes, divided by the number of terms minus unity, quotes the common difference.

Thus, in the series, 3, 5, 7, 9, 11, 13, the difference of the extremes 3 and 13 is 10, and 10 divided by the number of terms minus unity, *viz.* 5, quotes 2, the common difference.

The reason is the same here as in the former theorem.

THEOREM VI.

In an arithmetical series, the sum of all the terms may be made out several ways, as follows.

1. The sum of the extremes, multiplied by the number of terms, gives a product, which divided by 2, quotes the sum of all the terms.

Thus, in the series, 3, 6, 9, 12, 15, 18, 21, the sum of 3 and 21 is 24; and $24 \times 7 = 168$; and $2)168(84$, the sum of all the terms.

The reason will appear by inverting the series, and adding each equidistant pair of means, as follows.

$$\begin{array}{r}
 3, \quad 6, \quad 9, \quad 12, \quad 15, \quad 18, \quad 21 \\
 21, \quad 18, \quad 15, \quad 12, \quad 9, \quad 6, \quad 3 \\
 \hline
 24, \quad 24, \quad 24, \quad 24, \quad 24, \quad 24, \quad 24
 \end{array}$$

Now, as the sum of every pair is 24, and the number of terms 7, it is plain, the product of 7 into 24 will be double the sum of the series.

2. The sum of the extremes multiplied by half the number of terms, gives a product equal to the sum of all the terms.

Thus, in the series, 12, 10, 8, 6, 4, 2, the sum of 12 and 2 is 14, and $14 \times 3 = 42$, the sum of all the terms.

The reason is the same as before.

3. The half sum of the extremes multiplied by the number of terms, gives a product equal to the sum of all the terms.

Thus,

Thus, in the series, 1, 4, 7, 10, 13, 16, 19, the half sum of 1 and 19 is 10, and $10 \times 7 = 70$, the sum of all the terms.

The reason is the same as above.

4. When a series consists of an odd number of terms, the middle term multiplied by the number of terms, gives a product equal to the sum of all the terms.

Thus, in the series, 3, 6, (9), 12, 15, the middle term $9 \times 5 = 45$, the sum of all the terms.

The reason is plain; because the middle term, by Theorem II. is equal to half the sum of the extremes.

5. In a series of the natural numbers, 1, 2, 3, 4, 5, &c. the greatest given term multiplied into the next greater, gives a product, which divided by 2, quotes the sum of the whole series.

Thus, in the series, 1, 2, 3, 4, 5, 6, 7, 8, 9, the greatest given term, 9, multiplied into the next greater, 10, gives 90; and $2)90(45$, the sum of the whole series.

The reason is, because the greatest term is, in this case, the number of terms, and the next greater term is the sum of the extremes; therefore, &c.

6. In a series of the natural odd numbers, 1, 3, 5, 7, &c. the square of the number of terms is equal to the sum of the whole series.

Thus, in the series, 1, 3, 5, 7, 9, 11, 13, the number of terms is 7, and $7 \times 7 = 49$, the sum of the whole series.

The reason is, because, in this case, the half sum of the extremes is always equal to the number of terms.

7. In a series of the natural even numbers, 2, 4, 6, 8, &c. the number of terms multiplied into the number of terms plus unity, gives a product equal to the sum of the whole series.

Thus, in the series, 2, 4, 6, 8, 10, 12, the number of terms is 6, and $6 \times 7 = 42$, the sum of the whole series.

The reason is, because, in this case, the half sum of the extremes always exceeds the number of terms by unity.

THEOREM VII.

In an arithmetical series, the sum of all the terms divided by the number of terms, quotes half the sum of the extremes; and half the product of the common difference multiplied into the number of terms minus unity, is equal to half the difference of the extremes.

The reason of the first part of the proposition is evident; for, by Theorem VI. 2. the sum of the extremes multiplied into half the number of terms, gives a product equal to the sum of all the terms; and consequently the sum of all the terms divided by half the number of terms, quotes the sum of the extremes; and doubling the divisor, that is, dividing by the number of terms, the quot will be half the sum of the extremes.

The second part of the proposition is likewise plain: for, by Theorem V. the difference of the extremes, divided by the number of terms minus unity, quotes the common difference; but the quot multiplied into the divisor, produces the dividend; which, in this case, divided by 2, quotes half the difference of the extremes.

Hence the sum of a series, the number of terms, and the common difference being given, the extremes may be found: for the theorem gives their half sum, and half difference; and the half difference of two quantities added to half their sum, gives the greater quantity; and the half difference subtracted from half their sum, leaves the lesser quantity.

Of the five things that occur to be considered in an arithmetical series, if any three be given, the other two may be found; the exemplification whereof at large would open a wide field; but we propose only to give a brief specimen, in a few problems, whose solutions are founded upon, and flow directly from the above theorems.

P R O B. I.

Given the greatest term, the number of terms, and common difference, to find the least term; that is, Given II. III. IV. to find I.

R U L E.

R U L E.

Multiply the common difference into the number of terms minus unity, subtract the product from the greatest term, and the remainder is the least term, by Theorem I.

E X A M P L E I.

A man travels 6 days, and increases each day's journey by 4 miles; his last day's journey was 40 miles: What was the first? *Ans.* 20 miles.

$$4 \times 5 = 20, \text{ and } 40 - 20 = 20 \text{ miles. } \textit{Ans.}$$

Series 20, 24, 28, 32, 36, 40, complete.

E X A M P L E II.

A man takes out of his pocket, at eight several times, so many several numbers of shillings, every one exceeding the former by 6; the last was 54: What was the first? *Ans.* 12 shillings.

$$6 \times 7 = 42, \text{ and } 54 - 42 = 12 \text{ s. } \textit{Ans.}$$

P R O B. II.

Given the least term, the number of terms, and common difference, to find the greatest term. That is, Given I. III. IV. to find II.

R U L E.

Multiply the common difference into the number of terms minus unity, add the product to the least term, and the sum is the greatest term, by Theorem I.

E X A M P L E I.

There is an arithmetical series consisting of 18 terms or places; the first term is 4, the common difference 12: What is the greatest term? *Ans.* 208.

$$12 \times 17 = 204, \text{ and } 204 + 4 = 208 \text{ } \textit{Ans.}$$

EX-

EXAMPLE II.

A man had 12 children; the youngest was three years old, and the common difference of their ages was 4 years: What was the age of the eldest? *Ans.* 47 years.

$$4 \times 11 = 44, \text{ and } 44 + 3 = 47 \text{ years. } \textit{Ans.}$$

PROB. III.

Given the extremes, and common difference, to find the number of terms; that is, given I. II. IV. to find III.

RULE.

Divide the difference of the extremes by the common difference, and the quot plus unity is the number of terms, by Theorem IV.

EXAMPLE I.

A setting out on a journey, travels 12 miles the first day, and increases every day's journey by four miles, till at last he travelled 64 miles in one day: How many days did he travel? *Ans.* 14 days.

$$64 - 12 = 52, \text{ and } 4)52(13, \text{ and } 13 + 1 = 14 \text{ days. } \textit{Ans.}$$

EXAMPLE II.

The youngest child of a large family was 4 years old, the eldest 40, and the common difference of the childrens ages was 3 years: How many children were there? *Ans.* 13.

$$40 - 4 = 36, \text{ and } 3)36(12, \text{ and } 12 + 1 = 13 \text{ } \textit{Ans.}$$

PROB. IV.

Given the extremes, and number of terms, to find the common difference; that is, given I. II. III. to find IV.

RULE.

R U L E.

Divide the difference of the extremes by the number of terms minus unity, and the quot. is the common difference, by Theorem V.

E X A M P L E I.

A man had 12 sons, whose ages were in arithmetical progression; the youngest was 2 years old, and the eldest 35: What was the common difference of their ages? *Ans.* 3 years.

$$35 - 2 = 33, \text{ and } 11)33(3 \text{ years. } \textit{Ans.}$$

E X A M P L E II.

A gentleman distributes a sum of money in arithmetical progression among 20 beggars; to the first he gave 3 shillings, and to the last 41 shillings: What was the common difference? *Ans.* 2 shillings.

$$41 - 3 = 38, \text{ and } 19)38(2 \text{ s. } \textit{Ans.}$$

P R O B. V.

Given the extremes, and number of terms, to find the sum of the series; that is, given I. II. III. to find V.

The several solutions of this problem assigned in Theorem VI. supply the place of rules; and to these we shall constantly refer.

E X A M P L E I.

13 persons give their charity to a poor man in arithmetical progression; the first gave 2 d. the last 26 d.: How much did the poor man get? *Ans.* 182 d. or 15 s. 2 d. Theorem VI. 1.

$$2 + 26 = 28, \text{ and } 28 \times 13 = 364, \text{ and } 2)364(182 \text{ d. } \textit{Ans.}$$

E X A M P L E II.

Placing 100 eggs in a straight line, at a yard's distance

stance one from another, and the first a yard from a basket: How far must one travel to bring the eggs, one by one, into the basket? *Ans.* 10100 yards, or 5 English miles and 1300 yards. Theorem VI. 2.

$$2 + 200 = 202, \text{ and } 202 \times 50 = 10100 \text{ yards. } \textit{Ans.}$$

EXAMPLE III.

A man bought 12 growing trees, the first for 2 s. the last for 40 s.; and their prices being in arithmetical progression, what paid he for the whole? *Ans.* 252 s. or 12 l. 12 s. Theorem VI. 3.

$$2 + 40 = 42, \text{ and } 2)42(21, \text{ and } 21 \times 12 = 252 \text{ s. } \textit{Ans.}$$

EXAMPLE IV.

A person completes a journey in 13 days, the number of miles travelled each day being in arithmetical progression, the seventh day he travelled 22 miles: What was the length of his journey? *Ans.* 286 miles. Theorem VI. 4.

$$22 \times 13 = 286 \text{ miles. } \textit{Ans.}$$

EXAMPLE V.

How many strokes does a clock strike in one revolution of the index, viz. in 12 hours? *Ans.* 78 strokes. Theorem VI. 5.

$$12 \times 13 = 156, \text{ and } 2)156(78 \text{ strokes. } \textit{Ans.}$$

EXAMPLE VI.

A butcher bought a dozen of fat lambs, and was to pay 1 shilling for the first, 3 for the second, and 5 for the third, &c. still advancing 2 shillings more for every following one: What did the 12 lambs cost him? *Ans.* 144 shillings, or 7 l. 4 s. Theorem VI. 6.

$$12 \times 12 = 144 \text{ shillings. } \textit{Ans.}$$

EX-

EXAMPLE VII.

A man buys 10 horses, and is to pay 2 l. for the first, 4 for the second, 6 for the third, &c. still paying 2 l. more for every following one: What will the 10 horses cost him? *Ans.* 110 l. Theorem VI. 7.

$$10 \times 11 = 110 \text{ l. } \textit{Ans.}$$

PROB. VI.

Given the sum of the series, the number of terms, and common difference, to find the extremes; that is, given V. III. IV. to find I. and II.

R U L E.

Divide the sum of the series by the number of terms, and the quot is half the sum of the extremes; then multiply the common difference into the number of terms minus unity, and the product divided by 2 quotes half the difference of the extremes. The half difference added to the half sum gives the greater extreme, and subtracted leaves the lesser, by Theorem VII.

EXAMPLE.

A man received 300 l. at 12 payments, each payment exceeding the former by 4 l.: What was the first, and what the last payment?

$12)300(25$ half sum of the extremes.

$4 \times 11 = 44$, and $2)44(22$ half difference of extremes.

And $25 + 22 = 47$ greatest extreme, or last payment.

And $25 - 22 = 3$ least extreme, or first payment.

MIXT QUESTIONS.

Quest. 1. A man travels 15 days, increasing every day's journey by 3 miles, his last day's journey was 44 miles: What was his first day's journey? and how many miles did he travel?

$3 \times 14 = 42$, and $44 - 42 = 2$, the miles he travelled the first day, by Prob. I.

And $2 + 44 = 46$, and $2)46(23$, and $23 \times 15 = 345$, the miles travelled in all, by Prob. V. See Theorem VI. 3.

Quest. 2. A man clears a debt by payments in arithmetical proportion, the common difference being 5 l.; the first payment was 12 l. the last 57 l.: How many payments were there? and what the amount of the debt?

$57 - 12 = 45$, and $5)45(9$, and $9 + 1 = 10$, the number of payments, by Prob. III.

And $12 + 57 = 69$, and $69 \times 5 = 345$, the amount of the debt, by Prob. V. See Theorem VI. 2.

II. Geometrical progression.

A geometrical progression, or series, is formed from some number assumed as the first term, by continual multiplication or division; and is either increasing or decreasing.

Thus, $1 : 2 : 4 : 8 : 16 : 32 : 64$, is a geometrical progression or series, formed from 1 assumed as the first term, and increasing, being continually multiplied by 2; and may be continued upward to infinity.

And $243 : 81 : 27 : 9 : 3 : 1$, is a geometrical progression or series, formed from 243 assumed as the first term, and decreasing, being continually divided by 3, and may likewise be continued downward to infinity.

The multiplier or divisor whereby the series is continued upward or downward, is called the *common ratio*.

Note. The natural numbers, 1, 2, 3, 4, &c. are sometimes set over a geometrical series, to shew the distance of any term from unity, or from the first term; and in this case the natural numbers are called *indices*, or *exponents*; thus,

1. 2. 3. 4. 5. 6. exponents.
3 : 6 : 12 : 24 : 48 : 96 series.

But if the series proceed from unity, it is usual and convenient to place 0 over 1, thus,

0. 1. 2. 3. 4. 5. 6. exponents.
1 : 2 : 4 : 8 : 16 : 32 : 64 series.

In a geometrical progression, or series, five things occur to be considered; any three of which being given, the other two may be found. The five things are,

- | | | |
|------------------------------|---|-----------|
| I. The least term. | } | Extremes. |
| II. The greatest term. | | |
| III. The number of terms. | | |
| IV. The common ratio. | | |
| V. The sum of all the terms. | | |

The more useful properties of numbers in geometrical progression are explained in the following theorems.

THEOREM I.

Any term of a geometrical series is equal to the product of the least term, multiplied continually into the common ratio, repeated as often as there are terms before or after it.

Thus in the increasing series, 3 : 6 : 12 : 24 : 48 : 96, whose common ratio is 2, the term 96 is equal to $3 \times 2 \times 2 \times 2 \times 2 \times 2$.

Again, in the decreasing series 162 : 54 : 18 : 6 : 2, whose ratio is 3, the term 162 is equal to $2 \times 3 \times 3 \times 3 \times 3$.

The reason is evident from the nature of the series, and the manner of its formation.

In practice it will be convenient, in the first place, to multiply the common ratio continually into itself; that is, to raise it to a power whose index or exponent is equal to the number of terms minus unity, and then multiply this power into the least term.

Z 2

Thus,

Thus, in the first of the above examples, $2 \times 2 \times 2 \times 2 \times 2 = 32$, and $32 \times 3 = 96$. Again, in the second example, $3 \times 3 \times 3 \times 3 = 81$, and $81 \times 2 = 162$.

Hence, in any geometrical series, that power of the common ratio whose index is equal to the number of terms minus unity, multiplied into the least term, produces the greatest; and the greatest term divided by that power quotes the least.

THEOREM II.

In a geometrical series consisting of three, five, or any odd number of terms, the square of the middle term is equal to the product of the extremes, or to the product of any two means equally distant from the said middle term.

Thus, in the series $1 : 2 : 4 : (8) : 16 : 32 : 64$, the square of the middle term 8, viz. $8 \times 8 = 1 \times 64$, the extremes, or $= 2 \times 32$, or $= 4 \times 16$, the equidistant means.

The reason will appear by considering, that in four proportional numbers the product of the extremes is equal to the product of the means, as was demonstrated in Multiplication of Vulgar Fractions; and when there are only three proportional numbers given, the middle term supplies the place of two, being the consequent to the first, and the antecedent to the third term. Thus, $1 : 8 : 64$, expressed at more length is $1 : 8 :: 8 : 64$; and therefore $8 \times 8 = 1 \times 64$. Again, $2 : 8 : 32$, expressed more at large is, $2 : 8 :: 8 : 32$, and so $8 \times 8 = 2 \times 32$, &c.

COROLLARIES.

1. Hence a geometrical mean betwixt two given extremes is found by multiplying the extremes into one another, and extracting the square root of the product. Thus the mean betwixt 2 and 32 is 8; for $2 \times 32 = 64$. and the square root of 64 is 8. So $2 : 8 : 32$.

2. Hence also to an extreme and mean given, a third proportional, or the other extreme, is found by dividing the

the square of the mean by the given extreme. Thus, to $4 : 8$ given, the third proportional is 16; for $8 \times 8 = 64$, and $4)64(16$, so $4 : 8 : 16$. In like manner to $32 : 8$ given, the third proportional is 2; for $8 \times 8 = 64$, and $32)64(2$; so $32 : 8 : 2$.

3. Hence likewise, in a series not proceeding from unity, the square of any term divided by the first term quotes a term of a double exponent minus unity. Thus in the 1 series following, $12 \times 12 = 144$, and $3)144(48$, the fifth term; and in the 2 series, $48)144(3$, the fifth term.

| | | | | | |
|----------|----|----|----|----|---------------|
| | 1. | 2. | 3. | 4. | 5. |
| 1 series | 3 | : | 6 | : | 12 : 24 : 48. |
| 2 series | 48 | : | 24 | : | 12 : 6 : 3. |

THEOREM III.

In a geometrical series consisting of four, six, or any even number of terms, the product of the extremes is equal to the product of the two middle terms, or to the product of any two means equally distant from the extremes.

Thus in the series, $2 : 4 : (8 : 16) : 32 : 64$, the product of $2 \times 64 = 8 \times 16$, or $= 4 \times 32$.

The reason is plain: for in four proportional numbers the product of the extremes is equal to the product of the means; and any pair of equidistant means may be esteemed middle terms.

COROLLARIES.

1. Hence to an extreme and two means given, the other extreme, or fourth proportional, is found by dividing the product of the means by the given extreme. Thus to $4 : 8 : 16$, the fourth proportional is 32; for $8 \times 16 = 128$, and $4)128(32$; so $4 : 8 : 16 : 32$. And this takes place though the proportion be disjunct. Thus to $48 : 24 :: 6$, the fourth proportional is 3; for $24 \times 6 = 144$, and $48)144(3$; so $48 : 24 :: 6 : 3$.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by dividing the product of the two extremes by the given mean. Thus, to $2 : 4 : 16$, the other mean is 8; for $2 \times 16 = 32$; and $4)32(8$; so $2 : 4 : 8 : 16$. This likewise takes place tho' the proportion be disjunct. Thus, to $2 :: 32 : 64$, the other mean is 4; for $2 \times 64 = 128$, and $32)128(4$; so $2 : 4 :: 32 : 64$.

3. From this and the preceding problem, it follows, that in a series proceeding from unity, any term squared produces a term of a double exponent; and any two or more terms multiplied into one another, produce a term whose exponent is the sum of all their exponents. Thus, in the following series, $8 \times 8 = 64$, whose exponent is $6 = 2 \times 3$; and $4 \times 8 = 32$, whose exponent is $5 = 2 + 3$; and $2 \times 4 \times 8 = 64$, whose exponent is $6 = 1 + 2 + 3$.

| | | | | | | |
|----|----|----|----|----|----|----|
| 0. | 1. | 2. | 3. | 4. | 5. | 6. |
| 1 | : | 2 | : | 4 | : | 8 |
| : | : | : | : | : | : | : |
| 16 | : | 32 | : | 64 | : | |

THEOREM IV.

In a geometrical series, the quot of the greatest extreme divided by the least, is equal to that power of the common ratio whole exponent is the number of terms minus unity.

Thus, in the series, $3 : 6 : 12 : 24 : 48 : 96$, the quot of 3)96 ($32 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, the fifth power of 2, the common ratio of the series.

The reason is obvious: for, by Theorem I. the greatest term divided by such a power of the ratio, quotes the first term; and any dividend divided by the quot, gives the divisor.

COROLLARIES.

1. Hence the extremes and number of terms being given, the common ratio may be found; viz. divide the greatest extreme by the least, the quot is a power whose

whose exponent is the number of terms minus unity, and the root extracted is the common ratio.

2. Hence likewise we have a method of finding several mean proportionals betwixt two given numbers; *viz.* divide the greater by the lesser, esteem the quotient a power whose index is greater by unity than the number of means proposed, and the root of this power extracted is the ratio; by which multiply the lesser of the given numbers continually, and the several products are the means required.

THEOREM V.

In a geometrical series, the difference of the extremes divided by the common ratio minus unity, quotes the sum of all the terms except the greatest.

Thus, in the series, $3 : 9 : 27 : 81 : 243$, the greatest term $243 - 3 = 240$, and the ratio $3 - 1 = 2$, $240(120 = 3 + 9 + 27 + 81$.

The truth of the proposition will be evident from the following considerations, *viz.* In a series whose ratio is 2, any term minus the least is equal to the sum of all the lesser terms. Thus, in the series, $1 : 2 : 4 : 8 : 16$, the second term $2 - 1 = 1$; and $4 - 1 = 1 + 2$; and $8 - 1 = 1 + 2 + 4$; and $16 - 1 = 1 + 2 + 4 + 8$. If the ratio be 3, any term minus the least is double to the sum of all the lesser terms. Thus, in the series, $18 : 6 : 2$, the second term $\frac{6-2}{2} = 2$, and $\frac{18-2}{2} = 6 + 2$. If the ratio be 4, any term minus the least is triple of all the lesser terms, &c. And therefore, universally, the difference of the extremes divided by the common ratio minus unity, quotes the sum of all the terms except the greatest.

COROLLARIES.

1. The least term multiplied into that power of the ratio whose index is the number of terms, gives the next higher term of the series continued; from which, therefore, if the least term be subtracted, the remainder divided

vided by the ratio minus unity, will quote the sum of the given series.

2. If a decreasing series be supposed infinite, the least extreme vanishes, or becomes 0; and in this case, if the greatest extreme be multiplied into the common ratio, the product divided by the ratio minus unity, will quote the sum of the series. Thus, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$,

$$\&c. = \frac{1 \times 2}{2 - 1} = 2; \text{ and } 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}, \&c.$$

$$= \frac{1 \times 3}{3 - 1} = \frac{3}{2} = 1\frac{1}{2}.$$

3. The difference of the extremes of a series divided by the difference of the sum and greater extreme, quotes the ratio minus unity.

We shall now subjoin a few problems, whose solutions flow directly from the above theorems, or their corollaries.

P R O B. I.

Given the greatest term, the number of terms, and common ratio, to find the least term. That is, Given II. III. IV. to find I.

R U L E.

Raise the common ratio to a power whose index is the number of terms minus unity; by this divide the greatest term, and the quot is the least term, by Theorem I.

E X A M P L E.

A farmer buys six cows, whose prices were in geometrical progression, the common ratio being 2, the price of the last was 96 crowns: What was the price of the first? *Ans.* 3 crowns.

$$1, 2, 3, 4, 5$$

$$2 \times 2 \times 2 \times 2 \times 2 = 32, \text{ and } 32)96(3 \text{ crowns. } \textit{Ans.}$$

P R O B.

P R O B. II.

Given the least term, the number of terms, and common ratio, to find the greatest term; that is, Given I. III. IV. to find II.

R U L E.

Raise the common ratio to a power whose index is the number of terms minus unity, multiply this by the least term, and the product is the greatest, by Theorem I.

E X A M P L E.

A nobleman had nine sons, to whom he left his estate, divided into portions in geometrical progression, the common ratio being 3; the youngest son got the least portion, being only 50 l.: What did the eldest son get? *Ans.* 328050 l.

$$\begin{array}{l} 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \quad 7. \quad 8. \\ 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561, \text{ and} \\ 6561 \times 50 = 328050 \text{ l. } \textit{Ans.} \end{array}$$

P R O B. III.

The first term and common ratio of a series not proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

R U L E.

Find a few of the terms, by multiplying or dividing the first term continually by the common ratio, and over the terms thus found place their exponents; square the greatest of the found terms; which square, divided by the first term, will quote a term of a double exponent minus unity. Again, the square of the term last found, divided by the first, will quote another term of a double exponent minus unity. Thus proceed, till you either find the term sought, or one near it; and from a near term the one sought may be found by means of the ratio. Theorem II. cor. 3.

EXAMPLE.

A sum of money was divided among ten persons; their shares in geometrical progression, and the common ratio, 3; the first person got 4l.: What was the tenth person's share? *Ans.* 78732 l.

$$1. \quad 2. \quad 3. \quad 5. \quad 9. \quad 10. \\ 4 : 12 : 36 \text{ --- } 324 \text{ --- } 26244 : 78732. \text{ *Ans.*}$$

$$36 \times 36 = 1296 \div 4 = 324 \text{ the fifth term.}$$

$$324 \times 324 = 104976 \div 4 = 26244, \text{ the ninth term.}$$

The tenth term is found by multiplying the ninth into the common ratio 3.

P R O B. IV.

The common ratio of a series proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

R U L E.

Find some few terms by means of the ratio, over which place their exponents; then observe what exponents added will make up the exponent of the term sought, or of some term near it; and the terms standing under the said exponents, multiplied into one another, will produce the term answering to the sum of the exponents. Theorem III. cor. 3.

Note. In a series proceeding from unity, as 1 stands over 1, the exponent of every term will be one short of the number of terms from the beginning of the series. Thus, the exponent of the fifth term is 4, of the sixth term 5, &c.

EXAMPLE.

A grazier bought 14 sheep, at a farthing for the first, a halfpenny for the second, &c. still doubling the price for every subsequent one; and was to pay only the price of the last sheep for the whole: What sum had he to pay? *Ans.* L. 8 : 10 : 8.

0. 1. 2. 3. 4. 5.
1 ; 2 : 4 : 8 : 16 : 32

$$1 + 3 + 4 + 5 = 13 \quad L. s. d.$$

$$2 \times 8 \times 16 \times 32 = 8192 = 8^{10} 8 \text{ Ans.}$$

Or thus,

Or thus,

$$3 + 5 + 5 = 13 \quad 1 + 3 + 4 + 5 = 13$$

$$8 \times 32 \times 32 = 8192 \quad 2 \times 8 \times 16 \times 32 = 8192$$

P R O B. V.

Given the extremes and common ratio, to find the number of terms; that is, Given I. II. IV. to find III.

R U L E.

Divide the greatest extreme by the least, raise the common ratio to a power equal to the quot, and the index of that power plus 1 is the number of terms. Theorem IV.

E X A M P L E.

A gentleman purchased some acres of ground, whose prices were in geometrical progression, the common ratio being 3; the price of the first acre was 3 d. and of the last 59049 d.: How many acres did he purchase? *Ans.* 10 acres.

$$3)59049(19683$$

1. 2. 3. 4. 5. 6. 7. 8. 9. and 9 + 1 = 10 *Ans.*
And $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 19683$

P R O B. VI.

Given the extremes and common ratio, to find the sum of the series; that is, Given I. II. IV. to find V.

R U L E.

Divide the difference of the extremes by the ratio minus

A a 2

nus

nus unity, and the quot added to the greatest extreme, gives the sum of the series. Theorem V.

E X A M P L E.

A gentleman who had a daughter married on New-year's day, gave the husband towards her portion 4 shillings, promising to triple that sum the first day of every month, for nine months after the marriage; the sum paid on the first day of the ninth month was 26244 shillings: What was the lady's portion? *Ans.* 39364 shillings, or 1968 l. 4 s.

$$26244 - 4 = 26240, \text{ and } 3 - 1 = 2)26240(13120, \\ \text{and } 13120 + 26244 = 39364 \text{ s.} = 1968 \text{ l. 4 s.}$$

P R O B. VI.

Given the least extreme, the number of terms, and common ratio, to find the sum of the series; that is, Given I. III. IV. to find V.

R U L E.

Find, by Prob. III. or IV. the term next after the greatest extreme; from this term subtract the least term; and the remainder, divided by the ratio minus unity, will quote the sum of the series. Theorem V. cor. I.

E X A M P L E I.

A corn-merchant buys 12 stacks of wheat, and was to pay 2 d. for the first stack, 6 d. for the next, tripling the price for every following stack: What sum had he to pay? *Ans.* 531440 d. or L. 2214 : 6 : 8.

$$\begin{array}{ccccccc} 1. & 2. & 3. & 4. & & 7. & 13. \\ 2 : 6 : 18 : 54 & \text{---} & 1458 & \text{---} & 1062882 \\ 54 \times 54 = 2916 \div 2 = 1458 \\ 1458 \times 1458 = 2125764 \div 2 = 1062882 \\ 1062882 - 2 = 1062880, \text{ and } 3 - 1 = 2 \\ 2)1062880(531440 \text{ d.} = \text{L. } 2214 : 6 : 8. \end{array}$$

E X-

EXAMPLE II.

A gentleman buys a fine house, in which were 24 thresholds; and, in name of price, was to lay a farthing on the first threshold, a halfpenny on the second, a penny on the third, doubling the sum on every following threshold: What would the house cost him? *Ans.* 16777215 farthings, or L. 17476: 5: 3 $\frac{3}{4}$.

$$\begin{array}{cccccccc} o. & 1. & 2. & 3. & 4. & 5. & 9. & 24. \\ 1 & : & 2 & : & 4 & : & 8 & : & 16 & : & 32 & \text{---} & 512 & \text{---} & 16777216 \end{array}$$

$$4 + 5 = 9$$

$$16 \times 32 = 512$$

$$9 + 9 + 2 + 4 = 24$$

$$512 \times 512 \times 4 \times 16 = 16777216 \quad L. \quad s. \quad d.$$

$$16777216 - 1 = 16777215 f. = 17476 \quad 5 \quad 3\frac{3}{4}$$

The ratio minus unity being 1, there is no occasion to divide by it.

EXAMPLE III.

What will a horse cost by tripling the 32 nails in his shoes with a farthing? *Ans.* 926510094425920 farthings, or 965114682527 l.

EXAMPLE IV.

What will 40 drove of cattle cost, by tripling each drove with a farthing? *Ans.* 6332117426592150 l. 8 s. 4 d.

PROB. VIII.

Given the extremes, and sum of the series, to find the common ratio, and number of terms; that is, Given I. II. V. to find IV. III.

R U L E.

Divide the difference of the extremes by the difference of the sum and greater extreme, and the quot is the ratio minus unity, by Theorem V. cor. 3. The ratio

ratio being thus obtained, find the number of terms by Prob. V.

EXAMPLE.

A sold his estate for 21844 l. which was paid by several payments, in geometrical progression; the first was 4 l. the last 16384 l.: What was the ratio, and how many payments? *Ans.* the ratio 4, payments 7.

$$\begin{array}{r} 21844 \quad 16384 \\ 16384 \quad \underline{4} \\ \hline \end{array}$$

5460) 16380 (3 + 1 = 4 the common ratio.

$$4) 16384 (4096$$

1. 2. 3. 4. 5. 6. and 6 + 1 = 7 the number of
 $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096.$ payments.

CHAPTER XII.

INTEREST.

Interest is a small sum of money paid by the borrower to the lender for the use of a greater sum, at a certain rate *per cent. per annum*; and is either simple or compound.

I. Simple Interest.

Simple interest is that which arises purely or only from the principal, or sum lent.

The sum of principal and interest is called the *amount*. The rate by the laws of Britain, ever since 1714, cannot exceed 5 *per cent.* but may be less.

The year is supposed always to consist of 365 days, and the 29th of February in leap years is not reckoned; for no more interest can be legally charged for leap than for a common year.

The day computed from is not reckoned, but the day computed to is. Few chuse to compute by months,

a month being no aliquot part of a year. The computation by years and quarters is usual, but years and days still more so.

The operations may be rendered more simple by assuming the interest of 1 l. for a year, as the rate. Thus, at 4, $4\frac{1}{4}$, $4\frac{1}{2}$, $4\frac{3}{4}$, and 5 per cent. the interest or rate for 1 l. is .04, .0425, .045, .0475, .05, found by saying, as 100 : 4 :: 1 : .04, &c.

The principal and every year's amount make a series of arithmetical proportionals, the rate being the common difference; as under.

| | 1 | 2 | 3 | 4 | 5 years. |
|-------|--------------------------------------|-----------|---|---|----------|
| Prin. | 1 : 1.05 : 1.10 : 1.15 : 1.20 : 1.25 | .05 diff. | | | |
| Prin. | 100 : 105 : 110 : 115 : 120 : 125 | 5 diff. | | | |

P R O B. I.

Principal, rate, and time, in years, given, to find the interest, and consequently the amount.

R U L E.

Multiply the principal, the rate of 1 l. and the time continually, and the last product is the interest.

Or,

Multiply the principal, the rate of 100 l. and the time continually, and the last product divided by 100 quotes the interest.

E X A M P L E I.

What is the interest of 584 l. 6 s. 8 d. for $3\frac{3}{4}$ years, at $4\frac{1}{2}$ per cent.?

584

| | |
|--|---|
| $ \begin{array}{r} 584.3 \\ .045 \\ \hline 29216 \\ 233783 \\ \hline 26.2950 \\ 3\frac{3}{4} \\ \hline 4) 78.885 \\ 19.72125 \\ \hline \text{Int. } 98.60625 = 98 \text{ } 12 \text{ } 1\frac{1}{2} \\ \text{Principal } 584 \text{ } 6 \text{ } 8 \\ \hline \text{Amount } 682 \text{ } 18 \text{ } 9\frac{1}{2} \end{array} $ | $ \begin{array}{r} \text{Or thus,} \\ 584.3 \\ 4\frac{1}{2} \\ \hline 2337.83 \\ 292.16 \\ \hline 2629.50 \\ 3.75 \\ \hline 131475 \\ 184065 \\ \hline 78885 \end{array} $ |
|--|---|

100) 98.60625 interest.

The amount is found by adding the principal to the interest as above; but the amount may also be found thus: To the product of the rate and time add 1, and multiply by the principal.

| | |
|---|---|
| $ \begin{array}{r} \text{Time } 3.75 \\ \text{Rate } .045 \\ \hline 1875 \\ 1500 \\ \hline \text{Product } .16875 \end{array} $ | $ \begin{array}{r} 1.16875 \\ 584.3 \\ \hline 467500 \\ 935000 \\ \hline 584375 \\ 682.55000 \\ .389583 = \frac{1}{3} \\ \hline \text{Amount } 682.939583 = 682 \text{ } 18 \text{ } 9\frac{1}{2} \end{array} $ |
|---|---|

The interest may be found by subtracting the principal from the amount.

If the given time be days, or years and days, reduce the days to the decimal of a year, and then work as before. Or, reduce the years to days, then work, and divide the result by 365.

The reason both of this rule, and of those assigned in the subsequent problems, may be deduced from the compound proportion following.

Prin.

Prin. Ye. Int. Prin. Year. Int.
 $100 \times 1 : 4\frac{1}{2} :: 584\ 6\ 8 \times 3\frac{3}{4} : 98\ 12\ 1\frac{1}{2}$
 Or, $1 \times 1 : .045 :: 584.8 \times 3.75 : 98.60625$

But interest is more usually computed by some practical method: and these methods are numerous, and various in their own nature; and differ also as the rate varies, and as the given time is years or days.

I. *The given time years.*

R U L E S.

1. When the rate is 5 *per cent.* and the time 1 year, $\frac{1}{20}$ of the principal is the interest sought, because the rate 5 is $\frac{1}{20}$ of 100.

This is the most simple case that can happen; and because 5 *per cent.* is 1 s. *per l.* you may readily compute the interest by esteeming the pounds so many shillings, and the shillings so many half-pence, adding $\frac{1}{2}$, and if there be any pence, every 6 d. is $1\frac{1}{2}$ farthing. Thus the interest of 54 l. 16 s. for 1 year, at 5 *per cent.* is 54 s. $19\frac{1}{2}$ half-pence, or 2 l. 14 s. 9.6 d.

2. When the given time is any number of years, for 2 years take $\frac{2}{20} = \frac{1}{10}$; for 3 years take $\frac{1}{10}$ thrice; for 4 take $\frac{4}{20} = \frac{1}{5}$; for 5 take $\frac{5}{20} = \frac{1}{4}$; for 10 take $\frac{10}{20} = \frac{1}{2}$, &c.; or find the interest for 1 year, and multiply the interest so found by the years in the question, as in Ex. 2.

3. When the rate is any other than 5 *per cent.* first work for 5 *per cent.* and then to or from the interest thus found, add or subtract the correspondent part; or multiply $\frac{1}{5}$ of the interest at 5 *per cent.* by the given rate, as in Ex. 3.

4. When the given time is years and quarters, or years and aliquot parts, find the interest at 5 *per cent.* for the years, as taught above; for the quarters or parts take aliquot parts; and then multiply $\frac{1}{5}$ of the interest thus found by the given rate: Or, multiply the rate into the time, esteem the product a new rate, to which compute the interest, as in Ex. 4. 5. and 6.

EXAMPLE II.

What is the interest of 684l. 16 s. 8 d. for 15 years, at 5 per cent.?

| | L. | s. | d. |
|--------------------|-----|----|----|
| | 684 | 16 | 8 |
| Years. | | | |
| 10 = $\frac{1}{2}$ | 342 | 8 | 4 |
| 5 = $\frac{1}{4}$ | 171 | 4 | 2 |
| <u>Anf.</u> | 513 | 12 | 6 |

Or thus,

| | | | |
|-------------|-----|----|----|
| 20) | 684 | 16 | 8 |
| | 34 | 4 | 10 |
| 15 = 5 × 3 | | | 5 |
| | 171 | 4 | 2 |
| | | | 3 |
| <u>Anf.</u> | 513 | 12 | 6 |

EXAMPLE III.

What is the interest of 438 l. 14 s. 6 d. for 12 years, at $4\frac{1}{2}$ per cent.

| | L. | s. | d. |
|-----------------------|-----|----|------|
| | 438 | 14 | 6 |
| r. | | | |
| 10 = $\frac{1}{2}$ | 219 | 7 | 3 |
| 2 = $\frac{1}{10}$ | 43 | 17 | 5.4 |
| at 5 p. c. | 263 | 4 | 8.4 |
| Deduct $\frac{1}{10}$ | 26 | 6 | 5.64 |
| <u>Anf.</u> | 236 | 18 | 2.76 |

| | |
|-------------|-----------------|
| | Or thus, |
| 20) | 438 14 6 |
| | 21 18 8.7 |
| | 1 2 |
| 5) | 263 4 8.4 |
| | 52 12 11.28 |
| | 4 $\frac{1}{2}$ |
| | 210 11 9.12 |
| | 26 6 5.64 |
| <u>Anf.</u> | 236 18 2.76 |

EXAMPLE IV.

What is the interest of 317 l. 16 s. for $5\frac{3}{4}$ years, at $3\frac{1}{2}$ per cent.?

L.

| | | L. | s. |
|------------------------------|--|------------------------|-----------------|
| <i>r.</i> | | 317 | 16 |
| $5 = \frac{1}{4}$ | | 79 | 9 |
| $\frac{1}{2} = \frac{1}{10}$ | | 7 | 18 10.8 |
| $\frac{1}{4} = \frac{1}{2}$ | | 3 | 19 5.4 |
| | | 5)91 | 7 4.2 |
| | | 18 | 5 5.64 |
| | | | 3 $\frac{1}{2}$ |
| | | 54 | 16 4.92 |
| | | 9 | 2 8.82 |
| | | <i>Anf.</i> 63 19 1.74 | |

| | | Or thus, | | |
|------------------------------|--|--|-----|--------|
| | | $5\frac{3}{4} \times 3\frac{1}{2} = 20\frac{1}{8}$ | | |
| | | | 317 | 16 |
| <i>p. c.</i> | | 10 = $\frac{1}{10}$ | 31 | 15 7.2 |
| 5 = $\frac{1}{2}$ | | | 15 | 17 9.6 |
| 5 = $\frac{1}{2}$ | | | 15 | 17 9.6 |
| $\frac{1}{8} = \frac{1}{40}$ | | | 7 | 11.34 |
| | | <i>Anf.</i> 63 19 1.74 | | |

EXAMPLE V.

What is the interest of 79 l. 16 s. for $5\frac{1}{2}$ years, at $4\frac{3}{8}$ per cent.?

| | | L. | s. |
|------------------------------|--|-----------------------|-----------------|
| <i>r.</i> | | 79 | 16 |
| $5 = \frac{1}{4}$ | | 19 | 19 |
| $\frac{1}{2} = \frac{1}{10}$ | | 1 | 19 10.8 |
| | | 5)21 | 18 10.8 |
| | | 4 | 7 9.36 |
| | | | 4 $\frac{3}{8}$ |
| $\frac{2}{8} = \frac{1}{4}$ | | 17 | 11 1.44 |
| $\frac{1}{8} = \frac{1}{2}$ | | 1 | 1 11.34 |
| | | 10 | 11.67 |
| | | <i>Anf.</i> 19 4 0.45 | |

| | | Or thus, | | |
|-------------------------------|--|---|----|---------|
| | | $5\frac{1}{2} \times 4\frac{3}{8} = 24\frac{1}{16}$ | | |
| | | | 79 | 16 |
| <i>p. c.</i> | | 20 = $\frac{1}{5}$ | 15 | 19 2.4 |
| 4 = $\frac{1}{5}$ | | | 3 | 3 10.08 |
| $\frac{1}{16} = \frac{1}{64}$ | | | | 11.97 |
| | | <i>Anf.</i> 19 4 0.45 | | |

EXAMPLE VI.

What is the interest of 648 l. 8 s. for $6\frac{3}{4}$ years, at $4\frac{1}{4}$ per cent.?

| | L. |
|-------------------------------|--|
| r. | 648.4 |
| 5 = $\frac{1}{4}$ | 162.1 |
| 1 = $\frac{1}{5}$ | 32.42 |
| $\frac{1}{2}$ = $\frac{1}{2}$ | 16.21 |
| $\frac{1}{4}$ = $\frac{1}{2}$ | 8.105 |
| | <hr/> |
| | 5)218.835 |
| | <hr/> |
| | 43.767 |
| | <hr/> |
| | 4 $\frac{1}{4}$ |
| | <hr/> |
| | 175.068 |
| | <hr/> |
| | 10 94175 |
| | <hr/> |
| | Ans. 186.00975 = 186 0 2 $\frac{1}{4}$ |

| | Or thus, |
|---------------------------------|--|
| | $6\frac{3}{4} \times 4\frac{1}{4} = 28\frac{11}{16}$ |
| | <hr/> |
| p.c. | 648.4 |
| 20 = $\frac{1}{5}$ | 129.68 |
| 4 = $\frac{1}{5}$ | 25.936 |
| 4 = $\frac{1}{5}$ | 25.936 |
| $\frac{11}{16} = \frac{11}{16}$ | 4.45775 |
| | <hr/> |
| | Ans. 186.00975 |

II. The given time days.

R U L E S.

1. Multiply the principal by the number of days, and the product divided by 7300 quotes the interest at 5 *per cent.*; and if the rate be any other than 5 *per cent.* adjust the matter by aliquot parts, as in Ex. 7.

2. Divide any principal by 73, and the quot will be the interest for 100 days at 5 *per cent.* and for the remaining days take aliquot parts, as in Ex. 8.

3. If the number of the given days be 73, or any multiple of 73, such as, 146, 219, 292, 365, &c. divide the principal by 100, and the quot will be the interest for 73 days at 5 *per cent.*; which multiplied by the number denoting the multiple, will produce the interest for the given days, as in Ex. 9.

4. If the principal be 100 l. esteem the days so many pounds principal, divide by 73, and the quot will be the interest of 100 l. for the given number of days, at 5 *per cent.* as in Ex. 10.

The reason of the above rules will appear from the following

following compound proportion, in which p . is put to denote the principal, and d . the number of days.

Prin. Days. Int.

$$100 \times 365 : 5 :: p. \times d.$$

That is, $\frac{5 \times p. \times d.}{100 \times 365} = \frac{p. \times d.}{100 \times 73} = \frac{p. \times d.}{7300}$, Hence Rule 1.

Prin. Days. Int. Days.

Again, $100 \times 365 : 5 :: p. \times 100$

That is, $\frac{5 \times p. \times 100}{100 \times 365} = \frac{5 \times p.}{365} = \frac{p.}{73}$, Hence Rule 2.

The truth of Rule 3. is evident; for if 73 dividing any principal quotes the interest for 100 days, it follows, that 100 dividing any principal will quote the interest for 73 days.

The reason of Rule 4. is also obvious; for the interest of 100 l. for 50 days will be equal to the interest of 50 l. for 100 days.

EXAMPLE VII.

What is the interest of 245 l. 13 s. 4 d. from the 21st of March to the 22d of November, at $4\frac{3}{4}$ per cent.?

Days.

| Days. | L. | |
|----------|---------------|--------------------------------------|
| Mar. 10 | 245.6 | |
| Apr. 30 | 246 | |
| May 31 | <u>14740</u> | |
| June 30 | 98266 | |
| July 31 | 491333 | 20) |
| Aug. 31 | <u>584</u> | |
| Sept. 30 | 73 00)60434.0 | (8.2786, at 5 per cent. |
| Oct. 31 | <u>584</u> | .4139, at $\frac{1}{4}$ per cent. |
| Nov. 22 | 203 | 7.8647 = L. 7 17 $3\frac{1}{2}$ at |
| | <u>246</u> | $4\frac{3}{4}$ per cent. <i>Ans.</i> |
| | 146 | |
| | <u>574</u> | |
| | 511 | |
| | <u>630</u> | |
| | 584 | |
| | <u>460</u> | |
| | 438 | |
| | <u>(22)</u> | |

EXAMPLE VIII.

What is the interest of 378 l. 3 s. $8\frac{1}{2}$ d. for 275 days, at $4\frac{1}{2}$ per cent. ?

L.

L. s. d. L. s. d. Days.
73)378 3 8½ (5 3 7.349, for 100.

365

13

20

263

219

44

12

536.5

511

255

219

360

292

680

657

(23)

10 7 2.698, for 200

2 11 9.674, for 50 = $\frac{1}{2}$

1 0 8.669, for 20 = $\frac{1}{5}$

5 2.167, for 5 = $\frac{1}{4}$

10)14 4 11.208, at 5 per cent.

1 8 5.920, at $\frac{1}{2}$ per cent.

Ans. 12 16 5.288, at $4\frac{1}{2}$ per cent.

Or, L. 12 16 5 $\frac{1}{4}$

EXAMPLE IX.

What is the interest of 864 l. 19 s. 8 d. for 219 days, at $4\frac{5}{8}$ per cent.?

L. s. d. L. s. d. Days.
100)864 19 8(8 12 11.96, for 73

20

3

12|99

12

11|96

25 18 11.88, for 219, at 5 per cent.

1 18 11.09, at $\frac{3}{8}$ per cent. = $\frac{3}{40}$

24 0 0.79, at $4\frac{5}{8}$ per cent. Ans.

EXAMPLE X.

What is the interest of 100 l. for 254 days, at $4\frac{7}{8}$ per cent.?

L.

| | L. | s. | d. | |
|------------|------|----|--------|---|
| 73) 254 (| 3 | 9 | 7.068, | at 5 per cent. |
| | 219 | 1 | 8.876, | at $\frac{1}{8}$ per cent. = $\frac{1}{40}$ |
| | 35 | 3 | 7 | 10.192, at $4\frac{7}{8}$ p. c. for 254 days. <i>Ans.</i> |
| | 20 | | | |
| | 700 | | | |
| | 657 | | | |
| | 43 | | | |
| | 12 | | | |
| | 516 | | | |
| | 511 | | | |
| | 500 | | | |
| | 438 | | | |
| | 620 | | | |
| | 584 | | | |
| | (36) | | | |

If the given principal be any multiple of 100, work for 100 as above, and then multiply the result by the number denoting the multiple.

The computing of interest occurs so frequently in practice, that men of business, for the sake of ease and dispatch, generally use tables constructed for that purpose. The following tables give the interest at 5 per cent.; whence the interest at any other rate may easily be found.

The interest of one pound, at 5 per cent.

| | TABLE I.
per year. | TABLE II.
per day. | TABLE III.
per quarter. | TABLE IV.
per month. |
|---|-----------------------|-----------------------|----------------------------|-------------------------|
| 1 | .05 | .0001369863 | .0125 | .00416 |
| 2 | .1 | .0002739726 | .025 | .0083 |
| 3 | .15 | .0004109589 | .0375 | .0125 |
| 4 | .2 | .0005479452 | .05 | .0166 |
| 5 | .25 | .0006849315 | .0625 | .02083 |
| 6 | .3 | .0008219178 | .075 | .025 |
| 7 | .35 | .0009589041 | .0875 | .02916 |
| 8 | .4 | .0010958904 | .1 | .0333 |
| 9 | .45 | .0012328767 | .1125 | .0375 |

The

The above tables are constructed by the rules already assigned for finding the interest of any principal for any given time. The quarter in Table III. is $\frac{1}{4}$ of a year, or 91 days 6 hours; and the month in Table IV. is $\frac{1}{12}$ of a year, or 30 days 10 hours. In using these tables observe the following

R U L E S.

1. Resolve the given number of years or days into its constituent parts.
2. Seek the significant figure of each constituent part on the left; opposite to which, under *per year*, or *per day*, &c. according to the denomination of the given time, you have the decimal to be taken out.
3. Move the decimal point in each decimal so many places to the right as there are ciphers in the constituent part; and, in taking out for decimal figures, move the decimal point so many places to the left as the decimal figure is to the right.
4. Add the decimals thus collected from the tables, and their sum is the interest sought.
5. If the given time be of different denominations, such as years and days, find the interest for the years and days separately, and the sum of the results will be the interest sought. Or, reduce the years to days, and then collect from Table II. Do the like when the time is given in years and quarters, or years and months, &c.
6. If the given rate be not 5 *per cent.* first find the interest at 5 *per cent.* and then multiply one fifth of the interest so found by the given rate.

The decimals in the tables are complete, and consequently will, when all the figures are used, give a complete answer; but yet, in most cases, four or five figures will be sufficiently accurate.

E X A M P L E I.

What is the interest of 2485 l. for 1 year, at 4 $\frac{1}{2}$ *per cent.*?

In Table I.
opposite to —
you find

| | | |
|------|---|-----|
| 3000 | — | 150 |
| 400 | — | 20 |
| 80 | — | 4 |
| 5 | — | .25 |

5) 174.25 at 5 per cent.

34.85 at 1 per cent.

$4\frac{3}{4}$

139.40

17.425

8.7125

L. s. d.

Ans. 165.5375 = 165 10 9

EXAMPLE II.

What is the interest of 3485 l. for 1 day, at $4\frac{3}{4}$ per cent.?

In Table II.
opposite to —
you find

| | | |
|------|---|--------|
| 3000 | — | .41095 |
| 400 | — | .05479 |
| 80 | — | .01095 |
| 5 | — | .00068 |

5) .47737 at 5 per cent.

.09547 at 1 per cent.

$4\frac{3}{4}$

.38188

.04773

.02386

s. d.

.45347 = 9 $0\frac{3}{4}$ Ans.

EXAMPLE III.

What is the interest of 284 l. 5 s. for 15 years, at 5 per cent.?

Multiply the principal 284.25
by the number of years 15

Resolve the product 4263.75 into constituent parts, and then proceed as before.

4000

$$\begin{array}{r}
 4000 \text{ --- } 200 \\
 200 \text{ --- } 10 \\
 60 \text{ --- } 3 \\
 3 \text{ --- } .15 \\
 .7 \text{ --- } .035 \\
 .05 \text{ --- } .0025 \\
 \hline
 213.1875 = 213 \text{ L. } 3 \text{ s. } 9 \text{ d. } \text{ Ans.}
 \end{array}$$

EXAMPLE IV.

What is the interest of 564 l. for 238 days, at $4\frac{1}{2}$ per cent.?

Multiply the principal 564
by the number of days 238

Resolve the product 134232 into constituent parts,
and then proceed as formerly.

$$\begin{array}{r}
 100000 \text{ --- } 13.6986 \\
 30000 \text{ --- } 4.1095 \\
 4000 \text{ --- } .5479 \\
 200 \text{ --- } .0273 \\
 30 \text{ --- } .0041 \\
 2 \text{ --- } .0002
 \end{array}$$

5) 18.3876 at 5 per cent.

3.6775 at 1 per cent.

$$\begin{array}{r}
 4\frac{1}{2} \\
 \hline
 14.7100 \\
 1.8387 \\
 \hline
 16.5487 = 16 \text{ l. } 10 \text{ s. } 11\frac{1}{2} \text{ d. } \text{ Ans.}
 \end{array}$$

EXAMPLE V.

What is the interest of 78 l. for $4\frac{1}{4}$ years, at $4\frac{1}{2}$ per cent.?

$$78 \times 4.25 = 331.5$$

By Table I.

$$300 \text{ --- } 15$$

$$30 \text{ --- } 1.5$$

$$1 \text{ --- } .05$$

$$.5 \text{ --- } .025$$

$$5)16.575 \text{ at } 5 \text{ per cent.}$$

$$3.315 \text{ at } 1 \text{ per cent.}$$

$$4\frac{1}{8}$$

$$13.260$$

$$.414 = \frac{1}{8}$$

L. s. d.

$$\text{Ans. } 13.674 = 13 \text{ } 13 \text{ } 5\frac{3}{4}$$

$$\text{Or, } 78 \times 17 \text{ quarters} = 1326$$

By Table III.

$$1000 \text{ --- } 12.5$$

$$300 \text{ --- } 3.75$$

$$20 \text{ --- } .25$$

$$6 \text{ --- } .075$$

$$5)16.575 \text{ at } 5 \text{ per cent.}$$

$$3.315 \text{ at } 1 \text{ per cent.}$$

$$4\frac{1}{8}$$

$$13.260$$

$$.414 = \frac{1}{8}$$

$$\text{Ans. } 13.674 \text{ as before.}$$

EXAMPLE VI.

What is the interest of 85 l. 14 s. 6 d. for 3 years
11 months, at $4\frac{1}{2}$ per cent.?

$$85.725$$

$$85.725 \times 47 \text{ months} = 4029.075$$

By Table IV.

| | | |
|------|---|---------|
| 4000 | — | 16.6666 |
| 20 | — | .0833 |
| 9 | — | .0375 |
| .07 | — | .0002 |
| .005 | — | .0000 |

$$5)16.7876 \text{ at } 5 \text{ per cent.}$$

$$3.3575 \text{ at } 1 \text{ per cent.}$$

$$4\frac{5}{8}$$

$$13.4300$$

$$1.6787 = \frac{4}{3}$$

$$.4196 = \frac{1}{8}$$

L. s. d.

$$\text{Ans. } 15.5283 = 15 \text{ } 10 \text{ } 6\frac{3}{4}$$

In calculating interest on cash-accounts, or accounts-current where partial payments are made, and the account cleared within twelve months, multiply the principal and the several balances into the number of days they are at interest, and the sum of these products divided by 7300, will quote the interest at 5 per cent., and for any other rate multiply one fifth of the interest thus found by the given rate, and the product will be the interest sought; as in the following examples.

EXAMPLE I.

Lent A B, the 10th of June 1765, the sum of 800*l*. Sterling, and received the same back in partial payments, as follows: What interest is due at 5 per cent.?

1765.

1765.

| | | | L. | s. | d. | Da. | Products. |
|--------|----|---------------------------|-----|-----|----|-----|-----------|
| June | 10 | Principal lent | - | 800 | | 60 | 48000 |
| Aug. | 9 | Received | - | 200 | 10 | | |
| | | New principal | 599 | 10 | | 67 | 40166.5 |
| Octob. | 15 | Received | - | 328 | 15 | 6 | |
| | | New principal | 270 | 14 | 6 | 38 | 10287.55 |
| Nov. | 22 | Received | - | 170 | 14 | 6 | |
| | | New principal | 100 | | | 31 | 3100 |
| Dec. | 25 | Recd in full of principal | 100 | | | | |

101554.05

$$7300 = 73 \times 100$$

$$73)1015.5405(13.9115 = 13 \text{ l. } 18 \text{ s. } 2\frac{3}{4} \text{ d. Ans.}$$

Banks, or bankers, sometimes borrow at 4 *per cent.* but lend at 5 *per cent.*; and the person to whom they give a cash-credit, has no occasion to keep money by him, but gives it into the bank, and receives 4 *per cent.* interest for the balance of cash due to him; but when the balance runs against him, he pays interest at the rate of 5 *per cent.* In this case it will be proper to consider the money lent or paid by the bank as Dr, and the money received by the bank as Cr, and to make two columns for products, viz. one for the Dr products, and the other for the Cr products. The interest arising from the Dr products is to be computed at 5 *per cent.* and that from the Cr products at 4 *per cent.* and the difference of these two is the balance of interest due by the person to the bank, or by the bank to him.

EXAMPLE II.

Given A B a cash-credit at 5 *per cent.* for the balances due by him, allowing him 4 *per cent.* for such balances as may be due to him: What interest will be due, and by whom, in consequence of the following transactions?

1765.

Products.

| 1765. | | L. | s. | d. | Dal. | Dr. | Cr. |
|----------|----|------|----|----|----------|---------|-------|
| Jan. 8 | Dr | 845 | 6 | 8 | 102 | .86224 | |
| April 20 | Cr | 1265 | 6 | 8 | | | |
| | Cr | 420 | - | - | 86 | | 36120 |
| July 15 | Dr | 968 | 10 | 0 | | | |
| | Dr | 548 | 10 | 0 | 77 | 42234.5 | |
| Sept. 30 | Cr | 848 | 10 | 0 | | | |
| | Cr | 300 | - | - | 32 | | 9600 |
| Nov. 1. | Dr | 500 | - | - | | | |
| | Dr | 200 | - | - | 44 | .8800 | |
| Dec. 15. | Cr | 200 | - | - | | | |
| | | | | | 137258.5 | | 45720 |

$$\begin{array}{r} \text{At } 5 p. c. = L. 18 \ 15 \ 6 \\ \quad \quad \quad 5 \ 00 \ 0 \\ \hline 13 \ 15 \ 6 \text{ interest due by} \\ \text{A B.} \end{array}$$

If the rate of interest on both sides be the same, you have only to divide the difference of the sums of the Dr and Cr products by 7300.

When partial payments are made on bonds or bills at any interval greater than a year, the legal and usual method is, to add the interest at the times of payment to the principal, from that amount deducting the payment. Bankers avoid such dilatory payments by taking care to have all their cash-accounts settled within the year, this being the shortest or speediest way of converting interest into principal.

EXAMPLE III.

Borrowed on bond, 1761, June 1. the sum of 500 l. at 5 per cent. and made partial payments as follows: Required the state of the affair the 14th of August 1765, when the account comes to be cleared.

1761,

| | | L. | s. | d. |
|----------------|--------------------------------|-----|----|-----------------|
| 1761, June 1. | Principal borrowed | 500 | | |
| | Inter. for 1 year and 129 days | 33 | 16 | $8\frac{1}{2}$ |
| | Amount | 533 | 16 | $8\frac{1}{2}$ |
| 1762, Oct. 8. | Paid | 120 | | |
| | New principal | 413 | 16 | $8\frac{1}{2}$ |
| | Inter. for 1 year and 85 days | 25 | 10 | $2\frac{1}{4}$ |
| | Amount | 439 | 6 | $10\frac{3}{4}$ |
| 1764, Jan. 1. | Paid | 239 | 6 | $10\frac{3}{4}$ |
| | New principal | 200 | | |
| | Inter. for 1 year and 225 days | 16 | 3 | $3\frac{1}{4}$ |
| | Amount | 216 | 3 | $3\frac{1}{4}$ |
| 1765, Aug. 14. | Paid in full | 216 | 3 | $3\frac{1}{4}$ |

P R O B. II.

Amount, rate, and time in years given, to find the principal, or present worth.

R U L E.

Add 1 to the product of the time and rate of one pound, by which divide the amount.

E X A M P L E I.

What ready money will pay a bill of 682 l. 18 s. $9\frac{1}{2}$ d. due $3\frac{3}{4}$ years hence, discounting interest at $4\frac{1}{2}$ per cent.?

375

3.75 time.

.045 rate of 1 l.

$$\begin{array}{r}
 1875 \\
 1500 \\
 \hline
 1.16875 \times 682.939583 = 584.8 = 584 \text{ } 6 \text{ } 8 \text{ } \textit{Ans.} \\
 584375 \dots \\
 \hline
 985645 \\
 935000 \\
 \hline
 506458 \\
 467500 \\
 \hline
 389583 \\
 350625 \\
 \hline
 *38958
 \end{array}$$

Or say, As the amount of 100 l. at the rate and time given is to 100 l. principal; so is the given amount to the principal, or present worth, sought.

3.75 time.

4.5 rate.

$$\begin{array}{r}
 1875 \\
 1500 \\
 \hline
 16.875 \text{ interest of } 100 \text{ l. for the given time.} \\
 100 \\
 \hline
 116.875 \text{ amount of } 100 \text{ l. for the given time.}
 \end{array}$$

$$\begin{array}{cccc}
 L. & L. & L. & L. \\
 \text{Then say, } 116.875 : 100 :: 682.939583 : 584.8
 \end{array}$$

If the time be given in days, divide the product of the rate and time by 365, and then proceed as before.

The difference between the present worth and the amount, or sum of the bill, is called *rebate*, or *discount*; and is always less than the interest of the amount for the given time, being precisely equal to the interest of the present worth for the said time; so that if the present

worth be put to interest, it will, by the end of the given time, with the interest thence arising, be exactly equal to the given amount.

Thus, if I have a bill of 105 l. due 1 year hence, the discount at 5 per cent. will be 5 l. and the present worth 100 l.; for if I put the 100 l. to interest, it will, with the interest, by the end of the year be equal to 105 l. the given amount. But the interest of 105 l. for a year is 5 l. 5 s.; and if this be deduced from the sum of the bill, I shall then receive as present worth the sum only of 99 l. 15 s. which is 5 s. too little.

The discount is found by subtracting the present worth from the amount; but it may also be found as follows, viz. As the amount of 100 l. or of 1 l. for the time given, to the interest; so the given amount or sum of the bill to the discount sought. The discount of the former example found in this manner follows.

$$\begin{array}{rcl}
 116.875 : 16.875 :: 682.939582 & & \\
 23.375 : 3.375 & & 1027 \\
 4.675 : .675 & & 4780577082 \\
 .935 : .135 & & 13658791686 \\
 .187 : .027 & & \hline
 .187) 18.439368750 (98.60625 = 98 \text{ } 12 \text{ } 1\frac{1}{2} & & \text{L. s. d.} \\
 1683 \dots\dots & & \\
 \hline
 1609 & & \\
 1496 & & \\
 \hline
 1133 & & \\
 1122 & & \\
 \hline
 1168 & & \\
 1122 & & \\
 \hline
 467 & & \\
 374 & & \\
 \hline
 935 & & \\
 935 & & \\
 \hline
 \end{array}$$

The

The present worth is found by subtracting the discount from the amount, or sum of the bill.

$$\begin{array}{r}
 \text{L.} \quad \text{s.} \quad \text{d.} \\
 682 \quad 18 \quad 9\frac{1}{2} \text{ amount.} \\
 \underline{98 \quad 12 \quad 1\frac{1}{2} \text{ discount,}} \\
 584 \quad 6 \quad 8 \text{ present worth.}
 \end{array}$$

EXAMPLE II.

What is the present worth, and what the rebate, of a bill of 85 L 10 s, discounting for 66 days, at 5 per cent.?

73)66.0(.9041 interest of 100 L for 66 days.

$$\begin{array}{r}
 657 \\
 \hline
 300 \\
 -292 \\
 \hline
 80 \\
 73 \\
 \hline
 7
 \end{array}$$

Then 100.9041 : .9041 :: 85.5

$$\begin{array}{r}
 85.5 \\
 \hline
 45205 \\
 45205 \\
 \hline
 72328
 \end{array}$$

s. d.

100.9041)77.30055(.766 = 15 3 $\frac{3}{4}$ rebate.

7063287 85 10 amount.

6667680 84 14 8 $\frac{1}{2}$ present worth.

$$\begin{array}{r}
 6054246 \\
 \hline
 6134340 \\
 6054246 \\
 \hline
 (80094)
 \end{array}$$

D d a

EX.

EXAMPLE III.

A bill of 546 l. 16 s. dated January 10. payable the 8th of August, was presented March 15. for discount at $4\frac{3}{4}$ per cent. : What will the rebate come to?

73) 146(2 interest at 5 per cent.

.1 at $\frac{1}{4}$ p. c. = $\frac{1}{20}$

1.9 inter. of 100 l. for 146

100 days at $4\frac{3}{4}$ p. c.

101.9

March 16

April 30

May 31

June 30

July 31

Aug. 8

146 days

Then 101.9 : 1.9 :: 546.8

1.9

49212

5468

L. s. d.

101.9) 1038.92(10.195 = 10 3 10 $\frac{3}{4}$ Ans.

1019

1992

1019

9730

9171

5599

5095

(495)

Here it is to be observed, that an alteration, either in the days or the rate, does not produce a proportional alteration in the discount. Thus the discount of any sum for 40 days, will be more than half the discount for 80 days at the same rate; and the discount at 4 per cent, will be more than half the discount of the same sum for the same time at 8 per cent.

The reason is, because the first term, or divisor, consists

sists of two parts; the one variable, and the other invariable. The variable part is the interest of 100 l. for the given time, which increases proportionally with the rate or time; and when this interest comes to be doubled, the dividend is of course doubled also; but the divisor being by this means increased, will not give a double quotient. Thus, $100 \div 2 = 102$ 306 (3, but $100 \div 4 = 104$ dividing 612 will not quote 6. Hence it is that tables of discount cannot be accurate, unless calculated at every different rate, and for as many days as the case may require; because every day's discount varies, being still less as the days or rate increase.

P R O B. III.

Principal, amount, and time in years, given, to find the rate.

R U L E.

As the product of the principal and time, to the difference of the principal and amount, or total interest; so 100 to the rate *per cent*.

E X A M P L E. I.

At what rate of interest will 584 l. 6s. 8 d. amount to 682 l. 18s. 9½ d. in 3¼ years?

$$\begin{array}{r}
 584.3 \\
 \underline{3\frac{1}{4}} \\
 4)1753.0 \\
 \underline{438.25} \\
 2191.25 : 98.60625 :: 100 \\
 438.25 : 19.72125 \\
 87.65 : 3.94425 \\
 17.53 :) 78.885 (4.5 \text{ per cent. } \textit{Ans.} \\
 \underline{7012} \\
 8765 \\
 \underline{8765}
 \end{array}$$

If

If the time given be days, divide the product of the principal and time by 365, and then proceed as before.

EXAMPLE II.

At what rate of interest will 275 l. gain 8 l. 5 s. in 219 days?

$$\begin{array}{r}
 275 \\
 219 \\
 \hline
 2475 \\
 275 \\
 550 \\
 \hline
 365 \overline{) 60225} (165 : 8.25 :: 100 \\
 365 \quad 33 : 165 (5 \text{ per cent. } \text{Ans.} \\
 \hline
 2372 \quad 165 \\
 2190 \\
 \hline
 1825 \\
 1825 \\
 \hline
 \end{array}$$

P R O B. IV.

Principal, amount, and rate, given, to find the time in years.

R U L E.

Divide the difference of the principal and amount by the product of the principal and rate of one pound; that is, as one year's interest of the principal to 1 year, so the total interest to the time sought.

EXAMPLE I.

In what time will 584 l. 6 s. 8 d. amount to 682 l. 18 s. 9½ d. at 4½ per cent.?

584.8

$$\begin{array}{r} 584.3 \\ .045 \\ \hline 29216 \\ 233783 \end{array}$$

$$\begin{array}{r} 682.939588 \\ 584.333333 \\ \hline 98.606250 \end{array}$$

26.295)98.60625(3.75 years. *Ans.*

$$\begin{array}{r} 78885 \end{array}$$

$$\begin{array}{r} 197212 \end{array}$$

$$\begin{array}{r} 184065 \end{array}$$

$$\begin{array}{r} 131475 \end{array}$$

$$\begin{array}{r} 131475 \end{array}$$

If the time be required in days, multiply the difference of the principal and amount by 365, and then proceed as before.

EXAMPLE II.

In how many days will 275 l. principal gain 8 l. 5 s. at 5 per cent.?

$$\begin{array}{r} 275 \\ .05 \\ \hline \end{array}$$

13.75)3011.25(219 days. *Ans.*

$$\begin{array}{r} 2750 \end{array}$$

$$\begin{array}{r} 2612 \end{array}$$

$$\begin{array}{r} 1375 \end{array}$$

$$\begin{array}{r} 12375 \end{array}$$

$$\begin{array}{r} 12375 \end{array}$$

$$\begin{array}{r} 8.25 \end{array}$$

$$\begin{array}{r} 365 \end{array}$$

$$\begin{array}{r} 4125 \end{array}$$

$$\begin{array}{r} 4950 \end{array}$$

$$\begin{array}{r} 2475 \end{array}$$

$$\begin{array}{r} 3011.25 \end{array}$$

I shall conclude this section with the explication of a rule that stands connected with simple interest, viz.

Equation of Payments.

When two or more debts are payable at different times, the finding a mean time, at which all the debts may be paid at once, without loss to debtor or creditor,

is called *equating* the terms of payment; and is commonly performed by the following

R U L E.

Multiply the several debts into their respective times, divide the sum of the products by the total debts, and the quot is accounted the mean time.

E X A M P L E I.

A owes B 600 l. to pay at 40 days, 200 l. at 60 days, and 200 l. at 120 days: When may these debts be paid at once, without injury to either party?

| Debts. | Days. | Prod. |
|--------|-------|-----------------------------|
| 600 | × 40 | = 24000 |
| 200 | × 60 | = 12000 |
| 200 | × 120 | = 24000 |
| 1000 | |)60000(60 days. <i>Ans.</i> |

E X A M P L E II.

A owes B 640 l.; whereof 40 l. to be paid presently, 350 l. at the end of 3 months, and 250 l. at 9 months: Required the equated time for paying the whole.

| Debts. | Mon. | Prod. |
|--------|------|--|
| 40 | × 0 | = 0000 |
| 350 | × 3 | = 1050 |
| 250 | × 9 | = 2250 |
| 640 | |)3300(5 $\frac{5}{12}$ months. <i>Ans.</i> |

E X A M P L E III.

A owes B a certain sum; whereof $\frac{1}{3}$ to be paid in ready money, $\frac{1}{3}$ at the end of 6 months, and the other $\frac{1}{3}$ at 8 months: Required the equated time for paying the whole.

Debts.

Debts. Mon. Prod.

$$\begin{array}{r}
 \frac{1}{3} \times 0 = 0 \\
 \frac{1}{3} \times 6 = 2 \\
 \frac{1}{3} \times 8 = 2\frac{2}{3} \\
 \hline
 1
 \end{array}$$

) $4\frac{2}{3}$ ($4\frac{2}{3}$ months. *Ans.*

The above method is easy, and on that account commonly practised, but is not accurate: for a person, by keeping money unpaid after it becomes due, gains the interest thereof for that time; but by paying money before it is due, he does not lose the interest, as the rule supposes, but only the discount thereof for that time, which is always less than the interest.

They who incline to be more exact, may work by the following

R U L E.

Find, by Prob. 2. the present worth of each debt; and then, by Prob. 4. find in what time the sum of the present worths will amount to the sum of the debts.

E X A M P L E IV.

A owes B 50l.; whereof 20l. is payable 2 years hence, and 30l. 5 years hence: What is the equated time for paying both debts at once, discounting interest at 5 per cent.?

By Prob. 2.

L.

The present worth of 20l. for 2 years is, 18.18

The present worth of 30l. for 5 years is, 21.00

Sum 42.18

Total debts 50.00

Difference 7.82

By Prob. 4.

Principal 42.18

Rate .05

2.1090 7.8200 (3.7079 years.*Ans.* 3.7079 years, or 3 years and 258 days.

Some still complain, that the rule last assigned is not absolutely perfect; and argue, that the rule for finding the mean or equated time ought to be such as will make the interest of the debts paid after they are due exactly equal to the discount of the debts paid before they are due.

II. Compound Interest.

Compound interest arises both from the principal and interest; for at the end of one year the amount or sum of principal and interest becomes a new principal for the year ensuing.

The laws in Britain forbid the lending of money at compound interest; but then the lender may exact his interest at the year's end, lend it out again, and so, in effect, have compound interest on his money. Annuities too are commonly reckoned at compound interest; and this makes the knowledge of such computations in some measure necessary. I propose however to be very brief both on compound interest and annuities; and this the rather, because operations of this sort are performed to most advantage by logarithms.

In compound interest, the principal and the several years amounts make a series of geometrical proportionals, the common ratio being the amount of 1 l. for one year, as under.

| | 1 | 2 | 3 | 4 Years. | Ratio. |
|---------|-----|------|--------|----------|------------|
| Pr. | 1 | 1.05 | 1.1025 | 1.157625 | 1.21550625 |
| Pr. 100 | 100 | 105 | 110.25 | 115.7625 | 121.550625 |

P R O B.

P R O B. I.

Principal, rate, and time, given, to find the amount and interest.

R U L E.

Multiply the amount of 1 l. for a year so often into itself as are the number of years given save one; and the last product multiplied by the principal gives the amount; from which subtract the principal, and the remainder is the interest.

E X A M P L E I.

What is the amount and interest of 500 l. for 3 years, at 4 per cent. ?

1.04 amount of 1 l. for a year.

$$\begin{array}{r} 1.04 \\ \hline 1.0816 \\ 1.04 \\ \hline 1.124864 \end{array}$$

500 principal.

$$\begin{array}{r} L. \quad s. \quad d. \\ 562.432000 \text{ amount} = 562 \quad 8 \quad 7\frac{1}{2} \\ 500 \\ \hline \end{array}$$

$$62.432 \quad \text{interest} = 62 \quad 8 \quad 7\frac{1}{2}$$

Or, Multiply the given principal by the amount of 1 l. for a year, so often as there are years in the question.

500 principal given.

1.04 amount of 1 l. for a year.

520 amount for 1 year.

1.04

540.80 amount for 2 years.

1.04

562.4320 amount for 3 years.

E e 2

Or,

Or, Multiply the given principal by the rate of 1 l. and the product is the interest for one year; which, added to the principal, gives the amount for the first year; and work in like manner for each of the remaining years.

500 principal.

.04
20.00 interest.

520 amount.

.04
20.80 interest.

540.80 amount.

.04
21.6320 interest.

562.432 amount.

By practice, thus:

25)500 principal.
20 interest.

25)520 amount for 1 year.
20.8 interest.

25)540.8 amount for 2 years.
21.632 interest.

562.432 amount for 3 years.

If the given time consist of years and parts, or years and days, find the interest of the last amount for the given parts or days, and add the interest so found to the last amount.

EXAMPLE II.

What is the amount and interest of 354 l. 16 s. for $3\frac{1}{2}$ years, at 5 per cent.?

1.05

$$\begin{array}{r}
 1.05 \text{ amount of 1 l.} \\
 1.05 \\
 \hline
 1.1025 \\
 1.05 \\
 \hline
 1.157625 \\
 354.8 \\
 \hline
 40) 410.7253500 \\
 \underline{10.26813375} \\
 420.99348375 \text{ amount.} \\
 \underline{354.8} \\
 66.19348375 \text{ interest.}
 \end{array}$$

By practice :

| | |
|--------------------|---------------------|
| 20) 354.8 | principal. |
| <u>17.74</u> | interest. |
| 20) 372.54 | amount for 1 year. |
| <u>18.627</u> | interest. |
| 20) 391.167 | amount for 2 years. |
| <u>19.55835</u> | interest. |
| 40) 410.72535 | amount for 3 years. |
| <u>10.26813375</u> | interest. |
| 420.99348375 | total amount. |
| <u>354.8</u> | |
| 66.19348375 | total interest. |

The interest for the half-year, in the above example, is computed in the way of simple interest: but it must be observed, that the interest for the half-year found in this manner will be too much; for in simple interest the several amounts are in arithmetical proportion; but in compound interest, the amounts are in geometrical proportion; and consequently the amount of any principal at compound interest, for any number of years, will be more than at simple interest: for one year they will be equal;

equal; but for any time less than a year, the amount, at compound interest, will be less than at simple interest.

To compute the amount for any aliquot part of a year, in this way, observe the following rule, *viz.* Extract that root of the amount of 1 l. for a year, which is denoted by the denominator of the fraction; and the product of the principal into this root, is the amount for the part of the year required.

Thus, for $\frac{1}{4}$ of a year, extract the biquadrate root; for $\frac{1}{2}$ a year extract the square root; so, in our example, the square root of 1.05 is 1.024695; and $410.72535 \times 1.024695 = 420.86821251825$; which is somewhat less than the amount found above.

Again, for $\frac{3}{4}$ of a year, extract the biquadrate root, and raise this root to the third power, the numerator being 3. In like manner, if the given time be days, extract the 365th root, and raise this root to the power denoted by the numerator, or number of days given. But such extractions and involutions will prove a troublesome task, without the help of logarithms.

P R O B. II.

Amount, rate, and time, given, to find the principal or present worth.

R U L E

Divide the given amount by the amount of 1 l. for the given time and rate, and the quot is the principal or present worth; that is, as the amount of 1 l. to 1 l. principal, so the given amount to the principal sought.

E X A M P L E.

What ready money will pay a debt of 562.432 l. due 3 years hence, discounting at 4 per cent. compound interest?

1.04

$$\begin{array}{r}
 1.04 \\
 1.04 \\
 \hline
 1.0816 \\
 1.04 \\
 \hline
 1.124864
 \end{array}
 562.432000(500 \text{ l. } \textit{Ans.} \\
 \underline{5624320}$$

The difference of the given amount and present worth is the discount or rebate, which in the above example is 62.432 l.

If the given time be less than a year, the present worth may be found nearly in the way of simple interest; but, to be accurate, find the amount of 1 l. for the given time, in the way taught above, by which divide the given amount.

P R O B. III.

Principal, amount, and time, given, to find the rate.

R U L E.

Divide the amount by the principal, and that root of the quot which is denoted by the number of years will be the amount of 1 l. for a year, from which subtract 1, and there will remain the rate.

E X A M P L E.

At what rate of compound interest will 500 l. amount to 562.432 in 3 years?

500)562.432(1.124864; and the cube root of 1.124864 is 1.04, and hence .04 is the rate of 1 l.; that is, 4 per cent.

P R O B. IV.

Principal, amount, and rate, given, to find the time.

R U L E.

R U L E

Divide the amount by the principal, and again divide the quot by the amount of 1 l. for a year continually till the quot is 1, and the number of continual divisions is the number of years. Or, Raise the amount of 1 l. in a year to a power equal to the first quot, and the index of that power is the time in years.

E X A M P L E.

In what time will 500 l. amount to 562.432 l. at 4 per cent. compound interest?

$$500 \overline{) 562.4320}$$

$$1.04 \overline{) 1.124864}$$

$$1.04 \overline{) 1.0816}$$

$$1.04 \overline{) 1.04}$$

$$\text{Or, } 1.04$$

$$1.04$$

$$1.0816 \text{ square.}$$

$$1.04$$

$$1.124864 \text{ cube. } \text{Ans. 3 years.}$$

For ease and dispatch in calculations of compound interest, tables, framed for years and for days at all the different rates, are absolutely necessary, and accordingly are universally used. I shall here subjoin three tables of this kind, and then show their construction and use.

TABLE

TABLE I.

The amount of 1 l. for years, compound interest.

| T. | 2 per cent. | 2½ per c. | 3 per cent. | 3½ per c. |
|----|-------------|-----------|-------------|-----------|
| 1 | 1.0200000 | 1.0250000 | 1.0300000 | 1.0350000 |
| 2 | 1.0404000 | 1.0506250 | 1.0609000 | 1.0712250 |
| 3 | 1.0612080 | 1.0768906 | 1.0927270 | 1.1087178 |
| 4 | 1.0824321 | 1.1038128 | 1.1255088 | 1.1475230 |
| 5 | 1.1040808 | 1.1314082 | 1.1592740 | 1.1876863 |
| 6 | 1.1261624 | 1.1596934 | 1.1940523 | 1.2292553 |
| 7 | 1.1486856 | 1.1886857 | 1.2298738 | 1.2722792 |
| 8 | 1.1716593 | 1.2184029 | 1.2667700 | 1.3168089 |
| 9 | 1.1950925 | 1.2488629 | 1.3047731 | 1.3628973 |
| 10 | 1.2189944 | 1.2800845 | 1.3439163 | 1.4105987 |
| 11 | 1.2433743 | 1.3120866 | 1.3842338 | 1.4599697 |
| 12 | 1.2682417 | 1.3448888 | 1.4257608 | 1.5110686 |
| 13 | 1.2936066 | 1.3785110 | 1.4685337 | 1.5639560 |
| 14 | 1.3194787 | 1.4129738 | 1.5125897 | 1.6186945 |
| 15 | 1.3458683 | 1.4482981 | 1.5579674 | 1.6753488 |
| 16 | 1.3727857 | 1.4845056 | 1.6047064 | 1.7339860 |
| 17 | 1.4002414 | 1.5216182 | 1.6528476 | 1.7946755 |
| 18 | 1.4282462 | 1.5596587 | 1.7024330 | 1.8574892 |
| 19 | 1.4568111 | 1.5986501 | 1.7535060 | 1.9225013 |
| 20 | 1.4859474 | 1.6386164 | 1.8061112 | 1.9897888 |
| 21 | 1.5156663 | 1.6795818 | 1.8602945 | 2.0594314 |
| 22 | 1.5459796 | 1.7215714 | 1.9161034 | 2.1315115 |
| 23 | 1.5768992 | 1.7646106 | 1.9735865 | 2.2061144 |
| 24 | 1.6084372 | 1.8087259 | 2.0327941 | 2.2833284 |
| 25 | 1.6406059 | 1.8539441 | 2.0937779 | 2.3632449 |

TABLE I.

The amount of 1 l. for years, compound interest.

| Y. | 2 per cent. | 2½ per c. | 3 per cent. | 3½ per c. |
|----|-------------|-----------|-------------|-----------|
| 26 | 1.6734181 | 1.9002927 | 2.1565912 | 2.4459585 |
| 27 | 1.7068864 | 1.9478000 | 2.2212890 | 2.5315671 |
| 28 | 1.7410242 | 1.9964950 | 2.2879276 | 2.6201719 |
| 29 | 1.7758446 | 2.0464073 | 2.3565655 | 2.7118779 |
| 30 | 1.8113615 | 2.0975675 | 2.4272624 | 2.8067937 |
| 31 | 1.8475888 | 2.1500067 | 2.5000803 | 2.9050314 |
| 32 | 1.8845405 | 2.2037569 | 2.5750827 | 3.0067075 |
| 33 | 1.9222314 | 2.2588508 | 2.6523352 | 3.1119423 |
| 34 | 1.9606760 | 2.3153221 | 2.7319953 | 3.2208603 |
| 35 | 1.9998895 | 2.3732051 | 2.8138624 | 3.3335904 |
| 36 | 2.0398873 | 2.4325353 | 2.8982782 | 3.4502661 |
| 37 | 2.0806850 | 2.4933487 | 2.9852266 | 3.5710254 |
| 38 | 2.1222987 | 2.5556824 | 3.0747834 | 3.6960113 |
| 39 | 2.1647447 | 2.6195744 | 3.1670260 | 3.8253717 |
| 40 | 2.2080396 | 2.6850638 | 3.2620377 | 3.9592597 |
| 41 | 2.2522004 | 2.7521904 | 3.3598989 | 4.0978338 |
| 42 | 2.2972444 | 2.8209952 | 3.4606958 | 4.2412579 |
| 43 | 2.3431893 | 2.8915500 | 3.5645167 | 4.3897020 |
| 44 | 2.3900531 | 2.9638080 | 3.6714522 | 4.5433416 |
| 45 | 2.4378542 | 3.0379032 | 3.7815958 | 4.7023585 |
| 46 | 2.4866112 | 3.1138508 | 3.8950437 | 4.8669411 |
| 47 | 2.5363435 | 3.1916971 | 4.0118950 | 5.0372840 |
| 48 | 2.5870703 | 3.2714895 | 4.1322518 | 5.2135889 |
| 49 | 2.6388117 | 3.3532768 | 4.2562194 | 5.3960645 |
| 50 | 2.6915880 | 3.4371087 | 4.3839060 | 5.5849268 |

TABLE I.

The amount of 1 l. for years, compound interest.

| T. | 4 per cent. | 4 $\frac{1}{2}$ per c. | 5 per cent. | 6 per cent. |
|----|-------------|------------------------|-------------|-------------|
| 1 | 1.0400000 | 1.0450000 | 1.0500000 | 1.0600000 |
| 2 | 1.0816000 | 1.0920250 | 1.1025000 | 1.1236000 |
| 3 | 1.1248640 | 1.1411661 | 1.1576250 | 1.1910160 |
| 4 | 1.1698586 | 1.1925186 | 1.2155063 | 1.2624769 |
| 5 | 1.2166529 | 1.2461819 | 1.2762816 | 1.3382256 |
| 6 | 1.2653190 | 1.3022601 | 1.3400956 | 1.4185191 |
| 7 | 1.3159318 | 1.3608618 | 1.4071034 | 1.5036303 |
| 8 | 1.3685691 | 1.4221006 | 1.4774554 | 1.5938481 |
| 9 | 1.4233118 | 1.4860951 | 1.5513282 | 1.6894790 |
| 10 | 1.4802443 | 1.5529694 | 1.6288946 | 1.7908477 |
| 11 | 1.5394541 | 1.6228530 | 1.7103393 | 1.8982980 |
| 12 | 1.6010322 | 1.6958814 | 1.7958563 | 2.0121965 |
| 13 | 1.6650735 | 1.7721961 | 1.8856491 | 2.1329283 |
| 14 | 1.7316764 | 1.8519449 | 1.9799316 | 2.2609039 |
| 15 | 1.8009435 | 1.9352824 | 2.0789282 | 2.3965582 |
| 16 | 1.8729812 | 2.0223701 | 2.1828746 | 2.5402517 |
| 17 | 1.9479005 | 2.1133768 | 2.2920183 | 2.6927728 |
| 18 | 2.0258165 | 2.2084787 | 2.4066192 | 2.8543392 |
| 19 | 2.1068492 | 2.3078603 | 2.5269502 | 3.0255995 |
| 20 | 2.1911231 | 2.4117140 | 2.6532977 | 3.2071355 |
| 21 | 2.2787681 | 2.5202411 | 2.7859626 | 3.3995636 |
| 22 | 2.3699188 | 2.6336520 | 2.9252607 | 3.6035374 |
| 23 | 2.4647155 | 2.7521663 | 3.0715238 | 3.8197497 |
| 24 | 2.5633042 | 2.8760138 | 3.2251000 | 4.0489346 |
| 25 | 2.6658263 | 3.0054344 | 3.3863519 | 4.2918707 |

TABLE A

The amount of £1 for years, and compound interest.

| T. | 4 per cent. | 4½ per cent. | 5 per cent. | 6 per cent. |
|----|-------------|--------------|-------------|-------------|
| 26 | 2.7724697 | 3.1406799 | 3.5556727 | 4.5493829 |
| 27 | 2.8833685 | 3.2820995 | 3.7334563 | 4.8223459 |
| 28 | 2.9987933 | 3.4296999 | 3.9221291 | 5.1116866 |
| 29 | 3.1189514 | 3.5840364 | 4.1161356 | 5.4183878 |
| 30 | 3.2433975 | 3.7453181 | 4.3219424 | 5.7434911 |
| 31 | 3.3731334 | 3.9138574 | 4.5389395 | 6.0881006 |
| 32 | 3.5080587 | 4.0899819 | 4.7649415 | 6.4533866 |
| 33 | 3.6483811 | 4.2740301 | 5.0031885 | 6.8403898 |
| 34 | 3.7943163 | 4.4663615 | 5.2533480 | 7.2510252 |
| 35 | 3.9460889 | 4.6667347 | 5.5160154 | 7.6860867 |
| 36 | 4.1039325 | 4.8773784 | 5.7918161 | 8.1472519 |
| 37 | 4.2680898 | 5.0996604 | 6.0814069 | 8.6300870 |
| 38 | 4.4388134 | 5.3262192 | 6.3854773 | 9.1342523 |
| 39 | 4.6163659 | 5.5688990 | 6.7047511 | 9.6603574 |
| 40 | 4.8010200 | 5.8283645 | 7.0399887 | 10.2085717 |
| 41 | 4.9930614 | 6.0978100 | 7.3919881 | 10.7802860 |
| 42 | 5.1927839 | 6.3816154 | 7.7615875 | 11.3757032 |
| 43 | 5.4004952 | 6.674381 | 8.1496669 | 11.9954545 |
| 44 | 5.6165150 | 6.981229 | 8.5571503 | 12.6395481 |
| 45 | 5.8411756 | 7.2982484 | 8.9850078 | 13.3096107 |
| 46 | 6.0748227 | 7.6244196 | 9.4342582 | 14.0094873 |
| 47 | 6.3178156 | 7.9652684 | 9.9059711 | 14.7345916 |
| 48 | 6.5705282 | 8.3214555 | 10.4012696 | 15.4898716 |
| 49 | 6.8333493 | 8.6936710 | 10.9213331 | 16.2703539 |
| 50 | 7.1066833 | 9.0826362 | 11.4674000 | 17.0801541 |

TABLE II.

The amount of 1 l. for days, compound interest.

| | 2 per cent. | 2½ per cent. | 3 per cent. | 3½ per cent. |
|-----|-------------|--------------|-------------|--------------|
| 1 | 1.0000542 | 1.0000676 | 1.0000809 | 1.0000942 |
| 2 | 1.0001085 | 1.0001353 | 1.0001619 | 1.0001885 |
| 3 | 1.0001627 | 1.0002029 | 1.0002429 | 1.0002827 |
| 4 | 1.0002170 | 1.0002706 | 1.0003240 | 1.0003778 |
| 5 | 1.0002713 | 1.0003383 | 1.0004050 | 1.0004713 |
| 6 | 1.0003255 | 1.0004059 | 1.0004860 | 1.0005656 |
| 7 | 1.0003798 | 1.0004736 | 1.0005670 | 1.0006600 |
| 8 | 1.0004341 | 1.0005412 | 1.0006480 | 1.0007542 |
| 9 | 1.0004884 | 1.0006090 | 1.0007291 | 1.0008486 |
| 10 | 1.0005426 | 1.0006767 | 1.0008101 | 1.0009429 |
| 20 | 1.0010856 | 1.0013539 | 1.0016209 | 1.0018867 |
| 30 | 1.0016289 | 1.0020315 | 1.0024324 | 1.0028315 |
| 40 | 1.0021725 | 1.0027097 | 1.0032445 | 1.0037771 |
| 50 | 1.0027163 | 1.0033882 | 1.0040573 | 1.0047236 |
| 60 | 1.0032605 | 1.0040673 | 1.0048708 | 1.0056710 |
| 70 | 1.0038049 | 1.0047468 | 1.0056849 | 1.0066193 |
| 80 | 1.0043497 | 1.0054267 | 1.0064996 | 1.0075685 |
| 90 | 1.0048947 | 1.0061071 | 1.0073151 | 1.0085186 |
| 100 | 1.0054401 | 1.0067880 | 1.0081311 | 1.0094696 |
| 110 | 1.0059857 | 1.0074693 | 1.0089479 | 1.0104214 |
| 120 | 1.0065316 | 1.0081511 | 1.0097653 | 1.0113742 |
| 130 | 1.0070779 | 1.0088334 | 1.0105834 | 1.0123279 |
| 140 | 1.0076244 | 1.0095161 | 1.0114021 | 1.0132825 |
| 150 | 1.0081712 | 1.0101993 | 1.0122215 | 1.0142379 |
| 160 | 1.0087183 | 1.0108829 | 1.0130415 | 1.0151943 |

TABLE

TABLE

TABLE II.

The amount of 1. for days, compound interest.

| D. | 2 per cent. | 2½ per c. | 3 per cent. | 3½ per c. |
|------|-------------|-----------|-------------|-----------|
| 170 | 1.0092658 | 1.0115670 | 1.0138623 | 1.0161516 |
| 180 | 1.0098135 | 1.0122511 | 1.0146837 | 1.0171098 |
| 190 | 1.0103613 | 1.0129366 | 1.0155057 | 1.0180689 |
| 200 | 1.0109098 | 1.0136221 | 1.0163284 | 1.0190288 |
| 210 | 1.0114584 | 1.0143081 | 1.0171518 | 1.0199897 |
| 220 | 1.0120073 | 1.0149945 | 1.0179759 | 1.0209515 |
| 230 | 1.0125565 | 1.0156814 | 1.0188006 | 1.0219142 |
| 240 | 1.0131060 | 1.0163687 | 1.0196260 | 1.0228778 |
| 250 | 1.0136558 | 1.0170565 | 1.0204520 | 1.0238424 |
| 260 | 1.0142059 | 1.0177448 | 1.0212788 | 1.0248078 |
| 270 | 1.0147563 | 1.0184336 | 1.0221062 | 1.0257741 |
| 280 | 1.0153070 | 1.0191228 | 1.0229342 | 1.0267414 |
| 290 | 1.0158580 | 1.0198125 | 1.0237630 | 1.0277096 |
| 300 | 1.0164093 | 1.0205026 | 1.0245924 | 1.0286786 |
| 310 | 1.0169609 | 1.0211932 | 1.0254225 | 1.0296486 |
| 320 | 1.0175127 | 1.0218843 | 1.0262532 | 1.0306195 |
| 330 | 1.0180649 | 1.0225758 | 1.0270847 | 1.0315914 |
| 340 | 1.0186174 | 1.0232679 | 1.0279168 | 1.0325641 |
| 350 | 1.0191702 | 1.0239603 | 1.0287495 | 1.0335378 |
| 360 | 1.0197233 | 1.0246533 | 1.0295830 | 1.0345123 |
| 370 | 1.0202766 | 1.0253468 | 1.0304172 | 1.0354878 |
| 380 | 1.0208302 | 1.0260408 | 1.0312521 | 1.0364649 |
| 390 | 1.0213840 | 1.0267353 | 1.0320876 | 1.0374424 |
| 400 | 1.0219380 | 1.0274303 | 1.0329238 | 1.0384204 |
| 410 | 1.0224922 | 1.0281258 | 1.0337606 | 1.0393989 |
| 420 | 1.0230466 | 1.0288218 | 1.0345980 | 1.0403779 |
| 430 | 1.0236012 | 1.0295183 | 1.0354360 | 1.0413574 |
| 440 | 1.0241560 | 1.0302153 | 1.0362746 | 1.0423374 |
| 450 | 1.0247110 | 1.0309128 | 1.0371138 | 1.0433179 |
| 460 | 1.0252662 | 1.0316108 | 1.0379536 | 1.0442989 |
| 470 | 1.0258216 | 1.0323093 | 1.0387940 | 1.0452804 |
| 480 | 1.0263772 | 1.0330083 | 1.0396350 | 1.0462624 |
| 490 | 1.0269330 | 1.0337078 | 1.0404766 | 1.0472449 |
| 500 | 1.0274890 | 1.0344078 | 1.0413188 | 1.0482279 |
| 510 | 1.0280452 | 1.0351083 | 1.0421616 | 1.0492114 |
| 520 | 1.0286016 | 1.0358093 | 1.0430050 | 1.0501954 |
| 530 | 1.0291582 | 1.0365108 | 1.0438490 | 1.0511800 |
| 540 | 1.0297150 | 1.0372128 | 1.0446936 | 1.0521651 |
| 550 | 1.0302720 | 1.0379153 | 1.0455388 | 1.0531507 |
| 560 | 1.0308292 | 1.0386183 | 1.0463846 | 1.0541369 |
| 570 | 1.0313866 | 1.0393218 | 1.0472310 | 1.0551236 |
| 580 | 1.0319442 | 1.0400258 | 1.0480780 | 1.0561108 |
| 590 | 1.0325020 | 1.0407303 | 1.0489256 | 1.0570985 |
| 600 | 1.0330600 | 1.0414353 | 1.0497738 | 1.0580868 |
| 610 | 1.0336182 | 1.0421408 | 1.0506226 | 1.0590756 |
| 620 | 1.0341766 | 1.0428468 | 1.0514720 | 1.0600650 |
| 630 | 1.0347352 | 1.0435533 | 1.0523220 | 1.0610550 |
| 640 | 1.0352940 | 1.0442603 | 1.0531726 | 1.0620456 |
| 650 | 1.0358530 | 1.0449678 | 1.0540238 | 1.0630368 |
| 660 | 1.0364122 | 1.0456758 | 1.0548756 | 1.0640286 |
| 670 | 1.0369716 | 1.0463843 | 1.0557280 | 1.0650210 |
| 680 | 1.0375312 | 1.0470933 | 1.0565810 | 1.0660140 |
| 690 | 1.0380910 | 1.0478028 | 1.0574346 | 1.0670076 |
| 700 | 1.0386510 | 1.0485128 | 1.0582888 | 1.0680018 |
| 710 | 1.0392112 | 1.0492233 | 1.0591436 | 1.0689966 |
| 720 | 1.0397716 | 1.0499343 | 1.0600000 | 1.0699920 |
| 730 | 1.0403322 | 1.0506458 | 1.0608570 | 1.0709880 |
| 740 | 1.0408930 | 1.0513578 | 1.0617146 | 1.0719846 |
| 750 | 1.0414540 | 1.0520703 | 1.0625728 | 1.0729818 |
| 760 | 1.0420152 | 1.0527833 | 1.0634316 | 1.0739796 |
| 770 | 1.0425766 | 1.0534968 | 1.0642910 | 1.0749780 |
| 780 | 1.0431382 | 1.0542108 | 1.0651510 | 1.0759770 |
| 790 | 1.0436999 | 1.0549253 | 1.0660116 | 1.0769766 |
| 800 | 1.0442618 | 1.0556403 | 1.0668728 | 1.0779768 |
| 810 | 1.0448239 | 1.0563558 | 1.0677346 | 1.0789776 |
| 820 | 1.0453862 | 1.0570718 | 1.0685970 | 1.0799790 |
| 830 | 1.0459487 | 1.0577883 | 1.0694600 | 1.0809810 |
| 840 | 1.0465114 | 1.0585053 | 1.0703236 | 1.0819836 |
| 850 | 1.0470742 | 1.0592228 | 1.0711878 | 1.0829868 |
| 860 | 1.0476372 | 1.0599408 | 1.0720526 | 1.0839906 |
| 870 | 1.0482004 | 1.0606593 | 1.0729180 | 1.0849950 |
| 880 | 1.0487638 | 1.0613783 | 1.0737840 | 1.0859990 |
| 890 | 1.0493274 | 1.0620978 | 1.0746506 | 1.0870036 |
| 900 | 1.0498912 | 1.0628178 | 1.0755178 | 1.0880088 |
| 910 | 1.0504552 | 1.0635383 | 1.0763856 | 1.0890146 |
| 920 | 1.0510194 | 1.0642593 | 1.0772540 | 1.0900210 |
| 930 | 1.0515838 | 1.0649808 | 1.0781230 | 1.0910280 |
| 940 | 1.0521484 | 1.0657028 | 1.0789926 | 1.0920356 |
| 950 | 1.0527132 | 1.0664253 | 1.0798628 | 1.0930438 |
| 960 | 1.0532782 | 1.0671483 | 1.0807336 | 1.0940526 |
| 970 | 1.0538434 | 1.0678718 | 1.0816050 | 1.0950620 |
| 980 | 1.0544088 | 1.0685958 | 1.0824770 | 1.0960720 |
| 990 | 1.0549744 | 1.0693203 | 1.0833496 | 1.0970826 |
| 1000 | 1.0555402 | 1.0700453 | 1.0842228 | 1.0980938 |

TABLE II.

The amount of 1 l. for days, compound interest.

| D. | 4 per cent. | 4½ per c. | 5 per cent. | 6 per cent. |
|-----|-------------|-----------|-------------|-------------|
| 1 | 1.0001074 | 1.0001206 | 1.0001346 | 1.0001506 |
| 2 | 1.0002140 | 1.0002412 | 1.0002673 | 1.0003193 |
| 3 | 1.0003224 | 1.0003618 | 1.0004011 | 1.0004700 |
| 4 | 1.0004309 | 1.0004824 | 1.0005348 | 1.0006387 |
| 5 | 1.0005374 | 1.0006031 | 1.0006685 | 1.0007985 |
| 6 | 1.0006440 | 1.0007238 | 1.0008023 | 1.0009583 |
| 7 | 1.0007524 | 1.0008445 | 1.0009301 | 1.0011181 |
| 8 | 1.0008609 | 1.0009652 | 1.0010609 | 1.0012779 |
| 9 | 1.0009675 | 1.0010859 | 1.0012037 | 1.0014378 |
| 10 | 1.0010751 | 1.0012066 | 1.0013376 | 1.0015976 |
| 20 | 1.0021513 | 1.0024148 | 1.0026770 | 1.0031979 |
| 30 | 1.0032488 | 1.0036443 | 1.0040182 | 1.0048007 |
| 40 | 1.0043074 | 1.0048154 | 1.0053611 | 1.0064060 |
| 50 | 1.0053871 | 1.0060479 | 1.0067050 | 1.0080139 |
| 60 | 1.0064680 | 1.0072618 | 1.0080525 | 1.0099024 |
| 70 | 1.0075501 | 1.0084773 | 1.0094000 | 1.0112375 |
| 80 | 1.0086333 | 1.0096942 | 1.0107511 | 1.0128531 |
| 90 | 1.0097177 | 1.0109145 | 1.0121031 | 1.0144711 |
| 100 | 1.0108033 | 1.0121324 | 1.0134569 | 1.0160921 |
| 110 | 1.0118900 | 1.0133517 | 1.0148115 | 1.0177155 |
| 120 | 1.0129779 | 1.0145765 | 1.0161690 | 1.0193415 |
| 130 | 1.0140670 | 1.0158007 | 1.0175291 | 1.0209701 |
| 140 | 1.0151572 | 1.0170264 | 1.0188902 | 1.0226013 |
| 150 | 1.0162487 | 1.0182537 | 1.0202531 | 1.0242351 |
| 160 | 1.0173412 | 1.0194824 | 1.0216178 | 1.0258715 |

TABLE

TABLE

TABLE II.

The amount of 1l. for days, compound interest.

| D. | 4 per cent. | 4½ per c. | 5 per cent. | 6 per cent. |
|-----|-------------|-----------|-------------|-------------|
| 170 | 1.0184350 | 1.0207126 | 1.0229843 | 1.0275105 |
| 180 | 1.0195299 | 1.0219442 | 1.0243527 | 1.0291522 |
| 190 | 1.0206261 | 1.0231774 | 1.0257228 | 1.0307964 |
| 200 | 1.0217233 | 1.0244120 | 1.0270949 | 1.0324433 |
| 210 | 1.0228218 | 1.0256481 | 1.0284687 | 1.0340928 |
| 220 | 1.0239215 | 1.0268858 | 1.0298444 | 1.0357450 |
| 230 | 1.0250233 | 1.0281249 | 1.0312219 | 1.0373998 |
| 240 | 1.0261243 | 1.0293655 | 1.0326013 | 1.0390572 |
| 250 | 1.0272275 | 1.0306076 | 1.0339825 | 1.0407173 |
| 260 | 1.0283319 | 1.0318512 | 1.0353656 | 1.0423800 |
| 270 | 1.0294375 | 1.0330963 | 1.0367505 | 1.0440454 |
| 280 | 1.0305443 | 1.0343429 | 1.0381373 | 1.0457135 |
| 290 | 1.0316522 | 1.0355910 | 1.0395259 | 1.0473842 |
| 300 | 1.0327614 | 1.0368406 | 1.0409164 | 1.0490576 |
| 310 | 1.0338717 | 1.0383917 | 1.0423087 | 1.0507336 |
| 320 | 1.0349832 | 1.0393444 | 1.0437029 | 1.0524124 |
| 330 | 1.0360960 | 1.0405985 | 1.0450990 | 1.0540938 |
| 340 | 1.0372099 | 1.0418542 | 1.0464969 | 1.0557779 |
| 350 | 1.0383250 | 1.0431114 | 1.0478967 | 1.0574647 |
| 360 | 1.0394413 | 1.0443700 | 1.0492984 | 1.0591542 |
| 361 | 1.0395530 | 1.0444960 | 1.0494387 | 1.0593233 |
| 362 | 1.0396648 | 1.0446220 | 1.0495790 | 1.0594924 |
| 363 | 1.0397765 | 1.0447479 | 1.0497193 | 1.0596616 |
| 364 | 1.0398882 | 1.0448739 | 1.0498596 | 1.0598308 |
| 365 | 1.0400000 | 1.0450000 | 1.0500000 | 1.0600000 |

TABLE

TABLE III

The present worth of 1 l. for years, compound interest.

| T. | 2 per c. | 2½ per c. | 3 per c. | 3½ per c. | Q |
|----|----------|-----------|----------|-----------|----|
| 1 | .9803921 | .9750097 | .9708738 | .9661836 | 01 |
| 2 | .9611687 | .9518144 | .9425959 | .9335197 | 02 |
| 3 | .9423223 | .9285994 | .9151417 | .9019427 | 03 |
| 4 | .9238454 | .9050506 | .8884879 | .8714422 | 04 |
| 5 | .9057398 | .8838542 | .8626988 | .8410732 | 05 |
| 6 | .8879713 | .8622968 | .8374843 | .8135006 | 06 |
| 7 | .8705601 | .8412654 | .8130015 | .7859910 | 07 |
| 8 | .8534993 | .8207465 | .7894092 | .7594116 | 08 |
| 9 | .8367552 | .8007283 | .7664167 | .7337310 | 09 |
| 10 | .8203283 | .7811984 | .7440939 | .7089188 | 10 |
| 11 | .8042630 | .7621447 | .7224213 | .6849457 | 11 |
| 12 | .7884931 | .7435558 | .7013709 | .6617823 | 12 |
| 13 | .7730325 | .7254263 | .6809513 | .6394041 | 13 |
| 14 | .7578750 | .7077272 | .6611178 | .6177818 | 14 |
| 15 | .7430147 | .6904655 | .6418610 | .5968906 | 15 |
| 16 | .7284458 | .6736249 | .6231660 | .5767059 | 16 |
| 17 | .7141625 | .6571950 | .6050164 | .5572038 | 17 |
| 18 | .7001593 | .6411659 | .5873946 | .5383611 | 18 |
| 19 | .6864307 | .6255277 | .5702860 | .5201557 | 19 |
| 20 | .6729713 | .6102709 | .5536758 | .5025059 | 20 |
| 21 | .6597758 | .5953862 | .5375493 | .4855709 | 21 |
| 22 | .6468390 | .5808646 | .5218925 | .4691506 | 22 |
| 23 | .6341559 | .5666972 | .5066917 | .4532856 | 23 |
| 24 | .6217244 | .5528753 | .4919337 | .4379571 | 24 |
| 25 | .6095308 | .5393905 | .4776056 | .4231470 | 25 |

T A B L E III.

The present worth of 1 l. for years, compound interest.

| Y. | 2 per c. | 2½ per c. | 3 per c. | 3½ per c. |
|----|----------|-----------|----------|-----------|
| 26 | .5975793 | .5262347 | .4636947 | .4088378 |
| 27 | .5858620 | .5133997 | .4501891 | .3950123 |
| 28 | .5743746 | .5008778 | .4370768 | .3816543 |
| 29 | .5631123 | .4886613 | .4243464 | .3687482 |
| 30 | .5520709 | .4767427 | .4119868 | .3562784 |
| 31 | .5412460 | .4651148 | .3999871 | .3442304 |
| 32 | .5306333 | .4537706 | .3883370 | .3325897 |
| 33 | .5202287 | .4427030 | .3770263 | .3213427 |
| 34 | .5100282 | .4319053 | .3660449 | .3104761 |
| 35 | .5000276 | .4213711 | .3553834 | .2999769 |
| 36 | .4902232 | .4110937 | .3450324 | .2898327 |
| 37 | .4806109 | .4010671 | .3349829 | .2800316 |
| 38 | .4711872 | .3912849 | .3252262 | .2705619 |
| 39 | .4619482 | .3817414 | .3157536 | .2614125 |
| 40 | .4528904 | .3724306 | .3065568 | .2525725 |
| 41 | .4440102 | .3633470 | .2976280 | .2440314 |
| 42 | .4353041 | .3544848 | .2889592 | .2357791 |
| 43 | .4267688 | .3458389 | .2805429 | .2278059 |
| 44 | .4184008 | .3374038 | .2723718 | .2201023 |
| 45 | .4101968 | .3291744 | .2644386 | .2126594 |
| 46 | .4021537 | .3211458 | .2567365 | .2054679 |
| 47 | .3942684 | .3133129 | .2492588 | .1985197 |
| 48 | .3865376 | .3056712 | .2419988 | .1918065 |
| 49 | .3789584 | .2982158 | .2349503 | .1853202 |
| 50 | .3715279 | .2909422 | .2281071 | .1790534 |

TABLE

TABLE III.

The present worth of 1 l. for years, compound interest.

| Y. | 4 per c. | 4½ per c. | 5 per c. | 6 per c. |
|----|----------|-----------|----------|----------|
| 1 | .9615385 | .9569378 | .9523810 | .9433962 |
| 2 | .9245562 | .9157299 | .9070295 | .8899964 |
| 3 | .8889964 | .8762966 | .8638376 | .8396193 |
| 4 | .8548042 | .8385613 | .8227025 | .7920937 |
| 5 | .8219271 | .8024511 | .7835262 | .7472582 |
| 6 | .7903145 | .7678957 | .7462154 | .7049605 |
| 7 | .7599178 | .7348285 | .7106813 | .6650571 |
| 8 | .7306902 | .7031851 | .6768394 | .6274124 |
| 9 | .7025867 | .6729044 | .6446089 | .5918985 |
| 10 | .6755642 | .6439277 | .6139133 | .5583948 |
| 11 | .6495809 | .6161987 | .5846793 | .5267875 |
| 12 | .6245971 | .5896639 | .5568374 | .4969694 |
| 13 | .6005741 | .5642716 | .5303214 | .4688390 |
| 14 | .5774751 | .5399729 | .5050679 | .4423010 |
| 15 | .5552645 | .5167204 | .4810171 | .4172651 |
| 16 | .5339082 | .4944693 | .4581115 | .3936463 |
| 17 | .5133733 | .4731764 | .4362967 | .3713644 |
| 18 | .4936281 | .4528004 | .4155207 | .3503438 |
| 19 | .4746424 | .4333018 | .3957340 | .3305130 |
| 20 | .4563870 | .4146429 | .3768895 | .3118047 |
| 21 | .4388336 | .3967874 | .3589424 | .2941554 |
| 22 | .4219554 | .3797009 | .3418499 | .2775051 |
| 23 | .4057263 | .3633501 | .3255713 | .2617973 |
| 24 | .3901215 | .3477035 | .3100679 | .2469786 |
| 25 | .3751168 | .3327306 | .2953028 | .2329986 |

TABLE III.

The present worth of 1 l. for years, compound interest.

| Y. | 4 per c. | 4½ per c. | 5 per c. | 6 per c. |
|----|----------|-----------|----------|----------|
| 26 | .3606892 | .3184025 | .2812407 | .2198100 |
| 27 | .3468166 | .3046914 | .2678483 | .2073680 |
| 28 | .3334775 | .2915707 | .2550936 | .1956301 |
| 29 | .3206514 | .2790150 | .2429463 | .1845567 |
| 30 | .3083187 | .2670000 | .2313775 | .1741101 |
| 31 | .2964603 | .2555024 | .2203595 | .1642548 |
| 32 | .2850579 | .2444999 | .2098662 | .1549574 |
| 33 | .2740942 | .2339712 | .1998726 | .1461862 |
| 34 | .2635521 | .2238959 | .1903548 | .1379115 |
| 35 | .2534155 | .2142544 | .1812903 | .1301052 |
| 36 | .2436687 | .2050282 | .1726574 | .1227408 |
| 37 | .2342969 | .1961992 | .1644356 | .1157932 |
| 38 | .2252854 | .1877504 | .1566054 | .1092389 |
| 39 | .2166206 | .1796655 | .1491479 | .1030555 |
| 40 | .2082890 | .1719287 | .1420457 | .0972222 |
| 41 | .2002779 | .1645251 | .1352816 | .0917191 |
| 42 | .1925749 | .1574403 | .1288396 | .0865274 |
| 43 | .1851682 | .1506605 | .1227044 | .0816296 |
| 44 | .1180464 | .1441728 | .1168613 | .0770091 |
| 45 | .1711984 | .1379644 | .1112965 | .0726501 |
| 46 | .1646139 | .1320233 | .1059967 | .0685378 |
| 47 | .1582826 | .1263381 | .1009492 | .0646583 |
| 48 | .1521948 | .1208977 | .0961421 | .0609984 |
| 49 | .1463411 | .1156916 | .0915639 | .0575457 |
| 50 | .1407126 | .1107097 | .0872037 | .0542884 |

Table .

Table I. shews the amount of 1 l. principal for years; and is constructed by the rule assigned in Prob. 1. *viz.* Multiply the amount of 1 l. for a year by-itself continually. Thus, at 4 *per cent.* the amount of 1 l. is 1.04 for 1 year; and $1.04 \times 1.04 = 1.0816$ for 2 years; and $1.0816 \times 1.04 = 1.124864$ for 3 years, &c.

Table II. shews the amount of 1 l. for days; to construct which, find, by the directions given in the end of Prob. 1. the amount of 1 l. for one day; and this amount multiplied into itself continually, will give the amount for any number of days.

Example at 5 per cent.

1.00013368 the amount for 1 day.

1.00013368

Prod. 1.00026738 the amount for 2 days,

1.00013368

Prod. 1.00040110 the amount for 3 days.

Table III. shews the present worth of 1 l. principal for years; and is constructed by the rule assigned in Prob. 2. *viz.* Divide 1 l. by the amount of 1 l. for the given time and rate, as contained in Table I.

Example at 4 per cent.

1.04)1(.9615385 for 1 year.

1.0816)1(.9245562 for 2 years.

1.124864)1(.8889964 for 3 years, &c.

The table may, in this manner, be continued to any number of years whatever.

The use of Table I.

1. Principal, rate, and time, given, to find the amount. Multiply the given principal by the tabular number corresponding to the rate and time.

Ex.

Ex. What will 500 l. principal amount to in 15 years, at 5 per cent. compound interest?

$$\begin{array}{r} 2.0789282 \\ \hline 500 \\ 1039.4641000 = 1039 \quad 9 \quad 3\frac{1}{4} \end{array} \quad \begin{array}{l} L. \\ s. \\ d. \end{array}$$

If the given principal consist of pounds, shillings, pence, reduce the shillings and pence to a decimal, and then proceed as above.

2. Principal, amount, and time, given, to find the rate.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find in the table even with the given time, and you have the rate on the head.

Ex. At what rate of interest will 500 l. amount to 1039.4641 l. in 15 years?

$$\begin{array}{r} 500)1039.4641 \\ \hline 2.0789282 \text{ found under } 5 \text{ per cent.} \end{array}$$

3. Principal, amount, and rate, given, to find the time.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find under the given rate, and on the side you have the time.

Ex. In what time will 500 l. amount to 1039.4641 l. at 5 per cent. compound interest?

$$\begin{array}{r} 500)1039.4641 \\ \hline 2.0789282 \text{ found opposite to } 15 \text{ years.} \end{array}$$

4. Amount, rate, and time, given, to find the principal.

Divide the given amount by the tabular amount of 1 l. for the rate and time given, and the quot will be the principal.

Ex.

Ex. What principal will amount to 1039.4641 l. in 15 years, at 5 per cent.?

2.0789282)1039.4641(500 principal.

The use of Table II.

This table shews the amount of 1 l. for days, and is used in the same manner as the former.

But if the given number of days be not in this table; or if the given number of years be not in Table I. work as follows.

Divide the given number of days or years into two such numbers as are found in the tables, multiply the tabular amounts into one another, and their product is the amount of 1 l. for the time given.

Ex. What will 500 l. amount to in 75 years and 184 days, at 5 per cent.?

Amount for 50 years, at 5 per cent. is, 11.4674000

Amount for 25 years, at 5 per cent. is, 3.3863549

Their product is the amount for 75 years, 38.8326862

Amount for 180 days, is, — 1.0243527

Amount for 4 days, is, — 1.0005348

Their product is the amount for 184 days, 1.0249005

Amount for 75 years, is, — 38.8326862

Amount for 184 days, is, — 1.0249005

Their product is the amount of 1 l. for } 39.7996395
75 years and 184 days,

Multiply by the principal, 500

Amount sought, — — 19899.8197500

And so, 500 l. in 75 years and 184 days, at 5 per cent. will amount to 19899 l. 16s. 4½d. And in this manner may these two tables be extended to any number of years and days.

The

The use of Table III.

Amount, rate, and time, given, to find the principal or present worth.

Multiply the tabular number answering to the rate and time, by the given amount.

Ex. What ready money will pay a debt of 562.432 l. due 3 years hence, discounting at 4 per cent. compound interest?

$.8889964 \times 562.432 = 500$ l. the present worth sought.

I shall here subjoin two or three practical questions.

Quest. 1. What will L. 136 : 15 : 6 amount to in 20 years, at 6 per cent. compound interest?

By Table I.

3.2071355 amount of 1 l.
577.631 multiplier inverted.

32071355
 9621406
 1924281
 224499
 22449
 1603

438.65593 amount sought.

Quest. 2. What will 5 l. amount to in 400 years, at 5 per cent. compound interest?

By

By Table I.

The amount of 1 l. is, 11.4674 for 50 years.

11.4674

131.5013 for 100 years.

131.5013

17292.5919 for 200 years.

17292.5919

297650327.2679 for 400 years.

5

Amount of 5 l. is, 1488251636.3395 for 400 years.

The above example shows the rapid surprising increase of numbers in geometrical proportion; and for the learner's amusement, it may not be improper to observe, that if a single farthing had been lent on compound interest, at 5 *per cent.* in the first year of the Christian æra, or at the birth of Christ, and had continued to this present year 1766, the amount would have been such an immense sum as to surpass in value some millions of globes of solid gold, each as large as this earth.

Quest. 3. A owes B several sums, *viz.* 180 l. due 3 years hence; 200 l. due 5 years hence; and 300 l. due 7 years hence: What is the present worth of these debts, discounting at 4 *per cent.* compound interest?

By Table III.

$$.8889964 \times 180 = 160.0193520$$

$$.8219271 \times 200 = 164.3854200$$

$$.7599178 \times 300 = 227.9753400$$

$$\text{Present worth, } \underline{552.380112}$$

C H A P. XIII.

ANNUITIES.

AN annuity is a sum of money, payable yearly, half-yearly, or quarterly, to continue a certain number of years, for ever, or for life.

An annuity is said to be in arrear, when it continues unpaid after it falls due. And an annuity is said to be in reversion, when the purchaser, upon paying the price, does not immediately enter upon possession; the annuity not commencing till some time after.

Interest on annuities may be computed either in the way of simple or compound interest. But compound interest being found most equitable, both for buyer and seller, the computation by simple interest is universally disused.

I. Annuities for a certain time.

P R O B. I.

Annuity, rate, and time, given, to find the amount, or sum of yearly payments, and interest.

R U L E.

Make 1 the first term of a geometrical series, and the amount of 1 l. for a year the common ratio; continue this series to as many terms as there are years in the question; and the sum of this series is the amount of 1 l. annuity for the given years; which multiplied by the given annuity, will produce the amount sought.

E X A M P L E.

An annuity of 40 l. payable yearly is forborn and unpaid till the end of 5 years: What will then be due, reckoning compound interest at 5 per cent. on all the payments then in arrear?

1 2 3 4 5
 1 : 1.05 : 1.1025 : 1.157625 : 1.21550625; whose
 sum is 5.52563125 l.; and $5.52563125 \times 40 =$
 $221.02525 = 221 \text{ l. } 0 \text{ s. } 6 \text{ d.}$ the amount sought.

The amount may also be found thus: Multiply the given annuity by the amount of 1 l. for a year; to the product add the given annuity, and the sum is the amount in 2 years; which multiply by the amount of 1 l. for a year; to the product add the given annuity, and the sum is the amount in 3 years, &c. The former question wrought in this manner follows.

| | |
|-----------------------|---------------------------|
| 40 am. in 1 year. | 126.1 am. in 3 years. |
| <u>1.05</u> | <u>1.05</u> |
| 42.00 | 132.405 |
| <u>40</u> | <u>40</u> |
| 82 am. in 2 years. | 172.405 am. in 4 years. |
| <u>1.05</u> | <u>1.05</u> |
| 86.10 | 181.02525 |
| <u>40</u> | <u>40</u> |
| 126.1 am. in 3 years. | 221.02525 am. in 5 years. |

If the given time be years and quarters, find the amount for the whole years, as above; then find the amount of 1 l. for the given quarters; by which multiply the amount for the whole years; and to the product add such a part of the annuity as the given quarters are of a year.

If the given annuity be payable half-yearly, or quarterly, find the amount of 1 l. for half a year or a quarter; by which find the amount for the several half-years or quarters, in the same manner as the amount for the several years is found above.

P R O B. H.

Annuity, rate, and time, given, to find the present worth, or sum of money that will purchase the annuity.

H h 2

R U L E.

R U L E.

Find the amount of the given annuity by the former problem; and then, by Prob. 2. in compound interest, find the present worth of this amount, as a sum due at the end of the given time.

E X A M P L E.

What is the present worth of an annuity of 40*l.* to continue 5 years, discounting at 5 *per cent.* compound interest?

By the former problem, the amount of the given annuity for 5 years, at 5 *per cent.* is 221.02525; and by Prob. 1. in compound interest, the amount of 1*l.* for 5 years, at 5 *per cent.* is 1.2762815625

And, $1.2762815625)221.02525000(173.179 =$
173*l.* 3*s.* 7*d.* the present worth sought.

The present worth may also be found thus: By Prob. 2. of compound interest, find the present worth of each year by itself, and the sum of these is the present worth sought. The former example done in this way follows.

| | |
|---------------------------|----------|
| 1.2762815625)40.000000000 | (31.3410 |
| 1.21550625)40.0000000 | (32.9080 |
| 1.157625)40.00000 | (34.5535 |
| 1.1025)40.000 | (36.2811 |
| 1.05)40.0 | (38.0952 |
| Present worth, | 173.1788 |

A third way of finding the present worth, take as follows, *viz.* Find a principal sum, whereof the given annuity is one year's interest; then find the present worth of this principal, as a sum due at the end of the given time; subtract this present worth from its principal; and the remainder is the present worth sought. The former question done in this manner follows.

As

As .05 : 1 :: 40 : 800 l. principal.

$$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$$

$$\text{And } 1.2762815625 \times 800.00000000 = 626.8208$$

800

626.8208

173.1792 present worth sought.

If the annuity to be purchased be in reversion, find first the present worth of the annuity, as commencing immediately, by any of the three methods taught above; and then, by Prob. 2. of compound interest, find the present worth of that present worth, rebating for the time in reversion; and this last present worth is the answer.

Ex. What is the present worth of a yearly pension or rent of 75 l. to continue 4 years, but not to commence till 3 years hence, discounting at 5 per cent.?

$$.05 : 1 :: 75 : 1500$$

$$1.05 \times 1.05 \times 1.05 \times 1.05 = 1.21550625$$

$$1.21550625 \times 1500.00000 = 1234.05371$$

1500

1234.05371

265.94629 present worth of the annuity, if it was to commence immediately.

$$1.05 \times 1.05 \times 1.05 = 1.157625 \quad L. \quad s. \quad d.$$

$$1.157625 \times 265.94629 = 229.7344 = 229 \quad 14 \quad 8\frac{1}{4}$$

PROB. III.

Present worth, rate, and time, given, to find the annuity.

R U L E

By the preceding problem, find the present worth of 1 l. annuity for the rate and time given; and then say, As the present worth thus found to 1 l. annuity, so the present worth given to its annuity; that is, divide the given present worth by that of 1 l. annuity.

EX.

E X A M P L E.

What annuity, to continue 5 years, will 173 l. 3 s. 7 d. purchase, allowing compound interest at 5 per cent.?

$$.05 : 1 :: 1 : 201.$$

$$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$$

$$1.2762815625)20.000000000(15.6705$$

20

15.6705

4.3295 present worth of 1 l. annuity.

4.329)173.179(40 l. annuity. *Ans.*

The operations in annuities, as well as in compound interest, turn out heavy and troublesome; and on that account tables are universally used. I shall here annex three tables of that sort; and to the tables shall subjoin a brief illustration of their uses. And in regard there will sometimes be occasion for the promiscuous use of these tables on annuities, and the three former tables on compound interest, the following tables shall be numbered Table IV. V. VI.; by which means any table to be used will be easily referred to.

Table IV. shows the amount of 1 l. annuity, and may be constructed from Prob. I.; or rather, thus: To 1 l. the first year of this table, add the first year of Table I. and the sum is the second year of this table; to which add the second year of Table I. and the sum will be the third year of this table, &c. The reason is obvious.

Table V. shews the present worth of 1 l. annuity; and may be constructed from Prob. II. or more easily from Table III.; thus: The first year in Table III. and V. is the same; the sum of the first and second years in Table III. make the second year in Table V.; and the sum of the second and third years in Table III. make the third year in Table V. &c.

Table VI. shews the annuity which 1 l. will purchase; and may be constructed from Prob. III.; but more readily thus: Divide 1 by the first year of Table V. and the quot is the first year of Table VI. Again, divide 1 by the second year of Table V. and the quot is the second year of Table VI. &c.

TABLE

TABLE IV.

The amount of 1 l. annuity, compound interest.

| <i>T.</i> | <i>2 per cent.</i> | <i>2 $\frac{1}{2}$ per c.</i> | <i>3 per cent.</i> | <i>3 $\frac{1}{2}$ per c.</i> |
|-----------|--------------------|--|--------------------|--|
| 1 | 1.0000000 | 1.0000000 | 1.0000000 | 1.0000000 |
| 2 | 2.0200000 | 2.0250000 | 2.0300000 | 2.0350000 |
| 3 | 3.0604000 | 3.0756230 | 3.0909000 | 3.1062250 |
| 4 | 4.1216080 | 4.1525156 | 4.1836270 | 4.2149429 |
| 5 | 5.2040402 | 5.2563285 | 5.3091358 | 5.3624659 |
| 6 | 6.3081210 | 6.3877367 | 6.4684099 | 6.5501522 |
| 7 | 7.4342834 | 7.5474302 | 7.6624622 | 7.7794075 |
| 8 | 8.5829691 | 8.7361159 | 8.8923360 | 9.0516866 |
| 9 | 9.7546284 | 9.9545188 | 10.1591061 | 10.3684958 |
| 10 | 10.9497210 | 11.2033818 | 11.4638793 | 11.7313931 |
| 11 | 12.1687154 | 12.4834663 | 12.8077957 | 13.1419919 |
| 12 | 13.4120897 | 13.7955530 | 14.1920296 | 14.6019616 |
| 13 | 14.6803315 | 15.1404418 | 15.6177904 | 16.1130303 |
| 14 | 15.9739381 | 16.5189528 | 17.0863242 | 17.6769864 |
| 15 | 17.2934169 | 17.9319267 | 18.5989139 | 19.2956809 |
| 16 | 18.6392853 | 19.3802248 | 20.1568813 | 20.9710297 |
| 17 | 20.0120719 | 20.8647304 | 21.7615877 | 22.7050158 |
| 18 | 21.4123124 | 22.3863487 | 23.4144354 | 24.4996913 |
| 19 | 22.8405586 | 23.9460074 | 25.1168684 | 26.3571805 |
| 20 | 24.2973698 | 25.5446576 | 26.8703745 | 28.2796818 |
| 21 | 25.7833172 | 27.1832740 | 28.6764857 | 30.2694707 |
| 22 | 27.2989835 | 28.8628559 | 30.5367803 | 32.3289022 |
| 23 | 28.8449632 | 30.5844273 | 32.4528837 | 34.4604137 |
| 24 | 30.4218625 | 32.3490379 | 34.4264702 | 36.6665282 |
| 25 | 32.0302997 | 34.1577639 | 36.4592643 | 38.9498567 |

TABLE

TABLE IV.

The amount of 1 l. annuity, compound interest.

| <i>T.</i> | <i>2 per cent.</i> | <i>2½ per c.</i> | <i>3 per cent.</i> | <i>3½ per c.</i> |
|-----------|--------------------|------------------|--------------------|------------------|
| 26 | 33.6709057 | 36.0117080 | 38.5530422 | 41.3131017 |
| 27 | 35.3443238 | 37.9120007 | 40.7096335 | 43.7590602 |
| 28 | 37.0512103 | 39.8598008 | 42.9309225 | 46.2906273 |
| 29 | 38.7922345 | 41.8562958 | 45.2188502 | 48.9107993 |
| 30 | 40.5680792 | 43.9027032 | 47.5754157 | 51.6226773 |
| 31 | 42.3794408 | 46.0002707 | 50.0026782 | 54.4294710 |
| 32 | 44.2270296 | 48.1502775 | 52.5027585 | 57.3345025 |
| 33 | 46.1115702 | 50.3540345 | 55.0778413 | 60.3412101 |
| 34 | 48.0338016 | 52.6128653 | 57.7301765 | 63.4531524 |
| 35 | 49.9944776 | 54.9282074 | 60.4620818 | 66.6740127 |
| 36 | 51.9943672 | 57.3014126 | 63.2759443 | 70.0076032 |
| 37 | 54.0342545 | 59.7339479 | 66.1742226 | 73.4578693 |
| 38 | 56.1149396 | 62.2272966 | 69.1594493 | 77.0288947 |
| 39 | 58.2372384 | 64.7829791 | 72.2342327 | 80.7249060 |
| 40 | 60.4019832 | 67.4025535 | 75.4012597 | 84.5502778 |
| 41 | 62.6100228 | 70.0876174 | 78.6632975 | 88.5095375 |
| 42 | 64.6822233 | 72.8398078 | 82.0231964 | 92.6073713 |
| 43 | 67.1594678 | 75.6608030 | 85.4838923 | 96.8486293 |
| 44 | 69.5026511 | 78.5523231 | 89.0484191 | 101.2383313 |
| 45 | 71.8927103 | 81.5161312 | 92.7198614 | 105.7816729 |
| 46 | 74.3305645 | 84.5540344 | 96.5014172 | 110.4840315 |
| 47 | 76.8171758 | 87.6678853 | 100.3965009 | 115.3509726 |
| 48 | 79.3535193 | 90.8595824 | 104.4083960 | 120.3882566 |
| 49 | 81.9405697 | 94.1310729 | 108.5406479 | 125.6018456 |
| 50 | 84.5794015 | 97.4843488 | 112.7968673 | 130.9979102 |

TABLE

TABLE IV.

The amount of 1 l. annuity, compound interest.

| Y. | 4 per cent. | 4 $\frac{1}{2}$ per c. | 5 per cent. | 6 per cent. |
|----|-------------|------------------------|-------------|-------------|
| 1 | 1.0000000 | 1.0000000 | 1.0000000 | 1.0000000 |
| 2 | 2.0400000 | 2.0450000 | 2.0500000 | 2.0600000 |
| 3 | 3.1216000 | 3.1370250 | 3.1525000 | 3.1836000 |
| 4 | 4.2464640 | 4.2781911 | 4.3101250 | 4.3746016 |
| 5 | 5.4163226 | 5.4707097 | 5.5256312 | 5.6370930 |
| 6 | 6.6329755 | 6.7168917 | 6.8019128 | 6.9753187 |
| 7 | 7.8982945 | 8.0191518 | 8.1420084 | 8.3938378 |
| 8 | 9.2142263 | 9.3800136 | 9.5491089 | 9.8974681 |
| 9 | 10.5827953 | 10.8021142 | 11.0265643 | 11.4913162 |
| 10 | 12.0061071 | 12.2882094 | 12.5778925 | 13.1807958 |
| 11 | 13.4863514 | 13.8411788 | 14.2067871 | 14.9716435 |
| 12 | 15.0258055 | 15.4640318 | 15.9171265 | 16.8699420 |
| 13 | 16.6268377 | 17.1599133 | 17.7129828 | 18.8821385 |
| 14 | 18.2919112 | 18.9321094 | 19.5986320 | 21.0150667 |
| 15 | 20.0235876 | 20.7840543 | 21.5785636 | 23.2759707 |
| 16 | 21.8245311 | 22.7193367 | 23.6574918 | 25.6725289 |
| 17 | 23.6975124 | 24.7417069 | 25.8403664 | 28.2128806 |
| 18 | 25.6454129 | 26.8550837 | 28.1323847 | 30.9056534 |
| 19 | 27.6712294 | 29.0635625 | 30.5390039 | 33.7599925 |
| 20 | 29.7780786 | 31.3714228 | 33.0659541 | 36.7855920 |
| 21 | 31.9692017 | 33.7831368 | 35.7192518 | 39.9927275 |
| 22 | 34.2479698 | 36.3033779 | 38.5052144 | 43.3922911 |
| 23 | 36.6178886 | 38.9370299 | 41.4304751 | 46.9958285 |
| 24 | 39.0826041 | 41.6891963 | 44.5019989 | 50.8155782 |
| 25 | 41.6459083 | 44.5652101 | 47.7270988 | 54.8645128 |

TABLE IV.

The amount of 1 l. annuity, compound interest.

| T. | 4 per cent. | 4½ per c. | 5 per cent. | 6 per cent. |
|----|-------------|-------------|-------------|-------------|
| 26 | 44.3117446 | 47.5706446 | 51.1134538 | 59.1563827 |
| 27 | 47.0842144 | 50.7113236 | 54.6691265 | 63.7057657 |
| 28 | 49.9675830 | 53.9933332 | 58.4025828 | 68.5281116 |
| 29 | 52.9662863 | 57.4230332 | 62.3227119 | 73.6397983 |
| 30 | 56.0849377 | 61.0070698 | 66.4388475 | 69.0581862 |
| 31 | 59.3283352 | 64.7523878 | 70.7607899 | 84.8016774 |
| 32 | 62.7014687 | 68.6662452 | 75.2988294 | 90.8897780 |
| 33 | 66.2095274 | 72.7562263 | 80.0637708 | 97.3431647 |
| 34 | 69.8579085 | 77.0302565 | 85.0669594 | 104.1837546 |
| 35 | 73.6522248 | 81.4966180 | 90.3203073 | 111.4347799 |
| 36 | 77.5983138 | 86.1639658 | 95.8363227 | 119.1208667 |
| 37 | 81.7022464 | 91.0413443 | 101.6281388 | 127.2681187 |
| 38 | 85.9703362 | 96.1382048 | 107.7095458 | 135.9042058 |
| 39 | 90.4091497 | 101.4644240 | 114.0950231 | 145.0584581 |
| 40 | 95.0255157 | 107.0303231 | 120.7997742 | 154.7619656 |
| 41 | 99.8265363 | 112.8466876 | 127.8397829 | 165.0476836 |
| 42 | 104.8195978 | 118.9247885 | 135.2317511 | 175.9505446 |
| 43 | 110.0123817 | 125.2764040 | 142.9933386 | 187.5075772 |
| 44 | 115.4128169 | 131.9138422 | 151.1430056 | 199.7580319 |
| 45 | 121.0293920 | 138.8499651 | 159.7001559 | 212.7435138 |
| 46 | 126.8705677 | 146.0982135 | 168.6851637 | 226.5081246 |
| 47 | 132.9453904 | 153.6726331 | 178.1194218 | 241.0986121 |
| 48 | 139.2632060 | 161.5879016 | 188.0253929 | 256.5645288 |
| 49 | 145.8337342 | 169.8593572 | 198.4266626 | 272.9584006 |
| 50 | 152.6670836 | 178.5030282 | 209.3479957 | 290.3359046 |

TABLE

TABLE V.

The present worth of 1 l. annuity, compound interest.

| <i>T.</i> | <i>2 per cent.</i> | <i>2½ per c.</i> | <i>3 per cent.</i> | <i>3½ per c.</i> |
|-----------|--------------------|------------------|--------------------|------------------|
| 1 | 0.9803922 | 0.9756098 | 0.9708738 | 0.9661836 |
| 2 | 1.9415609 | 1.9274242 | 1.9134697 | 1.8996943 |
| 3 | 2.8838833 | 2.8560236 | 2.8286114 | 2.8016371 |
| 4 | 3.8077287 | 3.7619742 | 3.7170984 | 3.6730792 |
| 5 | 4.7134595 | 4.6458285 | 4.5797072 | 4.5150524 |
| 6 | 5.6014309 | 5.5081254 | 5.4171914 | 5.3285530 |
| 7 | 6.4719911 | 6.3493906 | 6.2302829 | 6.1145439 |
| 8 | 7.3254814 | 7.1701372 | 7.0196922 | 6.8739555 |
| 9 | 8.1622367 | 7.9708655 | 7.7861089 | 7.6076865 |
| 10 | 8.9825850 | 8.7520639 | 8.5302028 | 8.3166053 |
| 11 | 9.7868480 | 9.5142087 | 9.2526241 | 9.0015510 |
| 12 | 10.5753412 | 10.2577646 | 9.9540040 | 9.6633343 |
| 13 | 11.3483737 | 10.9831839 | 10.6349553 | 10.3027385 |
| 14 | 12.1062487 | 11.6909122 | 11.2960731 | 10.9205203 |
| 15 | 12.8492635 | 12.3813777 | 11.9379351 | 11.5174109 |
| 16 | 13.5777093 | 13.0550027 | 12.5611020 | 12.0941168 |
| 17 | 14.2918719 | 13.7121977 | 13.1661185 | 12.6513206 |
| 18 | 14.9920313 | 14.3533636 | 13.7535131 | 13.1896817 |
| 19 | 15.6784620 | 14.9788913 | 14.3237991 | 13.7098374 |
| 20 | 16.3514333 | 15.5891623 | 14.8774748 | 14.2124033 |
| 21 | 17.0112092 | 16.1845486 | 15.4150241 | 14.6979742 |
| 22 | 17.6580482 | 16.7654132 | 15.9369166 | 15.1671248 |
| 23 | 18.2922041 | 17.3321105 | 16.4436084 | 15.6204105 |
| 24 | 18.9139256 | 17.8849858 | 16.9355421 | 16.0583676 |
| 25 | 19.5234565 | 18.424764 | 17.4131477 | 16.4811146 |

TABLE V.

The present worth of 1 l. annuity, compound interest.

| T. | 2 per cent. | 2½ per c. | 3 per cent. | 3½ per c. |
|----|-------------|------------|-------------|------------|
| 26 | 20.1210358 | 18.9506111 | 17.8768429 | 16.8903523 |
| 27 | 20.7068978 | 19.4640109 | 18.3270315 | 17.2853645 |
| 28 | 21.2812724 | 19.9648887 | 18.7641082 | 17.6670188 |
| 29 | 21.8443847 | 20.4535499 | 19.1884546 | 18.0357670 |
| 30 | 22.3964556 | 20.9302926 | 19.6004413 | 18.3920454 |
| 31 | 22.9377015 | 21.3954074 | 20.0004285 | 18.7362758 |
| 32 | 23.4683348 | 21.8491780 | 20.3887655 | 19.0688656 |
| 33 | 23.9885636 | 22.2918809 | 20.7657918 | 19.3902082 |
| 34 | 24.4985917 | 22.7237863 | 21.1318367 | 19.7006842 |
| 35 | 24.9986193 | 23.1451573 | 21.4872200 | 20.0006612 |
| 36 | 25.4888425 | 23.5562511 | 21.8322525 | 20.2904938 |
| 37 | 25.9694534 | 23.9573181 | 22.1672354 | 20.5705254 |
| 38 | 26.4406406 | 24.3486030 | 22.4924616 | 20.8410874 |
| 39 | 26.9025888 | 24.7303444 | 22.8082151 | 21.1024999 |
| 40 | 27.3554792 | 25.1027751 | 23.1147719 | 21.3550723 |
| 41 | 27.7994895 | 25.4661220 | 23.4123999 | 21.5991037 |
| 42 | 28.2347936 | 25.8206068 | 23.7013592 | 21.8348828 |
| 43 | 28.6615623 | 26.1664457 | 23.9819021 | 22.0626887 |
| 44 | 29.0799631 | 26.5038495 | 24.2542739 | 22.2827910 |
| 45 | 29.4901599 | 26.8330239 | 24.5187125 | 22.4954503 |
| 46 | 29.8923136 | 27.1541696 | 24.7754490 | 22.7009181 |
| 47 | 30.2865820 | 27.4674826 | 25.0247078 | 22.8994378 |
| 48 | 30.6731196 | 27.7731537 | 25.2667066 | 23.0912443 |
| 49 | 31.0520780 | 28.0713695 | 25.5016569 | 23.2765645 |
| 50 | 31.4236059 | 28.3623117 | 25.7297640 | 23.4556179 |

TABLE

TABLE V.

The present worth of 1 l. annuity, compound interest.

| <i>T.</i> | <i>4 per cent.</i> | <i>4½ per cent.</i> | <i>5 per cent.</i> | <i>6 per cent.</i> |
|-----------|--------------------|---------------------|--------------------|--------------------|
| 1 | 0.9615385 | 0.9569378 | 0.9523809 | 0.9433962 |
| 2 | 1.8860947 | 1.8726678 | 1.8594104 | 1.8333926 |
| 3 | 2.7750910 | 2.7489644 | 2.7232480 | 2.6730119 |
| 4 | 3.6298952 | 3.5875257 | 3.5459505 | 3.4651056 |
| 5 | 4.4518223 | 4.3899767 | 4.3294767 | 4.2123638 |
| 6 | 5.2421369 | 5.1578725 | 5.0756921 | 4.9173244 |
| 7 | 6.0020547 | 5.8927009 | 5.7863734 | 5.5823815 |
| 8 | 6.7327448 | 6.5958861 | 6.4632128 | 6.2097939 |
| 9 | 7.4353314 | 7.2687905 | 7.1078217 | 6.8016923 |
| 10 | 8.1108955 | 7.9127182 | 7.7217349 | 7.3600871 |
| 11 | 8.7604763 | 8.5289169 | 8.3064142 | 7.8868747 |
| 12 | 9.3850733 | 9.1185808 | 8.8632516 | 8.3838440 |
| 13 | 9.9856473 | 9.6828524 | 9.3935730 | 8.8526831 |
| 14 | 10.5631223 | 10.2228253 | 9.8986409 | 9.2949840 |
| 15 | 11.1183868 | 10.7395457 | 10.3796580 | 9.7122491 |
| 16 | 11.6522949 | 11.2340151 | 10.8377695 | 10.1058953 |
| 17 | 12.1656680 | 11.7071914 | 11.2740662 | 10.4772597 |
| 18 | 12.6592961 | 12.1599918 | 11.6895869 | 10.8276035 |
| 19 | 13.1339385 | 12.5932936 | 12.0853208 | 11.1581165 |
| 20 | 13.5903253 | 13.0079365 | 12.4622103 | 11.4699213 |
| 21 | 14.0291589 | 13.4047239 | 12.8211527 | 11.7640767 |
| 22 | 14.4511142 | 13.7844248 | 13.1630026 | 12.0415818 |
| 23 | 14.8568405 | 14.1477749 | 13.4885739 | 12.3033790 |
| 24 | 15.2469619 | 14.4954784 | 13.7986418 | 12.5503576 |
| 25 | 15.6220787 | 14.8282089 | 14.0939445 | 12.7833562 |

TABLE

TABLE V.

The present worth of 1 l. annuity, compound interest.

| Y. | 4 per cent. | 4½ per cent. | 5 per cent. | 6 per cent. |
|----|-------------|--------------|-------------|-------------|
| 26 | 15.9827678 | 15.1466115 | 14.3751853 | 13.0031663 |
| 27 | 16.3295844 | 15.4513028 | 14.6430336 | 13.2105342 |
| 28 | 16.6630618 | 15.7428735 | 14.8981272 | 13.4061644 |
| 29 | 16.9837132 | 16.0218885 | 15.1410735 | 3 5907211 |
| 30 | 17.2920318 | 16.2888885 | 15.3724510 | 13.7648312 |
| 31 | 17.5884921 | 16.5443909 | 15.5928104 | 13 9290861 |
| 32 | 17.8735500 | 16.7888909 | 15.8026766 | 14.0840435 |
| 33 | 18.1476441 | 17.0228621 | 16.0025491 | 14.2302297 |
| 34 | 18.4111962 | 17.2467580 | 16.1929039 | 14.3681412 |
| 35 | 18.6646116 | 17.4610124 | 16.3741942 | 14.4982465 |
| 36 | 18.9082803 | 17.6660406 | 16.5468516 | 14.6209872 |
| 37 | 19.1425771 | 17.8622398 | 16.7112872 | 14.7367804 |
| 38 | 19.3678625 | 18.0499902 | 16.8678926 | 14.8460192 |
| 39 | 19.5844831 | 18.2296557 | 17.0170406 | 14.9490747 |
| 40 | 19.7927721 | 18.4015844 | 17.1590862 | 15.0462969 |
| 41 | 19.9930500 | 18.5661095 | 17.2943678 | 15.1380160 |
| 42 | 20.1856250 | 18.7235498 | 17.4232074 | 15.2245434 |
| 43 | 20.3707931 | 18.8742103 | 17.5459118 | 15.3061730 |
| 44 | 20.5488395 | 19.0183831 | 17.6627732 | 15.3831821 |
| 45 | 20.7200378 | 19.1563474 | 17.7740697 | 15.4558321 |
| 46 | 20.8846517 | 19.2883707 | 17.8800663 | 15.5243699 |
| 47 | 21.0429342 | 19.4147088 | 17.9610155 | 15.5890282 |
| 48 | 21.1951289 | 19.5356066 | 18.0771570 | 15.6500266 |
| 49 | 21.3414700 | 19.6512981 | 18.1687215 | 15.7075723 |
| 50 | 21.4821826 | 19.7620078 | 18.2559253 | 15.7618610 |

TABLE

TABLE VI.

The annuity which 1 l. will purchase, compound interest

| Y. | 2 per cent. | 2½ per c. | 3 per cent. | 3½ per c. |
|----|-------------|-----------|-------------|-----------|
| 1 | 1.0200000 | 1.0250000 | 1.0300000 | 1.0350000 |
| 2 | .5150495 | .5188272 | .5226108 | .5264005 |
| 3 | .3467547 | .3501372 | .3535304 | .3569342 |
| 4 | .2626238 | .2658179 | .2690271 | .2722511 |
| 5 | .2121584 | .2152469 | .2183546 | .2214814 |
| 6 | .1785258 | .1815499 | .1845975 | .1876682 |
| 7 | .1545120 | .1574954 | .1605064 | .1635445 |
| 8 | .1365098 | .1394674 | .1424564 | .1454767 |
| 9 | .1225154 | .1254569 | .1284339 | .1314460 |
| 10 | .1113265 | .1142588 | .1172305 | .1202414 |
| 11 | .1021779 | .1051060 | .1080775 | .1110920 |
| 12 | .0945596 | .0974871 | .1004621 | .1034840 |
| 13 | .0881183 | .0910483 | .0940295 | .0970616 |
| 14 | .0826020 | .0855365 | .0885263 | .0915707 |
| 15 | .0778255 | .0807665 | .0837666 | .0868251 |
| 16 | .0736501 | .0765990 | .0796109 | .0826848 |
| 17 | .0699698 | .0729278 | .0759525 | .0790431 |
| 18 | .0667021 | .0696701 | .0727087 | .0758168 |
| 19 | .0637818 | .0667606 | .0698139 | .0729403 |
| 20 | .0611567 | .0641471 | .0672157 | .0703610 |
| 21 | .0587847 | .0617873 | .0648718 | .0680366 |
| 22 | .0566314 | .0596466 | .0627474 | .0659321 |
| 23 | .0546681 | .0576964 | .0608139 | .0640188 |
| 24 | .0528511 | .0559128 | .0590474 | .0622728 |
| 25 | .0512204 | .0542759 | .0574279 | .0606740 |

TABLE

T A B L E VI.

The annuity which 1 l. will purchase, compound interest.

| <i>T.</i> | <i>2 per cent.</i> | <i>2½ per c.</i> | <i>3 per cent.</i> | <i>½ per c.</i> |
|-----------|--------------------|------------------|--------------------|-----------------|
| 26 | .0496992 | .0527688 | .0559383 | .0592054 |
| 27 | .0482931 | .0513769 | .0545642 | .0578524 |
| 28 | .0469897 | .0500879 | .0532932 | .0566027 |
| 29 | .0457784 | .0488913 | .0521147 | .0554454 |
| 30 | .0446499 | .0477776 | .0510193 | .0543713 |
| 31 | .0435964 | .0467190 | .0499989 | .0533724 |
| 32 | .0426106 | .0457683 | .0490466 | .0524415 |
| 33 | .0416865 | .0448591 | .0481561 | .0515724 |
| 34 | .0408187 | .0440068 | .0473220 | .0507597 |
| 35 | .0400022 | .0432056 | .0465193 | .0499984 |
| 36 | .0392329 | .0424516 | .0458038 | .0492842 |
| 37 | .0385068 | .0417409 | .0451116 | .0486133 |
| 38 | .0378206 | .0410701 | .0444593 | .0479821 |
| 39 | .0371711 | .0404362 | .0438439 | .0473878 |
| 40 | .0365558 | .0398362 | .0432624 | .0468273 |
| 41 | .0359719 | .0392679 | .0427124 | .0462982 |
| 42 | .0354173 | .0387288 | .0421917 | .0457983 |
| 43 | .0348899 | .0382169 | .0416981 | .0453254 |
| 44 | .0343879 | .0377304 | .0412299 | .0448777 |
| 45 | .0339096 | .0372675 | .0407852 | .0444534 |
| 46 | .0334534 | .0368268 | .0403625 | .0440511 |
| 47 | .0330179 | .0364067 | .0399605 | .0436692 |
| 48 | .0326018 | .0360060 | .0395778 | .0433065 |
| 49 | .0322040 | .0356235 | .0392131 | .0429617 |
| 50 | .0318032 | .0352581 | .0388655 | .0426337 |

T A B L E

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

| T. | 4 per cent. | 4½ per c. | 5 per cent. | 6 per cent. |
|----|-------------|-----------|-------------|-------------|
| 1 | 1.0400000 | 1.0450000 | 1.0500000 | 1.0600000 |
| 2 | .5301961 | .5339976 | .5378049 | .5454369 |
| 3 | .3603485 | .3637734 | .3672086 | .3741098 |
| 4 | .2754901 | .2787437 | .2820118 | .2885915 |
| 5 | .2246271 | .2277916 | .2309748 | .2373964 |
| 6 | .1907619 | .1938784 | .1970157 | .2033626 |
| 7 | .1666096 | .1697015 | .1728198 | .1791350 |
| 8 | .1485279 | .1516097 | .1547218 | .1610359 |
| 9 | .1344930 | .1375745 | .1406901 | .1470222 |
| 10 | .1232909 | .1263788 | .1295046 | .1358680 |
| 11 | .1141490 | .1172482 | .1203890 | .1267929 |
| 12 | .1065522 | .1096662 | .1128254 | .1192770 |
| 13 | .1001437 | .1032754 | .1064558 | .1129601 |
| 14 | .0946690 | .0978203 | .1010240 | .1075849 |
| 15 | .0899411 | .0931138 | .0963423 | .1029628 |
| 16 | .0858200 | .0890154 | .0921699 | .0989521 |
| 17 | .0821985 | .0854176 | .0886991 | .0954448 |
| 18 | .0789933 | .0822369 | .0855462 | .0923565 |
| 19 | .0761386 | .0794073 | .0827450 | .0896209 |
| 20 | .0735818 | .0768761 | .0802426 | .0871846 |
| 21 | .0712801 | .0746006 | .0779961 | .0850046 |
| 22 | .0691988 | .0725457 | .0759705 | .0830456 |
| 23 | .0673091 | .0706825 | .0741368 | .0812785 |
| 24 | .0655868 | .0689870 | .0724709 | .0796790 |
| 25 | .0640121 | .0674390 | .0709525 | .0782267 |

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

| Y. | 4 per cent. | 4½ per c. | 5 per cent. | 6 per cent. |
|----|-------------|-----------|-------------|-------------|
| 26 | .065674 | .0660214 | .0695643 | .0769044 |
| 27 | .0612385 | .0647195 | .0682919 | .0756972 |
| 28 | .0600130 | .0635208 | .0671225 | .0745926 |
| 29 | .0588799 | .0624146 | .0660455 | .0735796 |
| 30 | .0578301 | .0613915 | .0650514 | .0726489 |
| 31 | .0568554 | .0604435 | .0641321 | .0717522 |
| 32 | .0559486 | .0595632 | .0632804 | .0710023 |
| 33 | .0551036 | .0587445 | .0624900 | .0702729 |
| 34 | .0543148 | .0579819 | .0617554 | .0695984 |
| 35 | .0535773 | .0572705 | .0610717 | .0689739 |
| 36 | .0528869 | .0566058 | .0604345 | .0683948 |
| 37 | .0522396 | .0559840 | .0598398 | .0678574 |
| 38 | .0516319 | .0554017 | .0592842 | .0673581 |
| 39 | .0510608 | .0548557 | .0587640 | .0668938 |
| 40 | .0505235 | .0543431 | .0582782 | .0664915 |
| 41 | .0500174 | .0538616 | .0578223 | .0660589 |
| 42 | .0495402 | .0534087 | .0573947 | .0656834 |
| 43 | .0490899 | .0529824 | .0569933 | .0653331 |
| 44 | .0486645 | .0525807 | .0566163 | .0650061 |
| 45 | .0482625 | .0522020 | .0562617 | .0647005 |
| 46 | .0478821 | .0518447 | .0559282 | .0644149 |
| 47 | .0475219 | .0515073 | .0556142 | .0641477 |
| 48 | .0471807 | .0511886 | .0553184 | .0638977 |
| 49 | .0468571 | .0508874 | .0550397 | .0636636 |
| 50 | .0465502 | .0506021 | .0547767 | .0634443 |

The

The use of Table IV.

1. Annuity, rate, and time, given, to find the amount.

Multiply the amount of 1 l. annuity, for the rate and time given, by the given annuity, and the product is the amount sought.

Ex. What will an annuity of 85 l. 10 s. amount to in 20 years, at 5 per cent. compound interest?

$$\begin{array}{r}
 33.0659541 \text{ amount of 1 l. annuity.} \\
 \underline{85.5} \\
 1653297705 \\
 1653297705 \\
 \underline{2645276328} \\
 2827.13907555 \text{ amount sought.}
 \end{array}$$

2. Annuity, time, and amount, given, to find the rate.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find in the table even with the given time, and on the head you have the rate.

Ex. At what rate per cent. will 85.5 amount to 2827.13907555?

$$\begin{array}{r}
 85.5)2827.13907555 \\
 \underline{33.0659541} \text{ under 5 per cent.}
 \end{array}$$

3. Annuity, rate, and amount, given, to find the time.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find under the given rate, and on the side you have the time.

Ex. In what time will an annuity of 85.5 l. amount to 2827.13907555 l. at 5 per cent?

$$\begin{array}{r}
 85.5)2827.13907555 \\
 \underline{33.0659541} \text{ opposite to 20 years.} \\
 K k 2
 \end{array}$$

4. Amount

4. Amount, rate, and time, given, to find the annuity.

Divide the given amount by the amount of 1 l. for the rate and time given, and the quot is the annuity.

Ex. What annuity will amount to 2827.13907555 l. in 20 years, at 5 per cent.?

$$33.0659541 \overline{) 2827.13907555}$$

85.5 annuity.

The use of Table V.

1. Annuity, rate, and time, given, to find the present worth.

Multiply the present worth of 1 l. annuity, for the rate and time given, by the given annuity, and the product is the present worth sought.

Ex. What is the present worth of an annuity of 100 l. to continue 15 years, discounting at 5 per cent. compound interest?

$$10.3796580 \times 100 = 1037.9658 = 1037 \text{ } 19 \text{ } 3\frac{3}{4} \text{ } \textit{Ans.}$$

2. Annuity, time, and present worth, given, to find the rate.

Divide the present worth by the annuity, and the quot is the present worth of 1 l. annuity; which find opposite to the time, and on the head you have the rate.

Ex. The present worth of an annuity of 100 l. to continue 15 years, is 1037.9658 l.: At what rate is interest computed?

$$100 \overline{) 1037.9658} \text{ (10.379658 found under 5 per cent.)}$$

3. Annuity, rate, and present worth, given, to find the time.

Divide the present worth by the annuity, and the quot is the present worth of 1 l. annuity; which find under the given rate, and you have the time on the side.

Ex. The present worth of an annuity of 100 l. discounting

counting at 5 per cent. is 1037.9658 l.: How many years will the annuity continue?

100)1037.9658(10.379658 found opposite to 15 years.

4. Present worth, rate, and time, given, to find the annuity.

Divide the given present worth by the tabular number answering to the rate and time, and the quot is the annuity.

Ex. What annuity, to continue 15 years, will 1037.9658 l. purchase, computing compound interest at 5 per cent.?

10.379658)1037.965800(100 l. annuity.

The use of Table VI.

Price, or present worth, rate, and time, given, to find the annuity.

Multiply the price by the tabular number answering to the rate and time, and the product is the annuity sought.

Ex. What annuity, to continue 15 years, will 1037.9658 l. purchase, reckoning at 5 per cent. compound interest?

1037.9658 $\times$.0963423 = 100 l. annuity.

Having thus shown the use of each table separately, or by itself, I shall now subjoin a few practical questions; in solving several of which the promiscuous use of the tables will be necessary.

Quest. 1. A pays 2000 l. for an annuity of 100 l. to continue 50 years; B puts 2000 l. out at interest: Which of them will amount to the greatest sum at the end of the 50 years, at the rate of $4\frac{1}{2}$ per cent. compound interest?

By

| | | |
|--|---|-------------|
| By Table I. the amount of 2000 l. for | } | L. |
| 50 years, at $4\frac{1}{2}$ per cent. is, | | 18065.27240 |
| By Table IV. the amount of 100 l. annuity, for that time and rate, is, | } | 17850.30282 |
| B's amount exceeds that of A by | | 214.96958 |

By Table III. the present worth of 214.96958 l. is 23 l. 15 s. 11 $\frac{3}{4}$ d. discounting for 50 years at $4\frac{1}{2}$ per cent.; and so much was B's case better than that of A.

Quest. 2. A owes B 600 l. to be paid in 12 years, viz. 100 l. at the end of every 2 years; but agrees with B to pay him the whole in 6 years, by equal yearly payments, reckoning compound interest at 6 per cent.: What must the annual payment be?

By Table III. find the present worths of the 6 payments that were to be made in 12 years, and their sum will be 406.98272 l.

By Table VI. find what annuity for 6 years the said sum will purchase, viz. $406.98272 \times .2033626 = 82.765$ l. the annual payment.

Quest. 3. Which is most advantageous, a term of 15 years in an estate of 500 l. per annum, or the reversion of the same estate for ever, after the expiration of the said 15 years, reckoning compound interest at 5 per cent.?

| | | |
|--|---|-------|
| An estate of 500 l. per annum for ever, or, | } | L. |
| as it is usually expressed, in fee-simple, at | | 10000 |
| 5 per cent. is worth a sum whose interest is 500 l. yearly, viz. | | |

| | | |
|---|---|----------|
| By Table V. the present worth of the same | } | |
| estate, for a term of 15 years, at 5 per | | 5189.829 |
| cent. is, | | |

| | |
|-------------------------------|----------|
| And so the reversion is worth | 4810.171 |
|-------------------------------|----------|

| | | |
|--------------------------------------|---|----------|
| From the value of the term, viz. | - | 5189.829 |
| Subtract the value of the reversion, | - | 4810.171 |

| | |
|--|---------|
| The term is better than the reversion by | 379.658 |
|--|---------|

Quest.

Quest. 4. A has a term of 7 years in an estate of 100 l. *per annum*; B has a term of 14 years in the same estate in reversion, after the expiration of the 7 years; and C has a further term of 21 years in reversion, after A and B: Required the present worths of the several terms, discounting at $4\frac{1}{2}$ *per cent.* compound interest.

By Table V. find the present worth of the yearly rent or annuity, for 42 years, for 21 years, for 7 years; subtract these present worths from each other, and the remainders are the answers, as under.

| L. s. d. | | |
|-----------------------|--------------------------------------|-----------------------------|
| 42 years, | $18.7235498 \times 100 = 1872.35498$ | $= 1872 \ 7 \ 1$ |
| 21 years, | $13.4047239 \times 100 = 1340.47239$ | $= 1340 \ 9 \ 5\frac{1}{4}$ |
| 7 years, | $5.8927009 \times 100 = 589.27009$ | $= 589 \ 5 \ 4\frac{3}{4}$ |
| <hr/> | | |
| L. s. d. | L. s. d. | L. s. d. |
| 589 5 4 $\frac{3}{4}$ | 1340 9 5 $\frac{1}{4}$ | 1872 7 1 |
| A's term. | 589 5 4 $\frac{3}{4}$ | 1340 9 5 $\frac{1}{4}$ |
| | <hr/> | |
| | 751 4 0 $\frac{1}{2}$ | 531 17 7 $\frac{3}{4}$ |
| | B's term. | C's term. |

Quest. 5. A person having 9 years to run, in a lease for 49 years, of an estate of 80 l. *per annum*, would know what sum he ought to pay presently, in order to have the lease renewed or completed, by having 40 years added thereto, reckoning at 6 *per cent.* compound interest?

| | | |
|--|------|------------|
| By Table V. the present worth of 1 l. annuity, at 6 <i>per cent.</i> for 49 years, is, | } L. | 15.7075723 |
| By the same table, the value of 1 l. annuity, at 6 <i>per cent.</i> for 9 years, is, | | 6.8016923 |
| Their difference is, | | 8.9058800 |
| Which multiplied by | | 80 |
| Gives for answer, | | 712.4704 |

Quest. 6. A farmer gets a lease of 21 years renewed, for a fine or grassum of 400 l. and it is put in his option, either

either to pay the fine presently, or to pay yearly an additional rent equivalent thereto: Required the additional rent, reckoning compound interest at 5 *per cent.*

By Tab. VI. the annuity for 21 years that
 1 l. will purchase, at 5 *per cent.* is, } .0779961
 Which multiplied by 400
 Gives the additional rent 31.1984400

Quest. 7. What annuity to continue 19 years, and to commence presently, may be purchased by a bill of 2000 l. payable 3 years hence, reckoning at 6 *per cent.* compound interest?

By Table III. the present worth of 2000 l. } L.
 due 3 years hence, at 6 *per cent.* is, } 1679.238600
 By Table VI. the annuity which }
 1679.2386 l. will purchase for 19 } 150.494874
 years, at 6 *per cent.* is, - }

Quest. 8. What annuity would be sufficient to pay off a debt of 120 millions, in the space of 30 years, at 4 *per cent.* compound interest?

By Table VI. the annuity for 30 years }
 that 1 l. will purchase at 4 *per cent.* is, } .0578301
 Which multiply by the debt 120000000

The product is the annuity sought, *viz.* L. 6939612
 From which subtract the yearly interest }
 of 120 millions, at 4 *per cent.* *viz.* } 4800000

The remainder, if we suppose the 120 }
 millions to be the national debt, will be }
 a sinking fund sufficient to clear the } L. 2139612
 whole in 30 years. - - }

From the above example, appears the conveniency, or rather the necessity, of continuing the decimals in the tables to seven places at least.

Quest. 9. For a certain lease to continue 7 years, A offers 650 l. fine, and 200 l. *per annum*; B offers 150 l. fine,

fine, and 300 l. *per annum*; and C offers 1800 l. fine, without any yearly rent: Which of these three offers is the best, reckoning at 5 *per cent.* compound interest?

By Table V. the present worth of 200 l. } *L.*
per annum for 7 years, at 5 *per c.* is } 1157.27468
 To which add the fine 650

Value of A's offer 1807.27468

By Table V. the present worth of 300 l. }
per annum for 7 years, at 5 *per c.* is } 1735.91202
 To which add the fine 150

Value of B's offer 1885.91202

C's offer is 1800

By which it appears that B's offer is the best; and that, neglecting the decimal, he offers 78 l. more than A, and 85 l. more than C.

Quest. 10. A lease of an estate, to continue 14 years, is offered for 250 l. fine, and 44 l. yearly rent; but the tenant wants to reduce the rent to 20 l. *per annum*: What ought the fine to be, reckoning compound interest at 6 *per cent.*?

The difference between 44 and 20, the two rents, is 24 l.

By Table V. the present worth of 24 l. } *L.*
 for 14 years, at 6 *per cent.* is } 223.0796160
 To which add the first fine 250

The fine sought 473.0796160

II. Annuities for ever, or freehold estates.

In freehold estates, commonly called *annuities in fee-simple*, the things chiefly to be considered are, 1. The annuity or yearly rent. 2. The price or present worth. 3. The rate of interest. The questions that usually occur on this head will fall under one or other of the following problems.

Prob. 1. Annuity and rate of interest given, to find the price.

As the rate of 1 l. to 1 l. so the rent to the price.

Ex. The yearly rent of a small estate is 40 l.: What is it worth in ready money, computing interest at $3\frac{1}{2}$ per cent.?

$$\text{As } .035 : 1 :: 40 : 1142.857142 = 1142 \text{ } 17 \text{ } 1\frac{1}{2} \begin{matrix} L. & s. & d. \end{matrix}$$

Prob. 2. Price and rate of interest given, to find the rent or annuity.

As 1 l. to its rate, so the price to the rent.

Ex. A gentleman purchases an estate for 4000 l. and has $4\frac{1}{2}$ per cent. for his money: Required the rent.

$$\text{As } 1 : .045 :: 4000 : 180 \text{ l. rent sought.}$$

Prob. 3. Price and rent given, to find the rate of interest.

As the price to the rent, so 1 to the rate.

Ex. An estate of 180 l. yearly rent is bought for 4000 l.: What rate of interest has the purchaser for his money?

$$\text{As } 4000 : 180 :: 1 : .045 \text{ rate sought.}$$

Prob. 4. The rate of interest given, to find how many years purchase an estate is worth.

Divide 1 by the rate, and the quot is the number of years purchase the estate is worth.

Ex. A gentleman is willing to purchase an estate, provided he can have $2\frac{1}{2}$ per cent. for his money: How many years purchase may he offer?

$$.025)1.000(40 \text{ years purchase. } \textit{Ans}$$

Prob. 5. The number of years purchase at which an estate is bought or sold, given, to find the rate of interest.

Divide

Divide 1 by the number of years purchase, and the quot is the rate of interest.

Ex. A gentleman gives 40 years purchase for an estate: What interest has he for his money?

40) 1.000 (.025 rate sought.

The computations hitherto are all performed by a single division or multiplication, and the learner will scarcely perceive that the operations are conducted by the rules of compound interest; but when a reversion occurs, there will be occasion for the tables, as follows.

Prob. 6. The rate of interest, and the rent of a freehold estate in reversion, given, to find the present worth or value of the reversion.

By *Prob. 1.* find the price or present worth of the estate, as if possession was to commence presently; and then, by *Table V.* find the present value of the given annuity, or rent, for the years prior to the commencement, subtract this value from the former value, and the remainder is the value of the reversion.

Ex. A has the possession of an estate of 130 l. *per annum*, to continue 20 years; B has the reversion of the same estate from that time for ever: What is the value of the estate, what the value of the 20 years possession, and what the value of the reversion, reckoning compound interest at 6 *per cent.*?

By *Prob. 1.* .06) 130.00 (2166.6666 value of the estate.

By *Table V.* 1491.0896 value of the possession.

675.5770 value of the reversion.

Prob. 7. The price or value of a reversion, the time prior to the commencement, and rate of interest, given, to find the annuity or rent.

By *Table I.* find the amount of the price of the reversion for the years prior to the commencement; and

L 1 2

then,

then, by Prob. 2. find the annuity which that amount will purchase.

Ex. The reversion of a freehold estate, to commence 20 years hence, is bought for 675.577 l. compound interest being allowed at 6 *per cent.* : Required the annuity or rent.

By Table I. the amount of 675.577 l. ^{L.} } 2166.ø
for 20 years at 6 *per cent.* is

By Prob. 2. $2166.ø \times .06 = 130.0$ rent sought.

III. *Life Annuities.*

The value of annuities for life is determined from observations made on the bills of mortality. Dr Halley, Mr Simpson, and Monf. de Moivre, are gentlemen of distinguished merit in calculations of this kind.

Dr Halley had recourse to the bills of mortality at Breslaw, the capital of Silesia, as a proper standard for the other parts of Europe, being a place pretty central, at a distance from the sea, and not much crouded with traffickers or foreigners. He pitches upon 1000 persons all born in one year, and observes how many of these were alive every year, from their birth to the extinction of the last, and consequently how many died each year, as in the following table,

Dr

Dr Halley's table on the bills of mortality at Breslaw.

| Age. | Perf.
liv. | A. | Perf.
liv. | A. | Perf.
liv. | A. | Perf.
liv. |
|------|---------------|----|---------------|----|---------------|----|---------------|
| 1 | 1000 | 24 | 573 | 47 | 377 | 70 | 142 |
| 2 | 855 | 25 | 567 | 48 | 367 | 71 | 131 |
| 3 | 798 | 26 | 560 | 49 | 357 | 72 | 120 |
| 4 | 760 | 27 | 553 | 50 | 346 | 73 | 109 |
| 5 | 732 | 28 | 546 | 51 | 335 | 74 | 98 |
| 6 | 710 | 29 | 539 | 52 | 324 | 75 | 88 |
| 7 | 692 | 30 | 531 | 53 | 313 | 76 | 78 |
| 8 | 680 | 31 | 523 | 54 | 302 | 77 | 68 |
| 9 | 670 | 32 | 515 | 55 | 292 | 78 | 58 |
| 10 | 661 | 33 | 507 | 56 | 282 | 79 | 49 |
| 11 | 653 | 34 | 499 | 57 | 272 | 80 | 41 |
| 12 | 646 | 35 | 490 | 58 | 262 | 81 | 34 |
| 13 | 640 | 36 | 481 | 59 | 252 | 82 | 28 |
| 14 | 634 | 37 | 472 | 60 | 242 | 83 | 23 |
| 15 | 628 | 38 | 463 | 61 | 232 | 84 | 20 |
| 16 | 622 | 39 | 454 | 62 | 222 | 85 | 15 |
| 17 | 616 | 40 | 445 | 63 | 212 | 86 | 11 |
| 18 | 610 | 41 | 436 | 64 | 202 | 87 | 8 |
| 19 | 604 | 42 | 427 | 65 | 192 | 88 | 5 |
| 20 | 598 | 43 | 417 | 66 | 182 | 89 | 3 |
| 21 | 592 | 44 | 407 | 67 | 172 | 90 | 1 |
| 22 | 586 | 45 | 397 | 68 | 162 | 91 | 0 |
| 23 | 579 | 46 | 387 | 69 | 152 | | |

The above table is well adapted to Europe in general; but in the city of London, there is observed to be a greater disparity in the births and burials than in any other place, owing probably to the vast resort of people thither, in the way of commerce, from all parts of the known world. Mr Simpson, therefore, in order to have a table particularly suited to this populous city, pitches upon 1280 persons all born the same year, and records

records the number remaining alive each year, till none were in life; as in the following table.

Mr Simpson's table on the bills of mortality at London.

| <i>Age.</i> | <i>Perf.
liv.</i> | <i>A.</i> | <i>Perf.
liv.</i> | <i>A.</i> | <i>Perf.
liv.</i> | <i>A.</i> | <i>Perf.
liv.</i> |
|-------------|-----------------------|-----------|-----------------------|-----------|-----------------------|-----------|-----------------------|
| 0 | 1280 | 24 | 434 | 48 | 220 | 72 | 59 |
| 1 | 870 | 25 | 426 | 49 | 212 | 73 | 54 |
| 2 | 700 | 26 | 418 | 50 | 204 | 74 | 49 |
| 3 | 635 | 27 | 410 | 51 | 196 | 75 | 45 |
| 4 | 600 | 28 | 402 | 52 | 188 | 76 | 41 |
| 5 | 580 | 29 | 394 | 53 | 180 | 77 | 38 |
| 6 | 564 | 30 | 385 | 54 | 172 | 78 | 35 |
| 7 | 551 | 31 | 376 | 55 | 165 | 79 | 32 |
| 8 | 541 | 32 | 367 | 56 | 158 | 80 | 29 |
| 9 | 532 | 33 | 358 | 57 | 151 | 81 | 26 |
| 10 | 524 | 34 | 349 | 58 | 144 | 82 | 23 |
| 11 | 517 | 35 | 340 | 59 | 137 | 83 | 20 |
| 12 | 510 | 36 | 331 | 60 | 130 | 84 | 17 |
| 13 | 504 | 37 | 322 | 61 | 123 | 85 | 14 |
| 14 | 498 | 38 | 313 | 62 | 117 | 86 | 12 |
| 15 | 492 | 39 | 304 | 63 | 111 | 87 | 10 |
| 16 | 486 | 40 | 294 | 64 | 105 | 88 | 8 |
| 17 | 480 | 41 | 284 | 65 | 99 | 89 | 6 |
| 18 | 474 | 42 | 274 | 66 | 93 | 90 | 5 |
| 19 | 468 | 43 | 264 | 67 | 87 | 91 | 4 |
| 20 | 462 | 44 | 255 | 68 | 81 | 92 | 3 |
| 21 | 455 | 45 | 246 | 69 | 75 | 93 | 2 |
| 22 | 448 | 46 | 237 | 70 | 69 | 94 | 1 |
| 23 | 441 | 47 | 228 | 71 | 64 | 95 | 0 |

It may not be improper in this place to observe, that however perfect tables of this sort may be in themselves, and however well adapted to any particular climate, yet the conclusions deduced from them must always be uncertain, being nothing more than probabilities, or conjectures drawn from the usual period of human life. And the modish practice of buying and selling annuities
on

on lives, by rules founded on such principles, may be justly considered as a sort of lottery or chance-work, in which the parties concerned must often be deceived. But as estimates and computations of this kind are now become fashionable, I shall here give some brief account of such as appear most material.

From the above tables the probability of the continuance or extinction of human life is estimated as follows.

1. The probability that a person of a given age shall live a certain number of years, is measured by the proportion which the number of persons living at the proposed age has to the difference between the said number and the number of persons living at the given age.

Thus, if it be demanded, what chance a person of 40 years has to live 7 years longer? from 445, the number of persons living at 40 years of age in Dr Halley's table, subtract 377, the number of persons living at 47 years of age, and the remainder, 68, is the number of persons that died during these 7 years; and the probability or chance that the person in the question shall live these 7 years is as 377 to 68, or nearly as $5\frac{1}{2}$ to 1. But by Mr Simpson's table the chance is something less than that of 4 to 1.

2. If the year to which a person of a given age has an equal chance of arriving before he dies, be required, it may be found thus: Find half the number of persons living at the given age in the tables, and in the column of age you have the year required.

Thus, if the question be put with respect to a person of 30 years of age, the number of that age in Dr Halley's table is 531, the half whereof is 265, which is found in the table between 57 and 58 years; so that a person of 30 years has an equal chance of living between 27 and 28 years longer.

3. By the tables, the premium of insurance upon lives may in some measure be regulated.

Thus, the chance that a person of 25 years has to live another year, is, by Dr Halley's table, as 80 to 1; but the chance that a person of 50 years has to live a year

year longer is only 30 to 1. And, consequently, the premium for insuring the former ought to be to the premium for insuring the latter for one year, as 30 to 80, or as 3 to 8.

4. By the tables may be computed the value of an annuity for life.

Thus, if it be required to find the value of an annuity of 1 l. for the life of a person of 30 years of age, interest 5 per cent. by Table V. in *Annuities certain*, find the value of 1 l. annuity for 1 year, at 5 per cent. viz. .952. Now, it is obvious, that this .952 would be the true value of the first year's annuity, provided the purchaser was sure of receiving payment: but the person may die; and therefore this value must be lessened in proportion to the risk, by the following analogy from Dr Halley's table, viz. as 531 to 523, so .952 to .9375, the true value of the first year's annuity. In like manner may the value of the second year's annuity be found: for, by Table V. the value of 1 l. annuity for 2 years, at 5 per cent. is 1.8594; from which deduce the value of 1 year's annuity, viz. .9523, and there remains for the separate value of the second year's annuity .9071. Then, from the table, say, As 531 to 515, so .9071 to .8797, the true value of the second year's annuity. And by proceeding in this manner, the separate value of every year's annuity, to the utmost extent of life, may be found; the sum of all which will be the value of the annuity sought.

But the above method being tedious, and therefore unfit for practice, I shall here lay down another method, much shorter and easier, and shall deliver what seems necessary to be said on life-annuities in the way of problems. Previous to which it will be proper to observe, that in calculations of this kind, the age of 86 years is esteemed the utmost extent or limit of life; and the difference between the given age and 86 is called the *complement of life*.

P R O B.

P R O B . I.

To find the value of an annuity of 1 l. for the life of a single person of any given age.

Mons. de Moivre, by observing the decrease of the probabilities of life, as exhibited in the table, composed an algebraic theorem or canon, for computing the value of an annuity for life; which canon I shall here lay down by way of

R U L E

Find the complement of life; and, by Table V. in *Annuities Certain*, find the value of 1 l. annuity for the years denoted by the said complement; multiply this value by the amount of 1 l. for a year, and divide the product by the complement of life; then subtract the quot from 1; divide the remainder by the interest of 1 l. for a year; and this last quot will be the value of the annuity sought; or, in other words, the number of years purchase the annuity is worth.

Ex. What is the value of an annuity of 1 l. for an age of 50 years, interest at 5 per cent.?

86

50 age given.

36 complement of life.

By Table V. the value is, 16.5468

Amount of 1 l. for a year, 1.05

827340

165468

Complement of life 36) 17.374140 (.482615

From unity, viz. 1.000000

Subtract .482615

Interest of 1 l. .05) .517385 (10.3477 value sought.

By the preceding problem is constructed the following table.

VOL. III.

M m

The

The value of 1 l. annuity for a single life.

| Age. | 3 per c. | 3½ per c. | 4 per c. | 4½ per c. | 5 per c. | 6 per c. |
|--------|----------|-----------|----------|-----------|----------|----------|
| 9 = 10 | 19.87 | 18.27 | 16.88 | 15.67 | 14.60 | 12.80 |
| 8 = 11 | 19.74 | 18.16 | 16.79 | 15.59 | 14.53 | 12.75 |
| 7 = 12 | 19.60 | 18.05 | 16.64 | 15.51 | 14.47 | 12.70 |
| 13 | 19.47 | 17.94 | 16.60 | 15.43 | 14.41 | 12.65 |
| 6 = 14 | 19.33 | 17.82 | 16.50 | 15.35 | 14.34 | 12.60 |
| 15 | 19.19 | 17.71 | 16.41 | 15.27 | 14.27 | 12.55 |
| 16 | 19.05 | 17.59 | 16.31 | 15.19 | 14.20 | 12.50 |
| 5 = 17 | 18.90 | 17.46 | 16.21 | 15.10 | 14.12 | 12.45 |
| 18 | 18.76 | 17.33 | 16.10 | 15.01 | 14.05 | 12.40 |
| 19 | 18.61 | 17.21 | 15.99 | 14.92 | 13.97 | 12.35 |
| 4 = 20 | 18.46 | 17.09 | 15.89 | 14.83 | 13.89 | 12.30 |
| 21 | 18.30 | 16.96 | 15.78 | 14.73 | 13.81 | 12.20 |
| 22 | 18.15 | 16.83 | 15.67 | 14.64 | 13.72 | 12.15 |
| 23 | 17.99 | 16.69 | 15.55 | 14.54 | 13.64 | 12.10 |
| 3 = 24 | 17.83 | 16.56 | 15.43 | 14.44 | 13.55 | 12.00 |
| 25 | 17.66 | 16.42 | 15.31 | 14.34 | 13.46 | 11.95 |
| 26 | 17.50 | 16.28 | 15.19 | 14.23 | 13.37 | 11.90 |
| 27 | 17.33 | 16.13 | 15.04 | 14.12 | 13.28 | 11.80 |
| 28 | 17.16 | 15.98 | 14.94 | 14.02 | 13.18 | 11.75 |
| 29 | 16.98 | 15.83 | 14.81 | 13.90 | 13.09 | 11.65 |
| 30 | 16.80 | 15.68 | 14.68 | 13.79 | 12.99 | 11.60 |
| 2 = 31 | 16.62 | 15.53 | 14.54 | 13.67 | 12.88 | 11.50 |
| 32 | 16.44 | 15.37 | 14.41 | 13.55 | 12.78 | 11.40 |
| 33 | 16.25 | 15.21 | 14.27 | 13.43 | 12.67 | 11.35 |
| 34 | 16.06 | 15.05 | 14.12 | 13.30 | 12.56 | 11.25 |
| 35 | 15.86 | 14.89 | 13.98 | 13.17 | 12.45 | 11.15 |
| 36 | 15.67 | 14.71 | 13.82 | 13.04 | 12.33 | 11.05 |
| 37 | 15.46 | 14.52 | 13.67 | 12.90 | 12.21 | 11.00 |
| 38 | 15.29 | 14.34 | 13.52 | 12.77 | 12.09 | 10.90 |
| 1 = 39 | 15.05 | 14.16 | 13.36 | 12.63 | 11.96 | 10.80 |
| 40 | 14.84 | 13.98 | 13.20 | 12.48 | 11.83 | 10.70 |

The

The value of 1 l. annuity for a single life.

| A. | 3 per c. | 3½ per c. | 4 per c. | 4½ per c. | 5 per c. | 6 per c. |
|----|----------|-----------|----------|-----------|----------|----------|
| 41 | 14.63 | 13.79 | 13.02 | 12.33 | 11.70 | 10.55 |
| 42 | 14.41 | 13.59 | 12.85 | 12.18 | 11.57 | 10.45 |
| 43 | 14.19 | 13.40 | 12.68 | 12.02 | 11.43 | 10.35 |
| 44 | 13.96 | 13.20 | 12.50 | 11.87 | 11.29 | 10.25 |
| 45 | 13.73 | 12.99 | 12.32 | 11.70 | 11.14 | 10.10 |
| 46 | 13.49 | 12.78 | 12.13 | 11.54 | 10.99 | 10.00 |
| 47 | 13.25 | 12.56 | 11.94 | 11.37 | 10.84 | 9.85 |
| 48 | 13.01 | 12.36 | 11.74 | 11.19 | 10.68 | 9.75 |
| 49 | 12.76 | 12.14 | 11.54 | 11.00 | 10.51 | 9.60 |
| 50 | 12.51 | 11.92 | 11.34 | 10.82 | 10.35 | 9.45 |
| 51 | 12.26 | 11.69 | 11.13 | 10.64 | 10.17 | 9.30 |
| 52 | 12.00 | 11.45 | 10.92 | 10.44 | 9.99 | 9.20 |
| 53 | 11.73 | 11.20 | 10.70 | 10.24 | 9.82 | 9.00 |
| 54 | 11.46 | 10.95 | 10.47 | 10.04 | 9.63 | 8.85 |
| 55 | 11.18 | 10.69 | 10.24 | 9.82 | 9.44 | 8.70 |
| 56 | 10.90 | 10.44 | 10.01 | 9.61 | 9.24 | 8.55 |
| 57 | 10.61 | 10.18 | 9.77 | 9.39 | 9.04 | 8.35 |
| 58 | 10.32 | 9.91 | 9.52 | 9.16 | 8.83 | 8.20 |
| 59 | 10.03 | 9.64 | 9.27 | 8.93 | 8.61 | 8.00 |
| 60 | 9.73 | 9.36 | 9.01 | 8.69 | 8.39 | 7.80 |
| 61 | 9.42 | 9.08 | 8.75 | 8.44 | 8.16 | 7.60 |
| 62 | 9.11 | 8.79 | 8.48 | 8.19 | 7.93 | 7.40 |
| 63 | 8.79 | 8.49 | 8.20 | 7.94 | 7.68 | 7.20 |
| 64 | 8.46 | 8.19 | 7.92 | 7.67 | 7.43 | 6.95 |
| 65 | 8.13 | 7.88 | 7.63 | 7.39 | 7.18 | 6.75 |
| 66 | 7.79 | 7.56 | 7.33 | 7.12 | 6.91 | 6.50 |
| 67 | 7.45 | 7.24 | 7.02 | 6.83 | 6.64 | 6.25 |
| 68 | 7.10 | 6.91 | 6.75 | 6.54 | 6.36 | 6.00 |
| 69 | 6.75 | 6.57 | 6.39 | 6.23 | 6.07 | 5.75 |
| 70 | 6.38 | 6.22 | 6.06 | 5.92 | 5.77 | 5.50 |

The value of 1 l. annuity for a single life.

| A. | 3 per c. | 3½ per c. | 4 per c. | 4½ per c. | 5 per c. | 6 per c. |
|----|----------|-----------|----------|-----------|----------|----------|
| 71 | 6.01 | 5.87 | 5.72 | 5.59 | 5.47 | 5.20 |
| 72 | 5.63 | 5.51 | 5.38 | 5.26 | 5.15 | 4.90 |
| 73 | 5.25 | 5.14 | 5.02 | 4.92 | 4.82 | 4.60 |
| 74 | 4.85 | 4.77 | 4.66 | 4.57 | 4.49 | 4.30 |
| 75 | 4.45 | 4.38 | 4.29 | 4.22 | 4.14 | 4.00 |
| 76 | 4.05 | 3.98 | 3.91 | 3.84 | 3.78 | 3.65 |
| 77 | 3.63 | 3.57 | 3.52 | 3.47 | 3.41 | 3.30 |
| 78 | 3.21 | 3.16 | 3.11 | 3.07 | 3.03 | 2.95 |
| 79 | 2.78 | 2.74 | 2.70 | 2.67 | 2.64 | 2.55 |
| 80 | 2.34 | 2.31 | 2.28 | 2.26 | 2.23 | 2.15 |

The above table shews the value of an annuity of one pound for a single life, at all the current rates of interest; and is esteemed the best table of this kind extant, and preferable to any other of a different construction. But yet those who sell annuities have generally one and a half or two years more value, than specified in the table, from purchasers whose age is 20 years or upwards.

Annuities of this sort are commonly bought or sold at so many years purchase; and the value assigned in the table may be so reckoned. Thus the value of an annuity of one pound for an age of 50 years, at 3 per cent. interest, is 12.51; that is, 12 l. 10 s. or twelve and a half years purchase. The marginal figures on the left of the column of age serve to shorten the table, and signify, that the value of an annuity for the age denoted by them, is the same with the value of an annuity for the age denoted by the numbers before which they stand. Thus the value of an annuity for the age of 9 and 10 years is the same; and the value of an annuity for the age of 6 and 14, for the age of 3 and 24, &c. is the same. The further use of the table will appear in the questions and problems following.

Quest,

Quest. 1. A person of 50 years would purchase an annuity for life of 200 l.: What ready money ought he to pay, reckoning interest at $4\frac{1}{2}$ per cent.?

L.

By the table the value of 1 l. is 10.82

Multiply by 200

Value to be paid in ready money 2164.00 *Ans.*

Quest. 2. A young merchant marries a widow lady of 40 years of age, with a jointure of 300 l. a-year, and wants to dispose of the jointure for ready money: What sum ought he to receive, reckoning interest at $3\frac{1}{2}$ per cent.?

L.

By the table the value of 1 l. is 13.98

300

Value to be received in ready money 4194.00 *Ans.*

If an annuity for life, together with a reversion for a term of years, or the reversion by itself, is offered; in this case reduce the years purchase found in the table, to years certain, by Table V.; and to the years certain add the years in reversion; and the present worth corresponding to the sum of the years in Table V. will be the value of the life and reversion; from which, if the value of the life be subtracted, there will remain the value of the reversion.

Quest. 3. What is the present worth or value of an estate of 80 l. yearly rent, together with a reversion of 20 years after the death of the present possessor, supposed to be 40 years of age? and what is the value of the reversion by itself, reckoning interest at 5 per cent.?

The value of the life by the above table is 11.83; and the nearest value to this in Table V. under 5 per cent. is 11.6895869; opposite to which is 18 years, and $18 + 20 = 38$ years; and by Table V. the value of 1 l. annuity for 38 years at 5 per cent. is 16.8678926; which multiplied

multiplied by 80 gives 1349.4314080 l. for the value of the life and reversion.

To find the value of the reversion.

| | | |
|--------------------------|-------------|--------------------|
| | From | 16.8678926 |
| | Subtract | 11.6895869 |
| Value of r l. reversion | | <u>5.1783057</u> |
| | Multiply by | 80 |
| Value of 80 l. reversion | | <u>414.2644560</u> |

If the value of a reversion in fee-simple, or for ever, after a life of a given age, be required; from the value of the fee-simple, or perpetuity, subtract the value of the life in possession; and the remainder will be the value of the reversion.

Quest. 4. A widow lady of 60 years of age is in possession for life of an estate of L. 500 per annum: What is the value of the reversion in fee-simple, interest at 5 per cent.?

| | |
|-------------------------------------|-----------------|
| | L. |
| Value of r l. in fee, is, | 20 |
| Value of the life by the table, is, | <u>8.39</u> |
| Value of l. l. in reversion, | 11.61 |
| | Multiply by 500 |
| Value of 500 l. in reversion, | <u>5805.00</u> |

If the reversion depends on two joint lives, or on the longest of two lives, find the value of the joint lives, or of the longest of the two lives, by Prob. II. or Prob. III. following, and subtract the value so found from the value of the perpetuity, or estate in fee-simple.

PROB. II.

To find the value of an annuity for the joint continuance of two lives, one life failing, the annuity to cease.

Here there are two cases, according as the ages of the two persons are equal or unequal.

1. If the two persons be of the same age, work by the following

R U L E.

Take the value of any one of the lives from the table, multiply this value by the interest of 1 l. for a year, subtract the product from 2, divide the foresaid value by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchase sought.

Ex. What is the value of 100 l. annuity for the joint lives of two persons, of the age of 30 years each, reckoning interest at 4 per cent.?

| | | |
|---|---|---------------|
| By the table, one life of 30 years, is, | - | 14.68 |
| Multiply by | - | .04 |
| Subtract the product | | <u>.5872</u> |
| From | - | 2.0000 |
| Remains | | <u>1.4128</u> |

And $1.4128 \div 14.68 = 10.39$ value of 1 l. annuity.

And $10.39 \times 100 = 1039$ the value sought.

2. If the two persons are of different ages, work as directed in the following

R U L E.

Take the values of the two lives from the table, multiply them into one another, calling the result the first product; then multiply the said first product by the interest of 1 l. for a year, calling the result the second product; add the values of the two lives, and from their sum subtract the second product; divide the first product by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchase sought.

Ex. What is the value of 70 l. annuity for the joint lives of two persons, whereof one is 40 and the other 50 years of age, reckoning interest at 5 per cent.?

By

By the table, the value of 40 years is, 111.83
 And the value of 50 years is, 10.35

First product, 122.4405
 Multiply by .05

Second product, 6.122025

Sum of the two lives, 22.180000

Second product deduct, 6.122025

Remainder, 16.057975

And 16.057975) 122.4405 (7.62 value of 1 l. annuity.
 70

533.40 value sought.

P R O B. III.

To find the value of an annuity upon the longest of two lives; that is, to continue so long as either of the persons is in life.

R U L E.

From the sum of the values of the single lives, subtract the value of the joint lives, and the remainder will be the value sought.

Ex. What is the value of an annuity of 1 l. upon the longest of two lives, the one person being 30, and the other 40 years of age, interest at 4 per cent.?

By the table, 30 years is, 14.68
 40 years is, 13.20

Value of their joint lives, by Prob. II. } 27.88
 Case 2. is, } 9.62

Value sought, 18.26

If the annuity be any other than 1 l. multiply the answer found as above by the given annuity.

If the two persons be of equal age, find the value of their joint lives by Case 1. of Prob. II.

P R O B.

P R O B. IV.

To find the value of the next presentation to a living.

R U L E.

From the value of the successor's life, subtract the joint value of his and the incumbent's life, and the remainder will be the value of 1 l. annuity; which multiplied by the yearly income, will give the sum to be paid for the next presentation.

Ex. A enjoys a living of 100 l. *per annum*, and B would purchase the said living for his life after A's death: The question is, What he ought to pay for it, reckoning interest at 5 *per cent.* A being 60, and B 25 years of age?

| | | | |
|---|---|---|--------|
| | | | L. |
| By the table, B's life is, | - | - | 13.46 |
| Joint value of both lives, by Prob. II. is, | | | 6.97 |
| | | | <hr/> |
| The value of 1 l. annuity, | - | - | 6.49 |
| Multiply by | - | - | 100 |
| | | | <hr/> |
| Value of next presentation, | - | - | 649.00 |

The value of a direct presentation is the same as that of any other annuity for life, and is found for 1 l. by the table; which being multiplied by the yearly income, gives the value sought.

P R O B. V.

To find the value of a reversion for ever, after two successive lives; or to find the value of a living after the death of the present incumbent and his successor.

R U L E.

By Prob. III. find the value of the longest of the two lives, and subtract that value from the value of the perpetuity, and the remainder will be the value sought.

Ex. A, aged 50, enjoys an estate or living of 100 l. *per annum*; B, aged 30, is intitled to his lifetime of

the same estate after A's death; and it is proposed to sell the estate just now with the burden of A and B's lives on it: What is the reversion worth, reckoning interest at 4 per cent.?

| | |
|---|------------|
| | L. |
| By the table, A's life of 50 is, - - - | 11.34 |
| B's life of 30 is, - - - | 14.68 |
| | <hr/> |
| Sum, | 26.02 |
| Value of their joint lives, found by }
Prob. II. Case 2. is, - - - } | 8.60 |
| | <hr/> |
| Value of the longest life, - - - | 17.42 sub, |
| From the value of the perpetuity, - - - | 25.00 |
| | <hr/> |
| Remains the value of 1 l. reversion, - - - | 7.58 |
| Multiply by | 100 |
| | <hr/> |
| Value of the reversion, - - - | 758.00 |

P R O B. VI.

To find the value of the joint continuance of three lives, one life failing, the annuity to cease.

R U L E.

Find the single values of the three lives from the table; multiply these single values continually, calling the result the product of the three lives; multiply that product by the interest of 1 l. and that product again by 2, calling the result the double product; then, from the sum of the several products of the lives taken two and two, subtract the double product; divide the product of the three lives by the remainder, and the quot will be the value of the three joint lives.

Ex. A is 18 years of age, B 34, and C 56: What is the value of their joint lives, reckoning interest at 4 per cent.?

By the table, the value of A's life is 16.1, of B's 14.12, and of C's 10.01.

$16.1 \times 14.12 \times 10.01 = 2275.6$ product of the 3 lives.

.04

91.024

2

182.048 double product.

Product of A and B, $16.1 \times 14.12 = 227.33$

A and C, $16.1 \times 10.01 = 161.16$

B and C, $14.12 \times 10.01 = 141.34$

Sum of all, two and two, - - 529.83

Double product subtract, - - 182.048

Remainder, - 347.782

And $347.782 \div 2275.600 = 0.153$ value sought.

P R O B. VII.

To find the value of an annuity upon the longest of three lives.

R U L E.

From the sum of the values of the three single lives taken from the table, subtract the sum of all the joint lives taken two and two, as found by Prob. II. and to the remainder add the value of the three joint lives, as found by Prob. VI. and that sum will be the value of the longest life sought.

Ex. A is 18 years of age, B 34, and C 56: What is the value of the longest of these three lives, interest at 4 per cent.?

N n 2

By

| | |
|--|--------------|
| By the table, the single value of A's life is, | 16.1 |
| single value of B's life is, | 14.12 |
| single value of C's life is, | 10.01 |
| Sum of the single values, | <u>40.23</u> |

| | |
|---|--------------|
| By Prob. II. the joint value of A and B is, | 10.76 |
| joint value of A and C is, | 8.19 |
| joint value of B and C is, | 7.65 |
| Sum of the joint lives, | <u>26.60</u> |

Remainder, - - 13.63

By Prob. VI. the value of the 3 joint lives is, 6.54

Value of the longest of the three lives, - 20.17

Other problems might be added, but these adduced are the more useful, and sufficient for most purposes. The reader probably may wish that the reason of the rules, which, it must be owned, are intricate, had been assigned; but this could not be done without entering deeper into the subject than was practicable in this place. They who want further satisfaction on this head may have recourse to *Monf. de Moivre's Doctrine of Chances*, and other authors on this subject.

I shall now conclude *Life-annuities* by presenting the reader with a table, shewing how many years of an annuity, or annual income, will reimburse the purchaser or annuitant of the price or sum paid for the annuity, with the compound interest arising from the price as a capital or principal sum; that is, the table shews how many years and days possession will indemnify the purchaser, or save him from being a loser.

The

The possession that will reimburse the annuitant.

| Years purch. | 3 per c. | | $3\frac{1}{2}$ per c. | | 4 per c. | | $4\frac{1}{2}$ per c. | | 5 per c. | | 6 per c. | |
|-----------------|----------|-----|-----------------------|-----|----------|-----|-----------------------|-----|----------|-----|----------|-----|
| | R. | da. | R. | da. | R. | da. | R. | da. | R. | da. | R. | da. |
| 5 | 5 | 182 | 5 | 216 | 5 | 252 | 5 | 289 | 5 | 327 | 6 | 44 |
| $5\frac{1}{2}$ | 6 | 37 | 6 | 79 | 6 | 122 | 6 | 168 | 6 | 216 | 6 | 319 |
| 6 | 6 | 261 | 6 | 311 | 6 | 364 | 7 | 55 | 7 | 113 | 7 | 241 |
| $6\frac{1}{2}$ | 7 | 124 | 7 | 184 | 7 | 247 | 7 | 314 | 8 | 20 | 8 | 176 |
| 7 | 7 | 356 | 8 | 62 | 8 | 137 | 8 | 217 | 8 | 303 | 9 | 127 |
| $7\frac{1}{2}$ | 8 | 227 | 8 | 311 | 9 | 34 | 9 | 129 | 9 | 231 | 10 | 95 |
| 8 | 9 | 104 | 9 | 200 | 9 | 304 | 10 | 51 | 10 | 172 | 11 | 81 |
| $8\frac{1}{2}$ | 9 | 350 | 10 | 97 | 10 | 217 | 10 | 348 | 11 | 125 | 12 | 88 |
| 9 | 10 | 236 | 11 | | 11 | 138 | 11 | 290 | 12 | 92 | 13 | 119 |
| $9\frac{1}{2}$ | 11 | 128 | 11 | 274 | 12 | 69 | 12 | 245 | 13 | 75 | 14 | 177 |
| 10 | 12 | 24 | 12 | 191 | 13 | 9 | 13 | 212 | 14 | 75 | 15 | 265 |
| $10\frac{1}{2}$ | 12 | 292 | 13 | 115 | 13 | 324 | 14 | 194 | 15 | 94 | 17 | 23 |
| 11 | 13 | 200 | 14 | 48 | 14 | 286 | 15 | 190 | 16 | 134 | 18 | 188 |
| $11\frac{1}{2}$ | 14 | 115 | 14 | 354 | 15 | 259 | 16 | 203 | 17 | 196 | 20 | 36 |
| 12 | 15 | 36 | 15 | 305 | 16 | 246 | 17 | 234 | 18 | 285 | 21 | 309 |
| $12\frac{1}{2}$ | 15 | 329 | 16 | 265 | 17 | 246 | 18 | 285 | 20 | 38 | 23 | 289 |
| 13 | 16 | 264 | 17 | 235 | 18 | 261 | 19 | 358 | 21 | 189 | 25 | 360 |
| $13\frac{1}{2}$ | 17 | 206 | 18 | 216 | 19 | 292 | 21 | 90 | 23 | 13 | 28 | 183 |
| 14 | 18 | 156 | 19 | 209 | 20 | 340 | 22 | 215 | 24 | 247 | 31 | 164 |
| $14\frac{1}{2}$ | 19 | 115 | 20 | 215 | 22 | 43 | 24 | 5 | 26 | 168 | 35 | 5 |
| 15 | 20 | 82 | 21 | 234 | 23 | 132 | 25 | 195 | 28 | 151 | 39 | 189 |
| $15\frac{1}{2}$ | 21 | 59 | 22 | 267 | 24 | 245 | 27 | 60 | 30 | 209 | 45 | 233 |
| 16 | 22 | 45 | 23 | 316 | 26 | 18 | 28 | 336 | 32 | 360 | 55 | 88 |
| $16\frac{1}{2}$ | 23 | 41 | 25 | 16 | 27 | 185 | 30 | 300 | 35 | 264 | 79 | 14 |

The

The use of the Table.

The left hand column contains the number of years purchase paid for the annuity, or the number of pounds paid in ready money for each pound of the annuity; and the other columns show how long the annuitant must be in possession before he come to be reimbursed of the price paid, and the interest thence arising, reckoned at all the usual rates; the use whereof will appear in the solution of the following, or like questions.

Quest. 1. A person buys an annuity for 10 years purchase: How long must he enjoy it, in order to be reimbursed of the price, reckoning interest at $3\frac{1}{2}$ per cent.?

Under $3\frac{1}{2}$ per cent. and opposite to 10 on the left, you have the answer, 12 years and 191 days.

Quest. 2. How many years possession will reimburse an annuitant, who pays 16 $\frac{1}{2}$ years purchase, interest at 5 per cent.? *Ans.* 35 years, 264 days.

C H A P. XIV.

MENSURATION.

EVery magnitude is measured by some magnitude of the same kind. A line by a lineal foot, yard, &c. A superficies, or surface, by a square foot, yard, &c. A solid by a cubic foot, yard, &c.

The lineal or running measure is known to all, and needs neither direction nor example. There remains then the superficial and solid measure to be explained.

Superficial measure may be conceived, by imagining a floor paved with tiles, each a foot square; for then the number of tiles will be equal to the number of square feet in that floor. Now, if the floor be just one foot broad, the number of tiles, or square feet, will be equal to the number of lineal feet in the length of the floor. If the floor be two or three feet broad, the number of tiles will be twice or thrice as many. And universally, the number of feet in length multiplied by the

the number of feet in breadth will give the number of tiles or square feet in the floor.

Solid measure may be conceived by imagining a wall built with stones, each a cubic foot; for then the number of stones will be equal to the number of cubic feet in the wall. First then, if the wall be one foot thick, and one foot high, the number of stones, or cubic feet, will be equal to the number of lineal feet in the length of the wall. Secondly, if the wall be of the same length and height, as before, *viz* one foot, but two or three feet thick, then the number of stones, or cubic feet, will be twice or thrice as many as in the first supposition. Thirdly, if the length and thickness be the same as in the last supposition, but the height two or three feet, then the number of stones, or cubic feet, in the wall, will accordingly be twice or thrice as many as in the last supposition. And universally, the number of feet in length multiplied by the number of feet in thickness, and that product by the number of feet in height, will give the number of stones, or cubic feet, in the wall.

In treating of mensuration, I shall enumerate the various sorts of surfaces and solids, give short descriptions of them, with such draughts or figures as will in some measure show the reason of the rules. In the examples, I shall use absolute or abstract numbers, that may represent any sort of measure; as, inches, feet, yards, &c. After this general sketch of mensuration, I shall apply the doctrine to surveying, to the measuring of artificers work, to the measuring of board and timber, and to gauging.

I. Superficial Measure.

Problems, shewing how to find the superficial content, or area.

I. A square, or four-sided figure, whose angles are right, and sides equal; as, A B C D. Plate 1.

Rule. Multiply the side into itself, the product is the area, or content.

Ex.

Ex. Multiply the side $A B = 8$
by itself 8

Area or content 64

II. A rectangle, oblong, or rectangular parallelogram; *viz.* a four-sided figure, whose angles are right, and whose opposite sides only are equal; as, $A B C D$. Pl. 1.

Rule. Multiply the length by the breadth, and the product is the area.

Ex. Multiply the length $A B = 24$
by the breadth $B D = 8$

Area 192

III. A rhombus, or four-sided figure, whose sides are equal, but the angles not right; as, $A B C D$. Pl. 1.

Rule. Multiply one side into the perpendicular height, the product is the area.

Ex. Multiply the side $A B = 9.5$
by the height $A E$ or $B G = 7.8$

760

665

Area 74.10

IV. A rhomboides, or parallelogram not rectangular; *viz.* a four-sided figure, whose angles are not right, and whose opposite sides only are equal; as, $A B C D$. Pl. 1.

Rule. Multiply the length by the perpendicular height or breadth, the product is the area.

Ex. Multiply the length $A B = 12.6$
by the height $A E$ or $B G = 4.3$

50.0

$\frac{1}{3} = 4.2$

Area 54.8

V. A right-angled triangle, or triangle wherein one of the angles is right; as A B C. Pl. 1.

Rule. Multiply the perpendicular into the base, half the product is the area.

Or, Multiply half the perpendicular into the base, or half the base into the perpendicular, the product is the area.

Ex. Multiply the perpendicular A B = 6.4
by the base B C = 8.5

$$\begin{array}{r}
 320 \\
 512 \\
 \hline
 2)54.40 \\
 \hline
 27.2 \text{ area.}
 \end{array}$$

Or, Multiply half A B = 3.2
by the base B C = 8.5

$$\begin{array}{r}
 160 \\
 256 \\
 \hline
 27.20 \text{ area.}
 \end{array}$$

Or, Multiply half B C = 4.25
by A B = 6.4

$$\begin{array}{r}
 1700 \\
 2550 \\
 \hline
 27.200 \text{ area.}
 \end{array}$$

Note. In a right-angled triangle, if you add the squares of the base and perpendicular, the square root of that sum will be the hypotenuse, or longest side. *Euclid* I. 47.

VI. An oblique-angled triangle, or triangle wherein none of the angles is right; as, A B C. Pl. 1.

Rule. Drop a perpendicular from any one of the angles upon the base, produced, if need be, as in fig. 2.; then proceed as in right-angled triangles, viz. multiply the perpendicular

O O

perpendicular into the base, half the product is the area.

Or, Multiply half the perpendicular into the base, or half the base into the perpendicular, the product is the area.

$$\begin{array}{r}
 \text{Ex. Multiply the base } A B = 14.8 \\
 \text{by the perpendicular } C D = 5.4 \\
 \hline
 592 \\
 746 \\
 \hline
 2) 79.92 \\
 \hline
 39.96 \text{ area.}
 \end{array}$$

$$\begin{array}{r}
 \text{Or, Multiply the base } A B = 14.8 \\
 \text{by half the perpendicular } C D = 2.7 \\
 \hline
 1036 \\
 296 \\
 \hline
 39.96 \text{ area.}
 \end{array}$$

$$\begin{array}{r}
 \text{Or, Multiply half the base } A B = 7.4 \\
 \text{by the perpendicular } C D = 5.4 \\
 \hline
 296 \\
 370 \\
 \hline
 39.96 \text{ area.}
 \end{array}$$

VII. The three sides of any triangle being given, to find the area, without having recourse to the perpendicular. Pl. I.

Rule. Add the three given sides, and from their half-sum subtract the three sides severally; multiply the half-sum and the three remainders into one another continually; the square root of the last product is the area.

Ex.

Ex. A B 21

B C 17

A C 10

2)48

24 half-sum.

24

10

14 rem.

24

17

7 rem.

24

21

3 rem.

And $24 \times 14 \times 7 \div 3 = 7056$. And $7056 \div 84 \text{ root} = \text{area}$.

64

164)656

656

VIII. A trapezium, viz. any four-sided figure that is neither a square, rectangle, rhombus, nor rhomboides; as, A B C D. Pl. 1.

Rule. Multiply the diagonal into the half-sum of the two perpendiculars, or the contrary; the product is the area.

Ex. Multiply the diagonal B D = 28.4

by half of A a 7 + half of C a 6 = 6.5

1420

1704

Area 184.60

IX. A parallelopleuron, or trapezium that has two of its sides parallel, being a segment of a triangle cut by a line drawn parallel to the base; as, A B C D. Pl. 1.

Rule. Multiply the half-sum of the two parallel sides by the perpendicular height.

Ex. Multiply half A B 12 + half C D 22 = 17

by the perpendicular B E = 13

51

17

Area 221

Note. The parallelopleuron may also be a segment of a right angled-triangle; and in this case B D is the perpendicular, which multiplied into half the sum of A B and C D, will give the area.

X. A polygram, or irregular polygon, *viz.* a figure bounded by five or more unequal sides; as, A B C D E F. Pl. 1.

Rule. Divide all such many-sided, multangular, and irregular figures, into trapeziums and triangles, then measure them by Prob. 6. and 8.

Ex. Divide the polygram A B C D E F into the trapezium A B C F, and the two triangles C D F and F D E; drop the perpendiculars A a, C a, D r and D m; then measure the trapezium and triangles as taught in Prob. 8. and 6.; and the sum of their areas will be the area of the given polygram.

XI. A regular polygon, or ordinate figure whose sides and angles are all equal; as, A B C D E. Pl. 1.

Such polygons take their names from the number of their sides, or rather angles. Thus, if the polygon has 3 sides, or angles, it is called a *trigon*, or *equilateral triangle*; if the polygon have 4 sides, it is called a *tetragon*, or *square*; if 5, a *pentagon*; if 6, a *hexagon*; if 7, a *heptagon*; if 8, an *octagon*; if 9, an *enneagon*; if 10, a *decagon*; if 11, an *endecagon*; if 12, a *dodecagon*, &c.

Rule. Bisect any two adjacent angles, and from the point where the bisecting lines meet, drop a perpendicular upon the base, multiply half the sum of the sides into the perpendicular, and the product is the area of the polygon.

Ex.

Ex. The side of a pentagon is = 16.4
 Multiply by $\frac{1}{2}$ the number of sides 2.5

820

328

Half sum of the sides 41.00

Multiply by the perpend. m n 11.3

123

41

41

Area of the pentagon 463.3

Hence every polygon may be resolved into as many equal triangles as it has sides, the sum of whose areas will be equal to the area of a right-angled triangle, one of whose legs is the perimeter or sum of the sides, and the other the radius or femidiameter of the inscribed circle.

If the side of the several regular polygons be 1, and their respective areas be computed by trigonometry, these areas may be inserted in a table, and will be so many multipliers for the ready finding the area of any other like polygon. The table of areas or multipliers follows.

| <i>Names.</i> | <i>Multipliers.</i> | <i>Names.</i> | <i>Multipliers.</i> |
|---------------|---------------------|---------------|---------------------|
| Trigon | .433013 | Octagon | 4.828427 |
| Tetragon | 1.000000 | Enneagon | 6.181827 |
| Pentagon | 1.720475 | Decagon | 7.694209 |
| Hexagon | 2.598076 | Endecagon | 8.514250 |
| Heptagon | 3.633959 | Dodecagon | 9.330125 |

The areas of like polygons are to one another as the squares of their homologous or like sides, Euclid VI. 20.; but the side of the polygons in the table is 1, whose square is 1; and therefore the square of the side of any given polygon multiplied into the tabular area of the like polygon will produce the area thereof.

Ex.

Ex. Required the area of a hexagon whose side is 10.

The square of 10, or $10 \times 10 = 100$; and $100 \times 2.598076 = 259.8076$ the area sought; and in like manner may the area of any other regular polygon be found.

XII. A circle, the most capacious of all figures, is bounded by one curve line, called the *circumference* or *periphery*, to which all lines drawn from the middle point or centre are equal; as A D B E. Pl. I. fig. 1.

The line A B passing through the centre C, and terminated on both sides by the circumference, is called the *diameter*; and half that line, *viz.* A C, drawn from the centre to the circumference, is called the *semidiameter* or *radius*. The diameter divides the circle into two equal parts called *semicircles*; as A D B and A B E.

Rule. Multiply the radius into half the periphery, or the periphery into half the radius; the product is the area.

The reason is obvious: for a circle differs not from, or is the same with, a regular polygon of an infinite number of sides.

Ex. Required the area of a circle whose diameter is 20, and periphery 62.8318.

Half the diameter or radius is 10; and half the periphery is 31.4159; and $31.4159 \times 10 = 314.159$, the area sought.

Or, Half the radius is 5, and $62.8318 \times 5 = 314.159$, as before.

If the area of a semicircle, as A B D, be required, multiply the radius A C into half the semicircular arch A D B; or, multiply the square of the diameter A B into .3927; either of these products is the area: or, find the area of the whole circle, and then take its half.

If the area of a quadrant, as A C E, be demanded, multiply the radius A C into half the quadrantal arch A E; or, multiply the square of the diameter A B into .19635; either of these products is the area: or, find the area of the whole circle, and then take one fourth of it.

A great many other questions relative to the diameter, periphery, and area of circles, may occur; most of which may be solved by one or other of the proportions following.

1. As 7 to 22, nearly; or, as 113 to 355, more nearly; or, as 1 to 3.14159, still more nearly; so the diameter of a circle to the circumference.

2. As 22 to 7; or, as 355 to 113; or, as 3.14159 to 1; or, as 1 to .31831; so the circumference to the diameter.

3. As 1 to 3.14159, so the square of the radius to the area of the circle.

4. As 1 to .0795775, so the square of the periphery to the area of the circle.

5. As 14 to 11; or, as 452 to 355; or, as 1 to .785398, or, for practice, to .7854; so the square of the diameter, viz. the area of the circumscribed square A B C D, fig. 2. to the area of the circle.

Note. .7854 is the area of a circle whose diameter is unity, and circles are to one another as the squares of their diameters, Euclid XII. 2.

6. As 1 to .7071, so the diameter of a circle to the side of the inscribed square m n o p. Or, Multiply the square of the semidiameter by 2, and the square root of the product is the side of the inscribed square.

7. As 1 to .2251, so the periphery of a circle to the side of the inscribed square.

8. As 1 to 1.2732, so the area of a circle to the square of the diameter.

9. As 1 to 12.56637; or, as .0795775 to 1; so the area of a circle to the square of the periphery.

10. As 1 to .6366, so the area of a circle to the area of the inscribed square.

11. As 1 to 1.4142, so the side of a given square to the diameter of the circumscribing circle.

12. As 1 to 4.443, so the side of a square to the periphery of the circumscribing circle.

13. As 1 to .8862, so m n, the diameter of a circle,

to A B, the side of a square, equal to the circle in area; fig. 3.

14. As 1 to .2821, so the periphery of a circle to the side of a square equal in area

15. As 1 to 1.128, so the side of a square to the diameter of a circle equal to the square in area.

16. As 1 to 3.545, so the side of a square to the periphery of a circle equal in area.

XIII. A sector of a circle; as A C B E, or A C B D. Pl. 2. fig. 1.

Rule. Multiply the radius A C into half the arc A E B, the product is the area of the sector A C B E; which subtracted from the area of the whole circle, leaves the area of the sector A C B D.

Ex. Suppose the radius A C to be 10, and half the arc A E B to be 10.47198; then $10 \times 10.47198 = 104.7198$, the area of the sector A C B E; which subtracted from the area of the whole circle, found by Prob. 12. viz. 314.159, leaves 209.4393, the area of the sector A C B D.

If the area of the segment A B E, or A B D, be required, from the area of the sector A C B E, found above, subtract the area of the triangle A C B, and there will remain the area of the segment A B E; which subtracted from the area of the whole circle, will leave the area of the segment A B D.

The length of the arch line of any sector or segment, as A C B, fig. 2. may be found thus.

Divide the chord line A B into four equal parts, and set one of these parts from A to D, and from B to C; join D C; which line D C will be equal to half the arch line A C B.

Or, if the arch line be given in degrees, say, As 360, the periphery in degrees, to the periphery in measure, found by Prob. 12. so the degrees of the arch-line to the arch line in measure.

If A B, the chord of an arc A C B, fig. 3, be given, and also the versed sine D C, the diameter of the whole circle may be found thus.

As

$A : DC : DA :: DA : DE$.

And $DE + DC = CE$, the diameter.

Again, $DE : DA :: DA : DC$.

And the square root of $DC \times DE = DA$, or DB ;

Euclid III. 31. and VI. 8. Cor.

XIV. A lune, or space bounded by arcs of two circles of a different radius, like the falcated moon; as, $A DB E$. Pl. 2.

Rule. Find C and K , the centres of the two circles $A E B L$ and $A D B M$, and complete the circles.

Then, by Prob. 12. or 13. find the area of the semi-circle or segment $A C B E$, and also the area of the segment $ACBD$; subtract the one from the other, and the remainder is the area of the lune.

XV. A ring, or space included between the circumferences of two concentric circles; as, $a b c d$, Pl. 2. fig. 1.

Rule. Find the area of both circles, by Prob. 12.; subtract the lesser from the greater; the remainder is the area of the ring.

If such a portion of the ring as d be required, find the area of both sectors, and subtract the lesser from the greater.

If a portion of the ring, as b , be demanded, find the area of both segments, and subtract the lesser from the greater.

If the area of a mixed or compound figure be required, viz. a figure bounded partly by right lines and partly by circular arcs, as $A B C D$, fig. 2. resolve such a figure into triangles, trapezia, and into sectors or segments; then find the area of these severally; and their sum will be the area of the compound figure sought.

XVI. An ellipse or oval, viz. any thorough plane section of a cone, not parallel to the base; as $A B C D$, Pl. 2. fig. 1.

Rule. Multiply the longer axis or transverse diameter $A B$, into the shorter axis or conjugate diameter $C D$;

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then

then multiply that product by .7854; and this last product is the area.

Ex. Suppose the transverse diameter AB to be 26, and the conjugate DC 20, required the area,

$26 \times 20 = 520$, and $520 \times .7854 = 408.4080$, the area of the given ellipse.

The conic writers demonstrate the following proportions, *viz.*

1. The area of any ellipse is a mean-proportional between the area of the circumscribing and inscribed circle, *viz.* a circle on the transverse, and another on the conjugate diameter; as $ACBD$ and $cmdn$; fig. 2.

2. As the longer axis to the shorter, that is, as $CD (= AB)$ to cd , so any chord-line of the circle bb (parallel to cd) to rr . And as $mn (= cd)$ to AB , so oo (parallel to AB) to rt .

3. As CD to cd , or as bb to rr , so the area of the circular segment bAb to the area of the elliptical segment rAr .

4. As mn to AB , or as oo to rt , so the area of the circular segment $od o$ to the area of the elliptical segment rdt .

If the elliptical area included between two parallels be required, by the third or fourth of the above proportions, find the area of both segments, and subtract the lesser from the greater.

XVII. A parabola, *viz.* any section of a cone parallel to a plane, applied to the outside of the cone; as ACB . Pl. 2,

Rule. Multiply the base, or greatest ordinate, AB , into the perpendicular height DC , and two thirds of the product is the area, the parabola being equal to two thirds of the circumscribing rectangle $A E G B$.

Ex. Suppose AB to be 18.5, and DC 19; required the area of the parabola.

$$\begin{array}{r}
 18.5 \\
 19 \\
 \hline
 1665 \\
 185 \\
 \hline
 3)351.5 \\
 \hline
 117.1\bar{6} \\
 117.1\bar{6} \\
 \hline
 234.8 \text{ area.}
 \end{array}$$

Or, Multiply 351.5 by $\frac{6}{9} = \frac{2}{3}$.

$$\begin{array}{r}
 351.5 \\
 6 \\
 \hline
 9)2109.0(\\
 234.8 \text{ area.}
 \end{array}$$

If the parabolic area included between two parallels A B and m n, be required, find the area both of the parabola A C B and m C n, and subtract the lesser from the greater.

If a line be drawn from C to A and another from C to B, the triangle A C B will be three fourths of the parabola.

XVIII. A cycloid; as A D B E. Pl. 2.

Rule. Find, by Prob. 12. the area of the circle C, described on the axis D E, multiply that area by 3, and the product is the area of the cycloid.

Note. The cycloid is formed by applying any circle, as C, to the line A B, so as the point n may coincide with the point A; and then, rolling the circle, as a wheel, along the line A B, till the point n complete a revolution, and coincide with B. The path described by the point n is the cycloidal curve; A B the base, D E the axis, the point E the vertex, and C the generating circle.

XIX. The surface of a cylinder, or solid, like a rolling stone, formed by the revolution of a rectangle round one of its sides; as A B C D. Pl. 2.

The surface of a cylinder, excluding the bases, when spread out, becomes a rectangle, one of whose sides is the periphery of the circular base, and the other the same with the height or length of the cylinder.

Rule. Multiply the periphery of the base into the height,

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height, and to the product add twice the area of the base; the sum is the area sought.

XX. The surface of a cone, or solid like a sugar-loaf, formed by the revolution of a right-angled triangle round one of its legs; as $A B C$. Pl. 2.

The surface of a cone, excluding the circular base, when spread out, becomes a sector of a circle, whose terminating arc is the periphery of the base, and whose radius is $A B$ or $A C$, the length of the outside of the cone, or hypotenuse of the generating triangle. The point A is called the *vertex*, and $A o$ the *axis*, of the cone.

Rule. Multiply the periphery of the base into $A B$, the length of the side; and to half the product add the area of the base; the sum is the area sought. See Prob. 12. and 13.

If the surface of the frustum of a cone, cut by a plane parallel to the base, as $m n B C$, be required, find the curve surface of the whole cone $A B C$, and also of the cone $A m n$, cut off; from the former subtract the latter; to the remainder add the area of the two bases; and the sum is the area of the frustum.

The axis $A o$, or height of the whole cone, is found by saying, As the semidifference of the diameters of the greater and lesser base to the perpendicular height of the frustum; so the semidiameter of the greater base to $A o$, from which subtract the height of the frustum, and the remainder will be the height of the cone cut off; whence $A m$ or $A n$ may be found by the note in Prob. 5.

Or, $A m$ may be directly found by saying, As the semidifference of the two diameters to $B m$, so the semidiameter of the greater base to $A B$; from which subtract $B m$, and the remainder will be $A m$.

The area of the outside-surface of the frustum of a cone may be found more readily thus: Add the peripheries of the two bases, and multiply half their sum by the outside-height $B m$, and to the product add the area of the two bases.

XXI. The

XXI. The surface of a sphere or globe, viz. a round body formed by revolving a semicircle round its diameter; as, A B C D: Pl. 2.

Rule. Multiply the periphery of the globe into the axis or diameter A B, the product is the area.

Or, Find the area of a circle whose diameter is the same with that of the globe, and multiply the area so found by 4:

Example. Suppose the diameter A B to be 20, then by Prob. 12. the periphery will be 62.8318; and $62.8318 \times 20 = 1256.636$, the area of the surface of the globe.

Or, The area of a circle whose diameter is 20, by Prob. 12. is 314.159; and $314.159 \times 4 = 1256.636$, as before.

If the area of the surface of a segment, as m C n, be required, say,

As the axis or diameter of the sphere C D,
To the area of the whole globe;
So the height of the segment C r,
To the area of the segment, exclusive of the base.

If the diameter of the globe be wanted, divide the square of m r, the semidiameter of the frustum's base by C r its height, and the quot will give r D the height of the other frustum, and the sum of C r and r D is the diameter sought. See Prob. 13.

The area of the segment m C n may also be found thus: Square m r and r C, add these squares, and the square root of their sum is C m; then find the area of a circle whose radius is C m, and this will be equal to the area of the given segment, exclusive of the circular base.

If the area of that part of the surface of a sphere that lies between two parallel planes, as m n, and a o, be demanded, it may be found thus: Find, as above directed, the area of the two segments m C n and a C o, subtract the lesser area from the greater, and the remainder is the area sought.

XXII. A spherical triangle, or triangle formed on the surface of a globe by three great circles, viz. circles whose

whose centre is the same with that of the globe; as, A B C. Pl. 2.

Rule. From the sum of the three angles subtract 180 degrees, multiply the area of the whole surface of the globe by the remainder, divide the product by 720, the quot is the area of the triangle.

E X A M P L E.

$$\begin{array}{r}
 \text{Suppose the sum of the angles } A + B + C = 225 \text{ } 10 \\
 \text{Subtract} \quad \quad \quad 180 \\
 \hline
 \text{Rem.} \quad 45 \text{ } 10
 \end{array}$$

Suppose again the diameter of the given sphere or globe to be 20, then the area of the whole surface, as found above, will be 1256.636.

$$\begin{array}{r}
 \text{Multiply} \quad 1256.636 \\
 \text{by the rem.} \quad 45.10 \\
 \hline
 1256636 \\
 6283180 \\
 5026544 \\
 \hline
 56674.2836 \\
 418.87860 \\
 418.87860 \\
 \hline
 \end{array}$$

720) 57512.04093(79.877 area of the triangle.

The surfaces of prisms, pyramids, and other regular bodies, consist of triangles, parallelograms, or polygons; and so the way of measuring them is taught in the preceding problems.

II. *Mensuration of Solids.*

Problems, shewing how to find the solid content or solidity.

XXIII. A cube, or solid contained under six equal squares; as, A B C D. Pl. 3.

Rule.

Rule. Multiply the side of the cube into itself, and that product again by the side; this last product is the solid content or solidity of the cube.

Ex. Multiply the side $A, D = 4$
by itself $\cdot \quad \quad = 4$

Again, Multiply this product $\overline{16}$
by $\underline{4}$

Solidity of the cube $\underline{64}$

XXIV. A prism, or solid whose bases are equal similar parallel triangles, squares, parallelograms, or polygons, and the sides all parallelograms.

A prism whose bases are squares or parallelograms, is usually called a *parallelopiped*.

When the bases are triangles, the solid is called a *triangular prism*, and if the bases are polygons, it is called a *polygonal prism*.

Rule. Find the area of the base as taught in superficial measure, and multiply that by the length of the prism or parallelopiped.

Example. Required the solid content of the triangular prism $A B C D$. Pl. 3.

Multiply $B C \quad \quad = 18$
by half the per, $m n = \underline{6}$

Area of the base $\overline{108}$

Multiply by the length $A B = \underline{40}$

Solidity of the prism $\underline{4320}$

XXV. A cylinder. See a description of this solid. Prob. 19. Pl. 3. fig. 1.

Rule. The solidity of a cylinder is found the same way as that of a prism or parallelopiped, viz. Find the area of the circular base by Prob. 12. multiply that by the length or height, the product is the solid content.

Ex. Required the solidity of a cylinder, the diameter of whose base is 9, and length 15.

$$9 \times 9 \times .7854 \times 15 = 954.261 \text{ the solidity.}$$

If it be required to find how much liquor is contained in a cylindrical vessel that is in part empty, lying with its axis parallel to the horizon; such as a vessel whose base is the circle A B C D, fig. 2. the empty part being the segment A B D; find by Prob. 13. the area of the segment B C D, and multiply this area by the length.

XXVI. A pyramid, or solid whose base is a triangle, square, parallelogram, or polygon, and its sides all plane triangles, terminating in a point; as, A D B. Pl. 3.

Rule. Find the area of the base, and multiply that into one third of the altitude or height; the product is the solid content, a pyramid being equal to one third of the circumscribing prism. Euclid XI. 7.

Ex. Suppose the area of the base A B to be 4.25

Multiply that by $\frac{1}{3}$ of D C = 5.5

2125

2125

Solidity of the pyramid 23.375

If the pyramid be cut by a plane m n parallel to the base A B, and the solidity of the frustum A m n B be required; from the solidity of the whole pyramid A D B subtract the solidity of the pyramid m D n cut off, the remainder is the solidity of the frustum.

To find the height of the whole pyramid, say, As the difference between one of the sides of the greater base and the correspondent side of the lesser base, to the height of the frustum, so the side of the greater base to the height of the whole pyramid; from which subtract the height of the frustum, and the remainder will be the height of the pyramid cut off.

The solidity of the frustum of a pyramid may also be found by one or other of the three rules following.

Rule 1. To the product of any two correspondent sides

sides of the two bases, add the squares of these sides; multiply that sum by one third of the frustum's height, and the product will be the solidity, if the bases are squares.

Rule 2. Multiply the areas of the two bases together, and to the square root of the product add these two areas; multiply that sum by one third of the frustum's height, and the product is the solidity of the frustum, whether the bases be triangles, squares, or polygons.

Rule 3. To the product of any two correspondent sides of the two bases, add one third part of the square of their difference; and that sum will be the area of a mean base; which multiplied into the height, gives the solidity of the frustum, whether the bases be triangular, square, or multangular.

A solid, resembling the frustum of a pyramid, and having parallel bases, and these bases both rectangles, or the one a rectangle and the other a square, but disproportional, the sides of the one base not having the same proportion to one another as the correspondent sides of the other, is called a *prismoid*; as, A B C. Pl. 3. fig. 2. And its solidity may be found as follows.

To the longest side of the greater base add half the longest side of the lesser base, and multiply the sum by the breadth of the greater base, and reserve the product.

Again, to the longest side of the lesser base, add half the longest side of the greater base; and multiply the sum by the breadth of the lesser base; and to this product add the product formerly reserved; and multiply the sum by a third part of the height, and the product is the solidity of the prismoid.

If the area of the outside surface of the prismoid be required, multiply half the sum of the perimeters of both bases by the outside height, and to the product add the area of the two bases.

XXVII. A cone. See this solid described, Prob. 20.

A cone is one third of the circumscribing cylinder; or, A cone is the same with a pyramid of an infinite number of sides: and the solidity is found the same way, viz.

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Multiply

Multiply the area of the base by one third of the altitude or perpendicular height; Euclid XII. 10.

If the cone be cut by a plane $m n$ parallel to the base, the solidity of the frustum $m n B C$ is also found by subtracting the solidity of the cone cut off $A m n$ from the solidity of the whole cone $A B C$. Pl. 3. See Prob. 20.

The solidity of the frustum of a cone may also be found by any of the three rules following.

Rule 1. To the product of the diameters of the two bases add the squares of the said two diameters, and multiply the sum by .7854, the product will be the triple area of a mean base; which multiplied by one third of the perpendicular height, will give the solidity of the frustum.

Rule 2. Multiply the area of the greater base into the area of the lesser; extract the square root of the product; to this root add the areas of the two bases; and multiply the sum by one third of the frustum's height, and the product is the solidity.

Rule 3. To the product of the greater and lesser diameters add one third part of the square of their difference; and multiply the sum by .7854; the product is the area of a mean base; which multiplied by the perpendicular height, the product is the solidity.

If the base of a cone, or of a cylinder, be an ellipse at right angles to the axis, find the area of the base by Prob. 16.; and then proceed to find the solidity, as taught above. The solidity of the frustum of such a cone may also be found as above; only find the area of the elliptical bases by Prob. 16.

A solid, resembling the frustum of a cone, and having parallel bases, and these bases both elliptical, or the one base an ellipse and the other a circle, but disproportional, the diameters of the one base not having the same proportion to one another as the correspondent diameters of the other base, is called a *cylindroid*; as, $A B C$. Pl. 3. fig. 2. And its solidity may be found thus.

To the longest diameter of the greater base add half the longest diameter of the lesser base, and multiply the
sum

sum by the shortest diameter of the greater base, and reserve the product.

Again, to the longest diameter of the lesser base, add half the longest diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base; and to this product add the product formerly reserved, and that sum will be the triple square of a mean diameter; which multiply by .7854; and again multiply that product by a third part of the height; and this last product is the solidity of the cylindroid.

If the area of the outside surface of the cylindroid be required, measure the periphery of both bases, and multiply half their sum by the outside height, and to the product add the area of the two bases.

XXVIII. A sphere or globe. See the description of this solid, Prob. 21. Pl. 3.

Rule. A sphere or globe is equal to two thirds of the circumscribed cylinder; that is, a cylinder of equal diameter and altitude with the globe; and therefore find the area of a great circle; multiply this by the diameter; two thirds of the product is the solid content.

Ex. Suppose the diameter of a sphere or globe to be 20, then, by Prob. 12.

| | | | |
|--------------------------------|---|---|------------|
| The area of the circle will be | - | - | 314.159 |
| Multiply by the diameter, | - | - | 20 |
| | | | 3)6283.180 |
| | | | 2094.398 |
| | | | 2094.398 |
| Solidity of the globe, | - | - | 4188.786 |

A sphere may be considered as composed of an infinite number of cones, having their bases in the surface, and their common vertex in the centre; and so the solidity of a sphere may also be found thus: Multiply the diameter into the circumference; the product is the superficial content or area: then multiply this area by one sixth of the diameter, or

Q q 2 by

by one third of the radius ; the product is the solid content.

Ex. Suppose the diameter of a globe to be 20 ; then, by Prob. 12.

| | | | |
|-------------------------------------|---|---|-------------|
| The periphery will be | - | - | 62.8318 |
| Multiply by the diameter, | - | - | 20 |
| | | | 3) 1256.636 |
| Multiply by $\frac{1}{6}$ diameter, | | = | 3.8 |
| | | | 3769.908 |
| | | | 418.8786 |
| Solidity, as before, | - | - | 4188.7866 |

The solidity of a sphere may be found nearly, by multiplying the cube of the diameter into .5236, or, for greater accuracy, into .523598, the solidity of a sphere whose diameter is unity.

Thus, the cube of the diameter 20 is $20 \times 20 \times 20 = 8000$; and $.5236 \times 8000 = 4188.8$ the solidity nearly; or, $.523598 \times 8000 = 4188.784$, more nearly.

XXIX. The polar segment, or frustum of a sphere ; as, A C B D. Pl. 3.

Rule. From the triple product of the diameter of the sphere C E into the square of the frustum's height C D, subtract twice the cube of the segment's height C D; then divide the remainder by 1.91; or, multiply by .5236, or, for practice, by .5236, the quot or product is the solid content.

Ex. Suppose C E = 20 ; and C D = 5.

Then $5 \times 5 \times 20 \times 3 - 5 \times 5 \times 5 \times 2 = 1250$

And $1.91) 1250 (654.45$, the solidity sought.

Or, $1250 \times .52356 = 654.45$, as before.

The segment A C B D subtracted from the whole sphere, leaves the greater segment A E B D.

XXX. The

XXX. The middle segment or zone of a sphere; as, A B C D. Pl. 3.

Rule. Multiply the square of the sphere's diameter E G by 2, and to the product add the square of the diameter of the base A B or C D; divide this sum by 382; or, for greater accuracy, by 3.8197; and multiply the quote by the zone's thickness, or height m n, the product is the solid content.

Ex. Suppose E G = 20, A B or C D = 17.32, and m n = 10.

Then $20 \times 20 \times 2 + 17.32 \times 17.32 = 1099.9824$
And $3.82 \times 1099.9824 = 287.95$

And $287.95 \times 10 = 2879.5$ the solidity of the zone.

The solidity of the zone A B C D may also be found by subtracting twice the solidity of the polar segment A E B m from the solidity of the whole sphere.

Hence may be found the solidity of any segment of a sphere included between two parallel planes; as, a o B A; viz. Find the solidity of the segment A E B, and also that of the segment a E o, and subtract the lesser from the greater.

XXXI. A sector of a sphere; as, A B D C. Pl. 3.

Rule. A spherical sector, as already observed, consists of an infinite number of cones, having their bases in the surface of the sphere B D C, and their common vertex in the centre A: Wherefore, by Prob. 21. find the area of the segment B D C; and multiply this area into the third part of A B the radius of the sphere.

Hence we have another method of finding the solidity of a spherical segment, such as B D C, viz. From the sector A B D C subtract the cone A B C, and there will remain the solidity of the segment. But if the spherical segment be greater than a hemisphere, the cone must be added to the sector.

XXXII. A spheroid, or solid generated by revolving a semiellipse round its axis; as, A B C D. Pl. 3.

If

If the semiellipse be revolved round the longer axis A B, the solid thence generated is called an *oblong spheroid*, and resembles an egg.

If the revolution be round the shorter axis C D, the solid thence arising is called an *oblate spheroid*, being somewhat like a bias-bowl.

Rule. Multiply the axis round which the semiellipse was revolved into the square of the other axis; then multiply this product by .5236; this last product is the solidity of the spheroid, whether oblong or oblate.

Example. Suppose C D = 5.5; and A B = 9.

Then $5.5 \times 5.5 \times 9 \times .5236 = 142.5501$ the solidity of the spheroid.

A spheroid is two thirds of a cylinder whose height is the axis round which the semiellipse was revolved, and whose diameter is the other axis; and accordingly the solidity of the above spheroid may also be found thus;

$5.5 \times 5.5 \times .7854 \times 9 = 213.82515$; whereof two thirds is 142.5501, as before.

If the solidity of a segment, as, A m n, or of the middle frustum m n o p, be required, imagine a sphere described on the axis A B, and cut into segments similar to those of the spheroid, by extending the planes m n and o p; and then say, As the square of the axis A B to the square of the axis C D; so the solidity of the spherical segment or frustum found by Prob. 29. or 30. to the solidity of the segment or frustum of the spheroid. *N. B.* Twice the segment A m n subtracted from the whole spheroid, leaves the middle frustum or zone m n o p.

Note. The middle frustum of the spheroid m n o p differs little from the like frustum of a sphere described on the axis C D, and may have its solidity computed in the same manner, *viz.* To twice the square of C D add the square of m n or o p, divide the sum by 3.82, and multiply the quot by a r, the height.

XXXIII. A parabolic conoid, or solid, found by revolving a semiparabola $A n C$ round its axis $A o$; as, $A B C$. Pl. 3.

Rule. A parabolic conoid is one half of the circumscribed cylinder; multiply therefore the area of the circular base $B C$ into one half of the altitude $A o$, the product is the solid content.

Example. Suppose the diameter of the base $B o C$ to be 12, and the height $A o$ to be 18: Required the solid content.

$$12 \times 12 \times .7854 \times 9 = 1017.8784 \text{ solidity sought.}$$

If the parabolic conoid be cut by a plane $m n$ parallel to the base $B C$, the solidity of the frustum $m n B C$ may be found thus: To the square of the diameter of the greater base $B C$ add the square of the diameter of the lesser base $m n$; divide the sum by 2.5464; or, multiply by .3927; and then multiply the result by the height of the frustum; the product is the solid content. The frustum $A m n$ cut off is a complete conoid, and its solidity is found as taught above.

XXXIV. A parabolic spindle, or solid generated by revolving a parabola $A B C$ round its ordinate or base $A C$; as, $A B C D$. Pl. 3.

Rule. The parabolic spindle is equal to eight fifteenths of the circumscribing cylinder; find therefore the solidity of a cylinder, the diameter of whose base is $B D$, the greatest diameter of the spindle, and its height $A C$, the length of the spindle, or ordinate of the parabola, and $\frac{8}{15}$ of this is the solid content of the spindle.

Example. Suppose $B D = 6.5$, and $A C = 8$.

Then $6.5 \times 6.5 \times .7854 \times 8 \times \frac{8}{15} = 141.58144$ solidity.

Or, Multiply the square of $B D$ by $A C$, and that product by .41888 $= \frac{8}{15}$ of .7854.

Thus $6.5 \times 6.5 \times 8 \times .41888 = 141.58144$ as before.
The

The solidity of the middle frustum of the spindle, *viz.* $m n o p$ is found thus : To twice the square of the greatest diameter $B D$ add the square of the diameter of the base, *viz.* $m n$ or $o p$, and from the sum subtract four tenths of the square of the difference between these two diameters ; divide the remainder by 3.82 ; or, for greater accuracy, by 3.8197 ; and multiply the quot by $t r$, the height of the frustum, the product is the solidity.

Or thus, Multiply the square of the greatest diameter by 1.5708 ; and multiply the square of the lesser diameter by .7854 ; and multiply the square of the difference of these diameters by .31416 ; from the sum of the two former products subtract the latter product ; and multiply the remainder by one third part of the height ; and that product will be the solidity of the middle frustum required.

If the solidity of the middle frustum be subtracted from that of the whole spindle, half the remainder will be the solidity of the frustum $A m n$ or $C o p$.

XXXV. The five regular or Platonic bodies, Pl. 4. *viz.*

1. A tetraedron, or solid contained under four equal equilateral triangles.

This solid is a pyramid on a triangular base, and its solidity may be found by Prob. 26.

2. An hexaedron, or cube, *viz.* a solid contained under six equal squares.

This solid is a prism ; and its solidity may be found by Prob. 23. or 24.

3. An octaedron, or solid contained under eight equal equilateral triangles.

This solid consists of two pyramids on the same square base, and of the same height ; and consequently its solidity may be found by Prob. 26.

4. A dodecaedron, or solid contained under twelve equal equilateral pentagons.

This solid consists of twelve equal pyramids, having pentagonal bases, whose common vertex is in the middle

the point or centre; and so its solidity may be found by Prob. 26.

5. An icosaedron, or solid contained under twenty equal equilateral triangles.

This solid consists of twenty equal pyramids having triangular bases, and their common vertex in the centre; and so its solidity may likewise be found by Prob. 26.

But the area, or superficial content, as well as the solidity of any of these bodies, may easily and readily be found from the following table.

| <i>Names.</i> | <i>Area.</i> | <i>Solidity.</i> |
|---------------|--------------|------------------|
| Tetraedron | 1.732051 | 0.1178511 |
| Hexaedron | 6.000000 | 1.0000000 |
| Octaedron | 3.464102 | 0.4714045 |
| Dodecaedron | 20.645729 | 7.663119 |
| Icosaedron | 8.660254 | 2.181695 |

The table exhibits the area and the solidity of any of the above bodies, the side being unity. In using the table observe the following rules.

1. Multiply the proper tabular number by the square of the given side; the product is the area, or superficial content.

2. Multiply the proper tabular number by the cube of the given side; the product is the solidity.

Ex. Required the area and solidity of a dodecaedron, the side being 3.

$$3 \times 3 \times 20.645729 = 185.811561 \text{ the area.}$$

$$3 \times 3 \times 3 \times 7.663119 = 206.904213 \text{ the solidity.}$$

XXXVI. Any irregular body, such as, the bones in a horse's head, a thorn or whin bush, &c.

Rule. Take any vessel of a regular form, such as that of a prism or cylinder, and therein put the irregular solid; then pour in as much water as will cover the solid; this being done, take out the solid, and observe

how far the water sinks or falls on the side of the vessel, and compute the solidity by Prob. 24. or 25.

Or, Take any sort of vessel, fill it with water to the brim; then immerse the irregular body; receive the water that runs over, and pour it into some vessel of a regular form; and then proceed as above.

III. *Surveying.*

A surveyor, or person who measures land, ought to be provided with a theodolite and plain table, a case of instruments, with a set of plotting scales, and a gunter's chain.

The gunter's chain is divided into 100 equal links, and is in length equal to 4 rods or poles; and 160 square rods, or 10 square chains, make an acre.

The English rod or pole is $16\frac{1}{2}$ feet; and so the length of the English chain is 66 feet, or 22 yards. The Scotch rod, pole, or fall, is $18\frac{1}{2}$ feet, and the length of the chain 74 feet. And 4 Scotch acres are nearly equal to 5 English acres.

My design in this place is not to teach surveying at any great length, but only to give a general notion of the art, so far as to enable the reader to measure any park or plain field, and that with accuracy. For this purpose it will be necessary to show what lines are to be measured; how the measures ought to be taken; and in what manner the number of acres is computed.

If the ground to be measured lie in the form of a rectangle, you have only to multiply the length into the breadth: only observe, that if the two ends differ somewhat in breadth, you may add them, and take half their sum for a mean breadth; and do the same with respect to the sides, if they happen not to be equal.

But most fields lie in the form of some irregular polygon; and in this case you must go round the field, and erect a pole at every angle. Then make out an eye-draught, viz. a figure on paper as near the shape of the field as can be done by the guess of the eye, and divide this

this draught into triangles, trapezia, or other figures, as the shape of the draught and field directs. Draw also, in the triangles and trapezia, lines to represent the perpendiculars. Now, the lines to be measured are the diagonals and perpendiculars. But before they be measured, the points in the field where they intersect one another must be found as follows.

Provide yourself with a theodolite, or, in lieu thereof, with a cross, represented in Pl. 4. fig. 1. viz. a square board, with four pins or pegs erected in the corners, so that AB may be perpendicular, or at right angles, to CD ; and place the cross horizontally on the top of a staff FO fastened into a hole in the middle of the board at O . By means of this cross find the points in the field where the perpendiculars cut the diagonals, in the manner following.

Supposing the polygram $ABCDEFG$ in Pl. 4. fig. 2. to represent the field, you must step in a straight line from G to B ; and when you come to the place where you imagine the perpendicular Am will cut the diagonal GB , set up the cross directing CD to B ; and, if now BA on the cross point directly to A in the field, you have guessed right; but if it do not, go further on toward B , or return back toward G , till you hit upon the place, and there fix a pole. In like manner, find the points where all the other perpendiculars cut the diagonals, and mark the places thus found by fixing poles in them.

Then measure the several segments into which the diagonals are divided, and also the perpendiculars; and set down the measures so taken in a field-book, or on the eye-draught described above. When you come home, delineate the whole on paper, from a scale of equal parts, draw the outlines AB , BC , &c. and cast up the content, as follows.

The area of a triangle is found by Prob. 6. and that of a trapezium, by Prob. 8. as follows.

For the triangle A B G,
 Multiply the base G B = 11 + 17.20 = 28.2
 by half the perpendicular A m = 9
 Area 253.8

For the trapezium B C F G,
 Multiply the sum of the perpendiculars B n + F r = 34.54
 by half the diagonal G C = 25
 G C = 19 + 5 + 26 = 50
 17270
 6908
 Area 863.50

For the trapezium F C D E,
 Multiply the sum of the perpendiculars D o + F p = 31.86
 by half the diagonal E C = 23.02
 E C = 13.5 + 14 + 18.54 = 46.04
 6372
 9558
 6372
 Area 733.4172
 To which add { B C F G 863.5
 A B G 253.8
 Area of the field, - 1850.7172

The area of the field thus computed is 1850 square chains, and 7172 square links; and, because 10 square chains make an acre, the square chains are reduced to acres by dividing by 10; that is, by moving the decimal point one place toward the left; and the figures on the right are decimal parts of an acre, as follows.

The

The decimal is reduced to value by multiplying by 4, by 40, by $30\frac{1}{4}$, and by 9, as the table of land-measure directs,

Acres 185.07172

4

Roods .28688

40

Poles or Rods 11.47520

 $30\frac{1}{4}$

14.256

.1188

Square yards 14.3748

9

Square feet 3.3732

Some set down the measures taken in the field without inserting any decimal point between the chains and the links; and in this way the area of the above field will be 18507172 square links: which are easily reduced to acres; for the chain consists of 100 links, and consequently the square chain will contain 10000 square links, and 10 square chains make an acre; therefore the acre will contain 100000 square links. Divide then the area given in square links by 100000, and the quot will be acres; that is, cut off by a decimal point five places on the right, the figures on the left are so many acres, and the figures on the right are decimal parts of an acre. Thus, 18507172 square links becomes 185.07172 acres, as above.

It is usual to measure gardens, or small inclosures, with a rod-pole, viz. a rod one pole long; and in this case the rod-pole should be divided into 100 equal parts, that the parts in any measured line above a just number of rods may be decimal parts of a rod. Some use a half-rod-pole divided into 50 equal parts.

If we suppose the above field to have been measured by a rod-pole, the area will be 1850.7172 square rods, which are reduced to acres by dividing by 160, the number of square rods in an acre; or you may divide the square rods by 40, and the quot will be roods; and then divide

divide the roods by 4, and this last quot will be acres, as under.

$$\begin{array}{r}
 \text{Acres.} \\
 160)1850.7172(11.5669825 \\
 \underline{\hspace{1.5cm}4\hspace{1.5cm}} \\
 \text{Roods } 2.2679300 \\
 \underline{\hspace{1.5cm}40\hspace{1.5cm}} \\
 \text{Poles } 10.7172 \\
 \underline{\hspace{1.5cm}30\frac{1}{4}\hspace{1.5cm}} \\
 21.516 \\
 \underline{\hspace{1.5cm}.1793\hspace{1.5cm}} \\
 \text{Sq. yards } 21.6953 \\
 \underline{\hspace{1.5cm}9\hspace{1.5cm}} \\
 \text{Sq. feet } 6.2577
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus.} \\
 40)1850.7172(\\
 \underline{\hspace{1.5cm}4\hspace{1.5cm}} \\
 46.26793 \\
 \underline{\hspace{1.5cm}4\hspace{1.5cm}} \\
 \text{Acres } 11.5669825
 \end{array}$$

If a piece of ground lie in the form of a ridge, take the breadth at several places, add these several breadths, and divide their sum by the number of breadths taken, the quot will be a mean breadth, which multiplied into the length, will give the area.

If the measures of a field be taken in feet, the area will be square feet; which you may reduce into English acres by dividing by 43560, the number of square feet in an English acre; or it may be reduced to Scotch acres by dividing by 54760, the number of square feet in a Scotch acre.

If the measures of a field be taken in yards, the area will be square yards; and these may be reduced to English acres by dividing by 4840, the number of square yards in an English acre.

If the measures of an island, kingdom, or empire, be taken in English miles, the area will be square miles; and these may be reduced to English acres by multiplying by 640, the number of acres in a square mile.

And if the number of English acres on the whole surface of the terraqueous globe be required, find by Prob.

21. the area of the surface of a globe, whose diameter is 8000 English miles, and multiply the area in square miles so found by 640.

IV. Artificers Work.

1. Carpenters Work.

Carpenters measure their work by the square of 100 feet, viz. a square whose side is 10 feet. Their measurable work consists chiefly of flooring, partitioning, and roofing. See Prob. 2.

Ex. 1. If a floor be 38 feet 6 inches long, and 17 feet 9 inches broad, how many squares are in that floor?

| <i>Decimally.</i> | <i>Or thus.</i> |
|-------------------|-----------------|
| 17.75 | F. in. |
| 38.5 | 38 6 |
| <u>8875</u> | <u>17 9</u> |
| 14200 | 654 6 |
| <u>5325</u> | <u>28 10 6</u> |
| 100)6.83375 | 100)6 83 4 6 |

Anf. 6 squares, 83 feet.

Example 2. How many planks or deals that are 9 feet long and 8 inches broad, will floor a house that is 47 feet 6 inches long, and 24 feet 4 inches wide?

| <i>Inches.</i> | |
|----------------|-------------------|
| 8 = 8 | 24.8 |
| Length 9 | <u>47.5</u> |
| Product 6.0 | 1218 |
| | 17083 |
| | <u>97833</u> |
| | 6)1155.88(192.638 |
| | <u>9</u> |
| | 5.750 |

Anf. 192 planks, 5¾ feet.

Or,

Or, Reduce the dimensions to inches, then multiply the length of the house into the breadth, and divide the product by the square inches in one plank.

Example 3. How many squares are contained in a partition that is 64 feet 6 inches long, and 12 feet 3 inches high?

| | F. in. |
|-------------------|-------------------|
| 12.25 | 64 6 |
| 64.5 | 12 3 |
| <hr/> 6125 | <hr/> 774 |
| 4900 | 16 1 6 |
| 7350 | <hr/> 100)790 1 6 |
| <hr/> 100)7.90125 | |

Ans. 7 squares, 90 feet.

Example 4. How many squares of roofing will cover a house, whose length within the walls is 48 feet 6 inches, and breadth 18 feet 3 inches?

It is a received rule amongst workmen, That the flat and half-flat of any house, taken within the walls, is equal to the roof, though the measure of the roof varies a little, as the roof falls below or rises above the true pitch; that is, as the angle at which the two sides of the roof meets, is more or less than a right angle.

| | F. in. |
|--------------------|--------------------|
| 18.25 | 48 6 |
| 48.5 | 18 3 |
| <hr/> 9125 | <hr/> 873 0 |
| 14600 | 12 1 6 |
| 7300 | <hr/> 100)1327 |
| <hr/> Flat 885.125 | <hr/> Flat 885 1 6 |
| Half 442 | Half 442 |
| <hr/> 100)13.27 | <hr/> 100)13.27 |

Ans. 13 squares, 27 feet.

2. Bricklayers Work.

Bricklayers work consists chiefly of tiling; which is measured

measured by the square of 100 feet; and walling or chimney work, which are measured by the square rod of 272.25 square feet, the side of the square being 16.5 feet. But, in practice, the integer 272 is generally esteemed sufficiently accurate.

In roofs covered with tile it is usual to reckon the eaves double, which is done by adding the depth of the eaves to the whole depth of the roof.

Ex. 1. There is a roof covered with tiles, whose depth on both sides, the eaves being reckoned double, is 35 feet 3 inches, and the length 48 feet: How many squares of tiling are in it?

| | F. in. |
|--------------|-------------|
| 35.25 | 35 3 |
| <u>48</u> | <u>48 0</u> |
| 28200 | 12 |
| <u>14100</u> | 280 |
| 100)16.9200 | <u>140</u> |
| | 100)16 92 |

Anf. 16 squares 92 feet.

Ex. 2. A brick-wall is 598 feet long; 9 feet high, and $1\frac{1}{2}$ brickin thickness: How many roods of work are in it?

$$\begin{array}{r}
 598 \\
 \underline{9} \\
 272.25)5382.00(19 \text{ rods.} \\
 \underline{27225} \\
 265950 \\
 \underline{245025} \\
 68.06) 209.25(3 \text{ quarters of a rod.} \\
 \underline{20418} \\
 5.07
 \end{array}$$

Anf. 19 rods 3 quarters 5 feet.

Brick and half is reckoned standard thickness; and if the thickness of a wall be any other than brick and half, it may be reduced to standard thickness by the following

R U L E.

Multiply the number of superficial feet contained in the wall by the number of half-bricks in the thickness, and one third of the product will be the content reduced to standard thickness.

Ex. 3. A brick wall is 84 feet 6 inches long, and 17 feet 3 inches high, and 5 bricks and a half thick: How many rods of brick-work will be therein, when reduced to standard thickness?

$$\begin{array}{r}
 17.25 \\
 84.5 \\
 \hline
 8625 \\
 6900 \\
 \hline
 13800 \\
 1457.625 \\
 \hline
 11 \\
 \hline
 1457625 \\
 1457625 \\
 \hline
 3)16033.875 \\
 272.25)5344.625(19 \text{ rods.} \\
 27225 \\
 \hline
 262212 \\
 245025 \\
 \hline
 68)171.875(2 \text{ quarters.} \\
 136 \\
 \hline
 35.875
 \end{array}$$

Ans. 19 rods 2 quarters 35 feet.

But the operation may be shortened, by constructing a table of divisors, suited to the number of half-bricks in the thickness, as follows.

Divide 3, the number of half-bricks in $1\frac{1}{2}$, by the number of half-bricks in the thickness, and the quot will be a divisor for giving the answer in feet. And,

to

to obtain a divisor that will give the answer in rods, multiply 272.25 by the divisors found for feet. The table follows.

| <i>Thickness
of the wall
in bricks and
half-bricks.</i> | <i>Divisors
giving the
answer in
feet.</i> | <i>Divisors
giving the
answer in
rods.</i> |
|---|--|--|
| 1 | 1.5 | 408.375 |
| 1½ | 1 | 272.25 |
| 2 | .75 | 204.1875 |
| 2½ | .6 | 163.35 |
| 3 | .5 | 136.125 |
| 3½ | .4285 | 116.659 |
| 4 | .375 | 102.0937 |
| 4½ | .3 | 90.75 |
| 5 | .3 | 81.675 |
| 5½ | .27, | 74.25 |
| 6 | .25 | 68.09375 |

The former example done by the table follows.

74.25)1457.625(- - - -19.63 rods.

| | |
|-------|----------------|
| 7425 | 4 |
| 71512 | 2.52 quarters. |
| 66825 | 68 |
| 46875 | 416 |
| 44550 | 312 |
| 23250 | 35.36 feet. |
| 22275 | |
| 975 | |

If a brick chimney stand alone by itself, it must be girt round; and this is to be esteemed the length; the height of the story is the breadth; and the thickness is the same with that of the jambs. Every story is measured by itself, and the dimensions taken in the same manner;

manner; and the shaft or stalk above the roof is also measured by itself, and the content found by multiplying the girt into the height.

Ex. 4. The girt of a brick chimney in a low story is 32 feet 9 inches, and the height of the story 12 feet 6 inches: How many square feet of work are in the same?

| | |
|--------------|--------------|
| 32.75 | F. in. |
| 12.5 | 32 9 |
| <hr/> | 12 6 |
| 16375 | <hr/> |
| 6550 | 393 |
| 3275 | 16 4 6 |
| <hr/> | <hr/> |
| Feet 409.375 | Feet 409 4 6 |

If it be required to reduce the square feet thus found to feet or rods of standard thickness, divide by the divisor that suits the thickness of the jambs. And if the chimney be esteemed double work, as most chimneys are, double the result.

If a chimney have two or more funnels, the fenders or bridges that separate the funnels must be measured, and their content found by multiplying the length into the breadth.

If a chimney do not stand by itself, but be placed in a gavel or side-wall, the back of the chimney in this case is not measured, but accounted part of the gavel or side-wall; and the length of the chimney is found by adding the depth of the two jambs to the horizontal extent of the breast.

3. *Plaster-work.*

Plaster-work is either on the roof, called *ceiling*, or on the partitions or walls of a building; and is all measured by the square-yard, containing 9 square feet.

Example 1. If a ceiling be 54 feet 9 inches long, and 22 feet 6 inches broad, how many yards are in it?

$$\begin{array}{r}
 54.75 \\
 22.5 \\
 \hline
 27375 \\
 10950 \\
 10950 \\
 \hline
 9)1231.875 \\
 \hline
 \text{Yards } 136.875
 \end{array}$$

$$\begin{array}{r}
 \text{F. in.} \\
 54 \quad 9 \\
 22 \quad 6 \\
 \hline
 16 \quad 6 \\
 108 \\
 108 \\
 \hline
 27 \quad 4 \quad 6 \\
 \hline
 1231 \quad 10 \quad 6
 \end{array}$$

Example 2. The length of a partition plaistered on both sides is 44 feet 6 inches, and the height of the room 9 feet 3 inches: How many yards of plaister-work are in it?

$$\begin{array}{r}
 44.5 \\
 9.25 \\
 \hline
 2225 \\
 890 \\
 4005 \\
 \hline
 9)411.625 \\
 \hline
 \text{Yards } 45.736
 \end{array}$$

$$\begin{array}{r}
 \text{F. in.} \\
 44 \quad 6 \\
 9 \quad 3 \\
 \hline
 4 \quad 6 \\
 396 \\
 11 \quad 1 \quad 6 \\
 \hline
 411 \quad 7 \quad 6
 \end{array}$$

4. Joiners Work.

Joiners measure their work also by the square-yard; but in taking the height of any room, they use a small cord, with which they gird over the mouldings, panels, &c. alledging they ought to measure where their plane touches; but in measuring round the room they take it as it is upon the floor.

Doors, window-shutters, and such work as is wrought on both sides, are generally esteemed work and half-work.

Example. If the wainscoting of a room, girt downward over the mouldings, be 12 feet 9 inches high, and 118 feet 3 inches in compass, how many square yards of work are in it?

118.25

| | |
|-----------------|----------|
| 118.25 | F. in. |
| 12.75 | 118 3 |
| <hr/> | 12 9 |
| 59125 | <hr/> |
| 82775 | 1419 |
| 23650 | 59 1 6 |
| 11825 | 29 6 9 |
| <hr/> | <hr/> |
| 9)1507,6875 | 1507 8 3 |
| <hr/> | |
| Yards 167.52083 | |

The doors and windows of the above room must again be measured by themselves, and the half of their content must be added to the content of the whole room found above.

The content of chimneys, and other void places, must also be measured by themselves, and their content is to be deduced,

5. Painters Work.

Painters work is also measured by the square yard, and the height is likewise taken by girding over the mouldings and panels with a small cord.

Example 1. A room is painted, whose height, taken by girding over the mouldings, is 14 feet 6 inches, and the compass of the room 84 feet 6 inches: How many yards of painted work are in it?

| | |
|---------------|--------|
| 84.5 | F. in. |
| 14.5 | 84 6 |
| <hr/> | 14 6 |
| 4225 | <hr/> |
| 3380 | 343 |
| 845 | 84 |
| <hr/> | 42 3 |
| 9)1225.25 | <hr/> |
| <hr/> | 1225 3 |
| Yards 136.138 | |

Example 2. A round pillar is painted, whose height is 18 feet 4 inches, and the girt round 10 feet 6 inches: How many yards of work are in it?

18.3

$$\begin{array}{r}
 18.3 \\
 10.5 \\
 \hline
 91\phi \\
 18333 \\
 \hline
 9)192.50 \\
 \hline
 \text{Yards } 21.3\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{F. in.} \\
 18 \quad 4 \\
 10 \quad 6 \\
 \hline
 183 \quad 4 \\
 9 \quad 2 \\
 \hline
 192 \quad 6
 \end{array}$$

6. Glasiers Work.

Glasiers measure their work by the foot square, and take their dimensions to a quarter of an inch, the inch being supposed to be divided into 12 equal parts.

But it is more usual to measure by a foot decimally divided, and then the length or breadth consists of so many feet and decimal parts of a foot.

Round or oval windows, half-rounds, &c. are always measured at their full height and breadth, in consideration of the waste of glass, and trouble such work is attended with.

Example 1. Suppose 6 panes of glass, each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad, how many feet of glass are in the said 6 panes?

$$\begin{array}{r}
 4.646 \\
 1.437 \\
 \hline
 32522 \\
 13938 \\
 18584 \\
 4646 \\
 \hline
 6.676302 \\
 6
 \end{array}$$

$$\begin{array}{r}
 \text{F. in. p.} \\
 4 \quad 7 \quad 9 \\
 \hline
 1 \quad 5 \quad 3 \\
 1 \quad 1 \quad 11 \quad 3 \\
 1 \quad 11 \quad 2 \quad 9 \\
 4 \quad 7 \quad 9 \\
 \hline
 6 \quad 8 \quad 1 \quad 8 \quad 3 \\
 6
 \end{array}$$

Feet 40.057812

40 0 10 1 6

Example 2. Suppose 20 panes of glass, which measured by a decimal foot are each 2.458 feet long, and 1.75 feet broad, how many feet of glass do they contain?

2.458

$$\begin{array}{r}
 2.458 \\
 1.75 \\
 \hline
 12290 \\
 17206 \\
 2458 \\
 \hline
 4.30150 \\
 20 \\
 \hline
 \text{Feet } 86.030
 \end{array}$$

7. *Masons Work.*

Masons measure their work, sometimes by the foot solid, sometimes by the square foot or yard, and sometimes by the rod, that is, 21 feet long and 3 feet high, viz. 63 square feet. But the rod differs in different places.

Example 1. A wall of hewn stone is 24 feet 6 inches long, 9 feet 6 inches high, and 2 feet 3 inches thick: How many solid feet are in that wall?

| | |
|---------------|---------------|
| 24.5 | F. in. |
| <u>9.5</u> | 24 6 |
| 1225 | <u>9 6</u> |
| 2205 | 220 6 |
| <u>232.75</u> | <u>12 3</u> |
| 2.25 | 232 9 |
| <u>116375</u> | <u>2 3</u> |
| 46550 | 465 6 |
| <u>46550</u> | <u>58 2 3</u> |
| Feet 523.6875 | 523 8 3 |

Example 2. If a wall be 218 feet 9 inches long, and 12 feet 6 inches high, how many square feet, yards, and rods, are in it?

218.75

$$\begin{array}{r}
 218.75 \\
 \underline{12.5} \\
 109375 \\
 - 43750 \\
 \hline
 21875 \\
 9)2734.375 \text{ feet.} \\
 \hline
 7)303.8194 \text{ yards.} \\
 \hline
 43.4027 \text{ rods.}
 \end{array}$$

$$\begin{array}{r}
 \text{F. in.} \\
 218 \ 9 \\
 \underline{12 \ 6} \\
 2625 \\
 \underline{109 \ 4 \ 6} \\
 2734 \ 4 \ 6
 \end{array}$$

8. Paving.

Paving and caufeying are measured by the square yard, of 9 square feet.

Ex. 1. A paved walk is 92 feet 6 inches long, and 8 feet 4 inches broad : How many yards does it contain ?

| | |
|----------------|--------------|
| 92.5 | F. in. |
| <u>8.7</u> | 92 6 |
| 740.0 | <u>8 4</u> |
| <u>30.87</u> | 740 |
| 9)770.87 | <u>30 10</u> |
| Yards 85.6481, | 770 10 |

Ex. 2. A stable, 19 feet 6 inches long, and 12 feet 6 inches broad, is to be floored or caufeyed with hard bricks, called *clinkers*, each 6 inches long, and 3 inches wide : How many clinkers will it require ?

F. in. in.

$$19 \text{ } 6 = 234$$

$$12 \text{ } 6 = 150$$

$$\underline{1170}$$

$$\underline{234}$$

$$6 \times 3 = 18 \quad 35100 (1950 \text{ clinkers. } \textit{Ans:})$$

$$\underline{18 \dots}$$

$$\underline{171}$$

$$\underline{162}$$

$$\underline{90}$$

$$\underline{90}$$

V. Board and Timber Measure.

1. Board.

To measure board or plank is to find the superficial content or area of a rectangle. See Prob. 2.

R U L E.

Multiply the length in feet by the breadth in inches, divide the product by 12, and the quot is square feet, every remainder being one twelfth of a square foot.

Or, Reduce the length to inches; then multiply by the inches in the breadth; divide the product by 144, and the quot is square feet, the remainder being square inches.

Ex. 1. In a plank 14 feet long, and 8 inches broad, how many square feet?

Or thus.

$$\begin{array}{r} 14 \\ 8 \\ \hline 12 \overline{) 112} \end{array} \begin{array}{l} \text{Sq. feet.} \\ (9 \frac{1}{3}) = 9 \frac{1}{3} \text{ Ans.} \end{array}$$

$$\begin{array}{r} 14 \\ 12 \\ \hline 168 \\ 8 \\ \hline 144 \overline{) 1344} \end{array} \begin{array}{l} \\ \\ \\ \\ \\ (9 \frac{4}{11}) = 9 \frac{4}{11} \end{array}$$

By Gunter's scale.

Extend the compasses from 12 to the breadth 8; and that extent, set the same way, will reach from the length 14 feet to $9 \frac{1}{3}$.

By the sliding rule.

Set the breadth 8 inches on the slip or slider to 12 on the rule, and against the length 14 feet on the rule you have on the slip $9 \frac{1}{3}$.

Ex. 2. In a plank 18 feet 6 inches long, and 15 inches broad, how many square feet?

$$\begin{array}{r} 18.5 \\ 15 \\ \hline 925 \\ 185 \\ \hline 12 \overline{) 277.5} \end{array} \begin{array}{l} \text{Sq. ft.} \\ \\ \\ \end{array}$$

$$23.125 = 23 \frac{1}{8} \text{ Ans.}$$

Or thus.

$$\begin{array}{r} 18.6 \\ 12 \\ \hline 222 \\ 15 \\ \hline 1110 \end{array}$$

$$\begin{array}{r} 144 \overline{) 3330} \\ 288 \\ \hline 450 \\ 432 \\ \hline 18 \end{array} \begin{array}{l} (23 \frac{18}{144}) = 23 \frac{1}{8} \\ \\ \\ \end{array}$$

The operation by Gunter's scale and the sliding rule is the same as above.

If the two ends of a plank differ in breadth, add them, and take half their sum for a mean breadth.

If there be a number of planks of the same dimensions, measure one of them, and multiply the content by the number of planks.

Ex. 3. If a plank be 9 inches broad, how many inches in length will make a foot square?

9)144(16 inches in length. *Ans.*

Hence is constructed the following table, shewing how many inches in length make a square foot, at any breadth, not exceeding 40 inches.

| B. | L. | B. | L. | B. | L. | B. | L. | B. | L. |
|----|-------|----|-------|----|------|----|------|----|------|
| 1 | 144 | 9 | 16 | 17 | 8.47 | 25 | 5.76 | 33 | 4.36 |
| 2 | 72 | 10 | 14.4 | 18 | 8 | 26 | 5.54 | 34 | 4.23 |
| 3 | 48 | 11 | 13.09 | 19 | 7.58 | 27 | 5.3 | 35 | 4.01 |
| 4 | 36 | 12 | 12 | 20 | 7.2 | 28 | 5.14 | 36 | 4 |
| 5 | 28.8 | 13 | 11.07 | 21 | 6.85 | 29 | 4.96 | 37 | 3.89 |
| 6 | 24 | 14 | 10.28 | 22 | 6.54 | 30 | 4.8 | 38 | 3.79 |
| 7 | 20.57 | 15 | 9.6 | 23 | 6.26 | 31 | 4.64 | 39 | 3.69 |
| 8 | 18 | 16 | 9 | 24 | 6 | 32 | 4.5 | 40 | 3.6 |

The use of the table is obvious. Look for the breadth of the board or plank under B, and under L you have the inches in length that make a square foot: Take this length in the compasses from a scale of inches, and run that extent along the board, from end to end, and you have the number of square feet that board contains; or you may, by this means, cut off from that board any number of square feet required.

MORE

MORE EXAMPLES,

| Length.
Feet. | Breadth.
Inches. | Content.
Sq. feet. |
|------------------|---------------------|-----------------------|
| 14 | 18 | 21 |
| 9 | $17\frac{1}{2}$ | $13\frac{1}{8}$ |
| $11\frac{1}{4}$ | $7\frac{3}{4}$ | $7\frac{1}{4}$ |
| $9\frac{3}{4}$ | $13\frac{1}{4}$ | $10\frac{3}{4}$ |
| $8\frac{1}{4}$ | 22 | $15\frac{1}{8}$ |
| $14\frac{1}{2}$ | 20 | $24\frac{1}{8}$ |

2. Timber.

To measure timber is to find the solid content of a cube, parallelopiped, prism, or cylinder. See Prob. 23. 24. and 25.

Equal squared timber.

By equal squared timber is meant, such as is in the form of a parallelopiped, whose bases are equal squares or rectangles; and its solidity is found by multiplying the area of the base by the length.

Example 1. In a piece of squared timber 18 feet long, the side of the square base being 15 inches, how many solid feet?

| | |
|-----------------|--------|
| 15 | F. in. |
| 15 | 1 3 |
| 75 | 1 3 |
| 15 | 1 3 |
| 225 | 3 9 |
| 18 | 1 6 9 |
| 1800 | 9 |
| 225 | 14 0 9 |
| | 2 |
| 144)4050(28.125 | 28 1 6 |

Ans. 28 feet, and half a quarter.

Instead

Instead of dividing by 144, you may divide by the component parts 12×12 . Or you may reduce the length to inches, and then divide the product by 1728, or by $12 \times 12 \times 12$.

In working by feet and inches, instead of multiplying by 18, I multiply by the component parts 9×2 .

By Gunter's scale.

Extend the compasses from 12 to 15 inches, the side of the square; and that extent twice set will reach from the length 18 feet to the answer 28 feet, and something more.

By the sliding rule.

Set the length in feet on the slip to 12 on the girt line, and against the side of the square 15 inches, on the girt line, you have the answer on the slip, 28 feet and somewhat more.

Example 2. In a piece of squared timber 32 inches broad, 18 inches thick, and 14 feet 6 inches long, how many solid feet?

$$\begin{array}{r}
 32 \\
 18 \\
 \hline
 256 \\
 32 \\
 \hline
 576 \\
 14.5 \\
 \hline
 2880 \\
 2304 \\
 576 \\
 \hline
 144)8352.0(58 \text{ feet.}
 \end{array}$$

$$\begin{array}{r}
 \text{F. in.} \\
 2 \quad 8 \\
 1 \quad 6 \\
 \hline
 2 \quad 8 \\
 1 \quad 4 \\
 \hline
 4 \quad 0 \\
 14 \quad 6 \\
 \hline
 56 \\
 2 \\
 \hline
 58 \text{ feet. } \text{Ans.}
 \end{array}$$

The operation by Gunter's scale and the sliding rule is the same as above, only the side of a square equal to the area of the base must be found, by extracting the square root of $576 = 32 \times 18$, viz. 24; or it may be found

found by dividing the extent between 18 and 32 on the scale into two equal parts, and the middle point will be 24, the side of the square sought. But on the sliding rule, set the lines C and D even at the ends; and against any square number on C you have its square root on D.

Some persons, of inferior skill, add the depth and breadth together, and take half their sum for the side of the square. But this method is false; and if the depth and breadth differ widely, the error thence resulting will be considerable. In the above example $32 + 18 = 50$, and half of 50 is 25 for the side of the square; and $25 \times 25 = 625$ for the area of the base; which multiplied into the length, would give a product by far too great.

Divide 1728, the cubical inches in a solid foot, by the area of the base in inches, and the quot will be the inches in length that make a solid foot.

The above rule is general, and extends to timber of all sorts, that is of equal breadth and thickness from end to end, whether the base be square, triangular, multangular, or round.

Hence is constructed the following table, which shews how many inches in length make a solid foot, the side of the square, equal to the area of the base, not exceeding 30 inches.

| B. | Length. | B. | L. | B. | L. | B. | L. | B. | L. |
|----|---------|----|-------|----|-------|----|------|----|------|
| 1 | 1728 | 7 | 35.26 | 13 | 10.22 | 19 | 4.78 | 25 | 2.76 |
| 2 | 432 | 8 | 27 | 14 | 8.81 | 20 | 4.32 | 26 | 2.55 |
| 3 | 192 | 9 | 21.3 | 15 | 7.68 | 21 | 3.92 | 27 | 2.37 |
| 4 | 108 | 10 | 17.28 | 16 | 6.75 | 22 | 3.57 | 28 | 2.2 |
| 5 | 69.12 | 11 | 14.28 | 17 | 5.94 | 23 | 3.26 | 29 | 2.05 |
| 6 | 48 | 12 | 12 | 18 | 5.3 | 24 | 3 | 30 | 1.92 |

The use of the table is plain. Seek the side of the square, equal to the area of the base, under B; and under L you have the number of inches in length that make a solid foot. Take this length from a scale of inches,

es, and with it run the piece of timber from end to end, and you have the number of solid feet it contains. By this means too you may cut off any number of solid feet desired.

Unequal squared timber.

By unequal squared timber is meant, any piece of squared timber whose bases are unequal, whether they be squares or rectangles; and such are most timber trees, when hewn and brought to their squares, the root-end being generally grosser than the other.

The usual customary way of measuring such timber is to take the square or rectangle at the middle of the piece for a mean base, and then proceed as taught above.

Example 1. In a piece of squared timber 20 feet long, the side of the square base at the greater end is 25 inches, and at the lesser end 9 inches: How many feet of timber are in that tree?

$$\begin{array}{r} 25 \\ 9 \\ \hline 2)34 \\ \hline 17 \end{array}$$

17 the side of the square
in the middle.

$$\begin{array}{r} 17 \\ 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array}$$

20 Feet.

144)5780(40.13 Ans.

Example 2. In a piece of timber 18 feet long, the base at the greater end being 32 inches by 20, and at the lesser end 16 inches by 10: How many feet of timber?

$$\begin{array}{r}
 32 \\
 16 \\
 \hline
 2)48 \\
 \hline
 24
 \end{array}
 \begin{array}{r}
 20 \\
 10 \\
 \hline
 2)30 \\
 \hline
 15
 \end{array}
 \begin{array}{r}
 24 \\
 15 \\
 \hline
 120 \\
 24 \\
 \hline
 360 \text{ area of mean base.} \\
 18 \\
 \hline
 288 \\
 36 \\
 \hline
 \text{Feet.} \\
 144)6480(45 \text{ Ans.}
 \end{array}$$

This customary way is erroneous; and the greater the difference between the two bases is, the greater will the error be. But notwithstanding this, universal custom prevails, and purchasers will accept of no other sort of measure. I shall, however, here observe, that a piece of unequal squared timber is the frustum of a pyramid; and the content may be accurately computed by Prob. 26. The former example done in this way follows.

$$\begin{array}{r}
 32 \\
 20 \\
 \hline
 640 \\
 160 \\
 \hline
 384 \\
 64 \\
 \hline
 \text{Root.} \\
 102400 (320 \\
 9 \\
 \hline
 62)124 \\
 \hline
 124 \\
 \hline
 \text{Ans. } 46\frac{2}{3} \text{ feet.}
 \end{array}
 \begin{array}{r}
 16 \\
 10 \\
 \hline
 160 \\
 320 \\
 640 \\
 160 \\
 \hline
 3)1120(373.3 \\
 18 \\
 \hline
 2986\phi \\
 37333 \\
 \hline
 144)6720.0(46.6
 \end{array}$$

Round timber having equal bases.

The customary way of measuring such timber is, to gird the tree about the middle with a small cord, and

then take the fourth part of this girt for the side of a square, with which they proceed as in squared timber.

Example 1. If a tree be 72 inches in circumference, or girt, and 24 feet long, how many feet of timber are in it?

A fourth of 72 is 18

$$\begin{array}{r} 18 \\ \hline 144 \\ 18 \end{array}$$

Area of the base 324

$$\begin{array}{r} 24 \\ \hline 1296 \\ 648 \end{array}$$

144)7776(54 *Ans.*

F. in.

1 6

1 6

1 6

9

2 3

24

48

6

54

Example 2. If a piece of round timber be 54 inches in girt, and 22 feet 6 inches long, how many feet are contained in it?

$\frac{1}{4}$ of 54 is 13.5

$$\begin{array}{r} 13.5 \\ \hline 675 \end{array}$$

405

135

182.25

22.5

91125

36450

36450

144)4100.625(28.476 *A.*

F. in. p.

1 1 6

1 1 6

6 9 0

1 1 6

1 1 6

1 3 2 3

22 6

7 7 1 6

27 10 1 6

28 5 8 7 6

This customary way is false and erroneous; for if the circumference of a circle be 1, the area will be .07958;

now

now the fourth part of 1 is .25, the square whereof is .0625, which is considerably less than the true area .07958; and consequently the solidity thus computed will be considerably less than the true content.

But this customary way, false as it is, being established by long practice, is universally received, and merchants will not purchase timber at any other measure.

I shall here, however, show how the exact content may be found, by working the former example in a way that will give a true answer.

$$\begin{array}{r} 54 \\ 54 \\ \hline 216 \\ 270 \\ \hline 2916 \end{array}$$

$$\begin{array}{r} 2916 \\ .07958 \\ \hline 23328 \\ 14580 \\ \hline 26244 \\ 20412 \\ \hline 232.05528 \end{array}$$

$$\begin{array}{r} 232.055 \\ 22.5 \\ \hline 1160275 \\ 464110 \\ \hline 464110 \\ \hline 5221.2375 \end{array}$$

$$\begin{array}{r} \text{Feet.} \\ 144)5221.2375(36.25 \\ \hline \text{True content } 36.25 \\ \text{False content } 28.47 \\ \hline \text{Error } 7.78 \end{array}$$

Round timber having unequal bases, called tapering timber.

The customary way of measuring tapering timber is to divide the tree into parts of eight or ten feet long, and then take the girt at the middle of each part, by which, as taught above, is computed the solidity of each part, and their sum is the solid content of the whole tree.

Example. A tree 20 feet in length is divided into two parts, each 10 feet long, the middle girt of one of the parts being 64 inches, and of the other 36 inches: How many cubic feet of timber are in that tree?

$\frac{1}{4}$ of 64 is 16 $\frac{1}{4}$ of 36 is 9

16

9

9681

16

10

256810

19

2560

810

810

4

144)3370(23.4 feet *Ans.*

288

490

432

580

576

4

This way of dividing the tree into parts is far more accurate than working by the middle girt of the whole tree; and the shorter the parts are, the nearer the truth will the answer be.

But this way of measuring has been shown to be erroneous when the bases are equal; and it is still more so in timber that tapers; and the more tapering the tree is, the greater will the error be.

The truth is, a piece of tapering timber ought to be considered as the frustum of a cone, and the solidity computed by the directions given in Prob. 27.

MORE EXAMPLES.

| Length. | $\frac{1}{4}$ of the girt. | Content. |
|------------------|----------------------------|------------------|
| Feet. | Inches. | Cubic feet. |
| 18 | 8 | 8 |
| 12 | 15 | 18 $\frac{3}{4}$ |
| 22 | 8 $\frac{1}{2}$ | 11 |
| 27 $\frac{1}{2}$ | 19 | 68.9 |

VI. Gauging.

My design in this place is not to explain the principles or practice of gauging at any great length, but only to give the reader some general notion of that art, so as to enable him to find the content of any common cask or vessel, or of a cistern, or floor of malt.

P R O B. I.

To find divisors, multipliers, and gauge-points, with their uses.

282 cubic inches make an ale-gallon.

231 cubic inches make a wine gallon.

268.8 cubic inches make a corn-gallon.

2150.42 cubic inches make a corn or malt bushel.

The numbers, then, 282, 231, 268.8, 2150.42, are the divisors; and if 1 be divided by these, the quotients arising will be the equivalent multipliers, as exhibited in the following table.

TABLE I. For right-lined figures.

| <i>Divisors.</i> | | <i>Multipliers.</i> | |
|------------------|-------|---------------------|-------|
| 282 | A. G. | .003546 | A. G. |
| 231 | W. G. | .004329 | W. G. |
| 268.8 | C. G. | .0037202 | C. G. |
| 2150.42 | M. B. | .00046502 | M. B. |

If the cubic inches in any vessel be divided or multiplied by these divisors or multipliers, the result will be ale-gallons, wine-gallons, &c.

Again, if the superficial inches or area of any right-lined figure be divided or multiplied by these divisors or multipliers, the result will be the area in ale or wine gallons. &c.; that is, the number of ale or wine gallons contained in the same at 1 inch deep.

Ex.

Ex. Suppose a plain figure, in form of a rectangle, 232 inches in length, and 64 inches in breadth, what is the area in ale, wine gallons, &c.

$$232 \times 64 = 14848 \text{ area in inches.}$$

By division.

$$282 \quad)14848(52.652 \text{ ale-gallons.}$$

$$231 \quad)14848(64.277 \text{ wine-gallons.}$$

$$268.8 \quad)14848(55.238 \text{ corn-gallons.}$$

$$2150.42 \quad)14848(6.904 \text{ malt-bushels.}$$

By multiplication.

$$14848 \times .003546 = 52.651 \text{ ale-gallons.}$$

$$14848 \times .004329 = 64.277 \text{ wine-gallons.}$$

$$14848 \times .0037202 = 55.238 \text{ corn-gallons.}$$

$$14848 \times .00046502 = 6.904 \text{ malt-bushels.}$$

But for circular areas another set of divisors and multipliers must be obtained, in the following manner, *viz.* .785398 is the area of a circle whose diameter is 1; by which, if the numbers 282, 231, 268.8, 2150.42, be divided, the quots will be a set of divisors. And if .785398 be divided by the same numbers, *viz.* 282, 231, &c. the quots will be a set of multipliers, as in the following table.

TABLE II. For circular areas.

| <i>Divisors.</i> | | <i>Multipliers.</i> | |
|------------------|-------|---------------------|-------|
| 359.05 | A. G. | .002785 | A. G. |
| 294.12 | W. G. | .003399 | W. G. |
| 342.24 | C. G. | .002922 | C. G. |
| 2738 | M. B. | .00036523 | M. B. |

If the square of the diameter of any circle be divided or multiplied by these divisors or multipliers, the result will be the area in ale, wine, corn gallons, or malt-bushels;

bushels; that is, the number of ale, wine gallons, &c. contained in the same at 1 inch deep.

Ex. Suppose the diameter of a circle to be 72 inches, what is the area in ale, wine gallons, &c.?

$$72 \times 72 = 5184 \text{ the square of the diameter.}$$

By division.

$$359.05)5184(14.438 \text{ ale-gallons.}$$

$$294.12)5184(17.625 \text{ wine-gallons.}$$

$$342.24)5184(15.147 \text{ corn-gallons.}$$

$$2738)5184(1.893 \text{ malt-bushels.}$$

By multiplication.

$$5184 \times .002785 = 14.437 \text{ ale-gallons.}$$

$$5184 \times .003399 = 17.620 \text{ wine-gallons.}$$

$$5184 \times .002922 = 15.147 \text{ corn-gallons.}$$

$$5184 \times .00036523 = 1.893 \text{ malt-bushels.}$$

If the area in ale, wine gallons, &c. of an ellipse, be required, multiply the greater and lesser diameter together; and then divide or multiply their product by the divisors or multipliers for circular areas; and the result will be the area of the ellipse in ale, wine gallons, &c.

Gauge-points are the sides of squares, or the diameters of circles, whose content, at 1 inch deep, is one gallon, one bushel, &c. and consequently are the square roots of the divisors exhibited in the two preceding tables. These gauge-points are marked on the sliding rule, for rendering the operation by that instrument more simple, and are as follows.

| Gauge-points for squares. | | Gauge-points for circular areas. | |
|---------------------------|-------|----------------------------------|-------|
| 16.79 | A. G. | 18.95 | A. G. |
| 15.19 | W. G. | 17.15 | W. G. |
| 16.39 | C. G. | 18.5 | C. G. |
| 46.37 | M. B. | 52.32 | M. B. |

The

The way of using these gauge-points will appear in what follows.

P R O B. II.

To find the content in ale, wine gallons, &c. of any parallelopiped.

R U L E.

Find the solid content in inches, by multiplying the three dimensions continually; and this product, divided or multiplied by the divisors or multipliers in Table I. will give the content in ale, wine gallons, &c.

Or, Find the area of the base in ale, wine gallons, &c. by Prob. I.; and then multiply that area by the height or depth.

Ex. Suppose a parallelopiped, trough or cistern, to be 81 inches long, 25 inches broad, and 26 inches deep: Required the content in ale, wine gallons, &c.

$$\begin{array}{rcl}
 81 & 282 &)52650(186.7 \text{ ale-gallons.} \\
 25 & 231 &)52650(227.92 \text{ wine-gallons.} \\
 \hline
 405 & 2150.42 &)52650(24.48 \text{ malt-bushels.} \\
 162 & &
 \end{array}$$

Or thus,

Area 2025

$$282)2025(7.18$$

26

12150

$$\text{And } 7.18 \times 26 = 186.68 \text{ ale-gallons, \&c.}$$

4050

52650

By the sliding rule.

In order to work by the sliding rule, you must find the side of a square equal to the area of the base, by extracting the square root of 2025; or, by the rule, thus: Set the lines C and D even at the ends; and against any square-number on C, you have the root on D, which in this example is 45; and accordingly,

Set

Set the gauge-point 16.79 upon D, to the depth 26 upon C, and against the side of the square 45 upon D you have on C 186.7, the content in ale-gallons.

The operation for wine-gallons, malt-bushels, &c. is the same, only use the proper gauge-points.

P R O B. III.

To find the content in ale, wine gallons, &c. of a vessel whose bases are similar and parallel, but unequal, being the frustum of a pyramid.

R U L E.

Find the area of each base, and a mean proportional between them, and multiply the sum of these three by one third of the depth or height, and the product is the content in cubic inches; which, divided or multiplied by the divisors or multipliers in Table I. gives the content in ale, wine gallons, &c.

Ex. Suppose a vessel, whose bases are rectangles, the length of the greater base being 100 inches, and its breadth 70 inches, the length of the lesser base 80, and its breadth 56, and the depth of the vessel 42 inches: Required the content in ale, wine gallons, &c.

$$\begin{array}{l} 100 \times 70 = 7000 \\ 80 \times 56 = 4480 \\ \text{Mean proportional } 5600 \end{array} \left. \vphantom{\begin{array}{l} 100 \times 70 = 7000 \\ 80 \times 56 = 4480 \\ \text{Mean proportional } 5600 \end{array}} \right\} \begin{array}{l} 7000 \times 4480 = 31360000, \\ \text{whose square root is } 5600. \end{array}$$

$$\begin{array}{r} 17080 \\ \text{A thd of the depth } 14 \\ \hline 6832 \\ 1708 \end{array}$$

282)239120(847.94 ale-gallons,

231)239120(1035.15 wine-gallons.

By the sliding rule.

Set the gauge-point 16.79 upon D, to 14, one third of the depth, on C; and against 130.7, the square root of

of the triple mean base 17080, upon D, you have on C 847.94, the content in ale-gallons.

P R O B. IV.

To find the content of a cylinder in ale, wine gallons, &c.

R U L E.

Multiply the square of the diameter by the depth or height, and divide the product by the divisors for circular areas in Table II.

Ex. Suppose the diameter of a cylinder to be 56.5 inches, and the height 96 inches: Required its content in ale, wine gallons, &c.

$$56.5 \times 56.5 \times 96 = 306456$$

$$359.05)306456(853.52 \text{ ale-gallons.}$$

$$294.12)306456(1041.94 \text{ wine-gallons.}$$

$$2738)306456(111.92 \text{ malt-bushels.}$$

By the sliding rule.

Set the proper gauge-point on D to the depth on C, and against the diameter on D you have the content on C.

P R O B. V.

To find the content of a vessel in the form of the frustum of a cone, in ale, wine gallons, &c.

R U L E.

This may be done, as in Prob. 3. by multiplying the sum of the areas of the two bases and a mean proportional, by the third part of the depth; but more easily in the following manner.

To the product of the two diameters add one third part of the square of their difference; that sum is the square of a mean diameter; which multiply by the height or depth, and the product, divided by the divisors

vifors in Table II. gives the content in ale, wine gallons, &c.

Ex. Suppose the greater diameter be 56.5 inches, the lesser diameter 19 inches, and the height 62 inches: Required the content in ale, wine gallons, &c.

$$56.5 \times 19 = 1073.5 \text{ product of the diameters.}$$

19

$$\text{Diff. } 37.5 \times 37.5 = 1406.25 \text{ square of their difference.}$$

$$3)1406.25(468.75$$

$$\underline{1073.5}$$

$$1542.25 \text{ square of a mean diameter.}$$

62

$$308450$$

$$\underline{925350}$$

$$359.05)95619.50(266.31 \text{ ale-gallons.}$$

$$294.12)95619.50(325.1 \text{ wine-gallons.}$$

$$2738)95619.50(34.92 \text{ malt-bushels.}$$

By the sliding rule.

Set the proper gauge-point on D to the depth on C; and against 39.27, the mean diameter or square root of 1542.25 on D, you have the content on C.

Gaugers usually inch all such vessels; that is, they compute the content at every inch deep, and enter these contents in a table; and when they come to survey, they have only to take the depth, and by comparing the wet inches with the table, they have the content by inspection.

P R O B. VI.

To find the drip or fall of a tun.

The drip or fall of a tun is when the vessel is set a little reclining, for conveniency of cleansing, there being a plug-hole at C for the wash to run off at.

X x 2.

By

By this position it will happen, that the liquor will rise to E on one side, when the bottom is but just covered at D on the other; and when the liquor rises to B, the part A B G will be empty. Now, if the content of the empty part A B G be subtracted from the content of the whole vessel in a perpendicular position, the remainder will be the content of the vessel in this oblique position. It is also required to find how much liquor will cover the bottom, or fill the part D C E.

| | Inches. |
|----------------------|-------------|
| Suppose the diameter | A B = 48 |
| the diameter | D C = 40 |
| the height | m n = 60 |
| the diameter | G H = 45.5 |
| the diameter | F E = 42.1 |
| the perpendicular | A L = 17.46 |
| the height | n o = 15.53 |

The content of the whole tun in a perpendicular position, found by Prob. 5. is 324.2 ale-gallons.

To find the content of the part A B G, observe the following rule.

To the square of the top-diameter A B 48, add one half of the product of the said top-diameter A B and bottom-diameter G H 45.5; multiply this sum by the perpendicular A L 17.46; and divide the product by triple the divisors in Table II. viz. by 1077.15 for ale, and by 882.36 for wine.

Square of A B 48 = 2304 and A B 48 x G H 45.5 = 2184
Half of 2184 = 1092

$$3396 \times 17.46 = 58294.16$$

1077.15) 58294.16 (54.12 ale-gallons.

Tun's content 342.2 A. G.

Content of A B G 54.12 A. G.

Remains G B C D 288.08 A. G.

To find how much liquor will cover the bottom, or fill DCE, work as follows.

To the square of the bottom-diameter DC 40, add half the product of the said bottom-diameter and top-diameter FE 42.1; multiply this sum by the perpendicular height no 15.53, and divide the product by triple the divisors in Table II. viz. by 1077.15 for A. G. and by 882.36 for W. G.

Square of DC 40 = 1600, and DC 40 $\times$ FE 42.1 = 1684

Half of 1684 = 842

2442 $\times$ 15.53 = 37924.26

1077.15)37924.26(35.207 ale-gallons.

If the frustum of a pyramid stand reclined, instead of the divisors assigned above, you must use the triple of the divisors in Table I. viz. 846 for ale, and 693 for wine, the rest of the work being the same.

P R O B. VII.

To gauge a copper with a rising crown.

Take the dimensions thus.

Extend a pack-thread over the middle from A to B.

Set one end of a rod in C and D, and move it forward and backward, till you find the shortest depth EC and GD; or, instead of a rod, you may hang a plummet.

Next, set the end of the rod on the top of the crown at o, and you have F o, which subtracted from EC, leaves m C the height of the crown.

EG is equal to CD the diameter at the bottom of the crown, and the diameter m n may be measured.

Now suppose the dimensions thus taken to be as follows.

Inches,

| <i>Inches.</i> | <i>Inches.</i> |
|----------------|----------------|
| $EC = 47$ | $AB = 99$ |
| $FO = 42$ | $CD = 64$ |
| $mC = 5$ | $mn = 65$ |

Now to find the content of the copper from the crown upwards, the depth being 42 inches, take the diameter in the middle of every four, or of every six inches, (the more diameters you take the better), and insert these diameters in the second column of the following table. Then by Prob. 1. find the areas of the circles answering to these diameters in ale-gallons, and insert them in the third column; and next multiply each of these areas by 6, and insert the products in the fourth column; and their sum will be the content of $ABmn$; that is, so much as the copper will hold after the crown is covered.

The crown is to be considered as the frustum of a sphere, and its content by Prob. 29. will be found to be 28.75 ale-gallons. But the content of the crown may more readily, and with sufficient accuracy, be found as follows.

The diameter CD is 64, and the area to this diameter is 11.408, which multiply by half the height of the crown, *viz.* 2. 5, and the product is 28.52 for the content of the crown.

The content of the part $mnDC$, considered as the frustum of a cone, will be found to be 57.935 gallons; from which subtract the content of the crown 28.52, and the remainder is 29.415 gallons, the quantity of liquor that will just cover the crown.

Parts

| Parts
of the
depth. | Diameters. | Areas in
ale-gall. | Content
of every
6 inches. |
|---------------------------|------------|-----------------------|----------------------------------|
| 6 | 95.3 | 25.2945 | 151.767 |
| 6 | 90.1 | 22.6095 | 135.657 |
| 6 | 85.0 | 20.1223 | 120.734 |
| 6 | 80 | 17.8246 | 106.947 |
| 6 | 75.2 | 15.7499 | 94.499 |
| 6 | 70.5 | 13.8426 | 83.056 |
| 6 | 66 | 12.1319 | 72.791 |
| Sum | | - | 765.451 |
| To just cover the crown | | | 29.415 |
| Content of the copper | | | 794.866 |

P R O B. VIII,

To gauge a cask.

The finding the content of casks is a difficult part in gauging, the shape of casks being so different, that not any one rule will serve for all. Gaugers therefore commonly distinguish casks into four forms or varieties, *viz.*

1. Such as resemble the middle frustum of a spheroid;
2. the middle frustum of a parabolic spindle; 3. the lower frustums of two equal parabolic conoids; 4. the lower frustums of two equal cones, as represented in fig. 1.

1. The utmost line in the figure, which exhibits the staves as very much curved, or arching, represents a cask of the first form; and is considered as the middle frustum of a spheroid, being the most capacious sort of cask, or what will hold more liquor than a cask of any other form.

2. The curved line next to the utmost line, represents a cask of the second form; and is considered as the middle frustum of a parabolic spindle, being less capacious than the spheroid.

3. The third curved line represents a cask of the third form,

form, which is considered as the lower frustums of two equal parabolic conoids abutting upon one common base; and is less capacious than any of the two former.

4. The innermost line, which is quite straight from bung to head, represents a cask of the fourth form, and is considered as the lower frustums of two equal cones abutting upon one common base, being of all sorts of casks the least capacious.

Various rules are assigned in books of gauging for ascertaining the contents of these casks, but the most easy and practical way is, to find such a mean diameter as will reduce the proposed cask to a cylinder; and this may be done pretty accurately as follows.

Multiply the difference of the bung and head diameter by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones, and add the product to the head diameter, and the sum is the mean diameter.

Example. Suppose the bung diameter be 32 inches, the head diameter 24 inches, and the length 40 inches; the content in each variety is required.

The difference between the bung and head diameter is 8, and

Mean diameter.

$8 \times .7 = 5.6$, and $24 + 5.6 = 29.6$ for the spheroid.
 $8 \times .65 = 5.2$, and $24 + 5.2 = 29.2$ for the spindle.
 $8 \times .6 = 4.8$, and $24 + 4.8 = 28.8$ for the conoids.
 $8 \times .55 = 4.4$, and $24 + 4.4 = 28.4$ for the cones.

The content in ale-gallons, found by Prob. 4. follows.

A. G.

$29.6 \times 29.6 \times .002785 \times 40 = 97.6$ for the spheroid.
 $29.2 \times 29.2 \times .002785 \times 40 = 94.98$ for the spindle.
 $28.8 \times 28.8 \times .002785 \times 40 = 92.4$ for the conoids.
 $28.4 \times 28.4 \times .002785 \times 40 = 89.85$ for the cones.

By the sliding rule.

Set the gauge-point upon D to the length of the cask upon

upon C; and against the mean diameter on D you have the content upon C.

It may not be improper to observe here, that however geometrical and accurate the rules for cask-gauging may be in themselves, yet the content found by them can only come near the truth, because casks have only some resemblance, but do not answer exactly in shape, to the forms described above.

Hitherto we have considered casks as quite full; it now remains to point out a method of finding how much liquor is in casks that are not full. This is called *ullaging*; the quantity of liquor in the cask being the ullage; which, subtracted from the content of the whole cask, leaves the quantity that would fill the empty space or vacuity.

Now, the cask may either be supposed to lie with its axis parallel to the horizon, as represented in Pl. 4. fig. 2.; or it may stand upright, having its axis perpendicular to the horizon. The former of these cases we shall consider in the first place, and the latter afterwards. And in both cases the rules shall be adapted to the spheroidal cask; because, in the practice of excise, most casks are considered as such.

It is usual to find the ullage by means of a table of segments suited to a cylindrical cask; but as every one may not be provided with such a table, we shall here assign a rule for finding the ullage, and that pretty accurately, without any table, as follows.

R U L E.

Divide the wet inches, taken at the bung, by the bung-diameter; and if the quot exceed .5, subtract .5 from it, and add one fourth part of the remainder to the former quot, and then multiply this sum by the content of the whole cask; the product is the ullage.

But if, upon dividing the wet inches by the bung-diameter, the quot be less than .5, subtract the quot from .5, and from the said quot subtract one fourth part of the remainder, and then multiply this last remainder by the content of the whole cask.

EXAMPLE.

| | <i>Inches.</i> |
|-------------------------------|-----------------------|
| Suppose the dimensions of the | { Length 32.5 |
| given cask to be | { Bung-diam. 31 |
| | { Wet 21 |
| | { Dry 10 |
| | { Content 75.37 A. G. |

$$31 \overline{) 21.000000} \begin{matrix} .677419 \\ .5 \end{matrix}$$

$$4 \overline{) .177419} \begin{matrix} .044355 \\ .677419 \end{matrix}$$

.721774 sum.

And $.721774 \times 75.37 = 54.40010638$ ale-gallons.

Subtract the ullage 54.4 from 75.37, the content of the cask, and the remainder will be the quantity of liquor that would fill the vacuity. Or, find the content of the vacuity thus: Divide the dry inches by the bung-diameter, and then proceed in all respects as you did in finding the ullage.

If the cask stand on end, as represented in Pl. 4. fig. 3. the quantity of liquor it contains may be found by the following rules.

1. Find the diameter D F at the surface of the liquor, by laying two rulers on different sides of the cask, and at the height of the liquor, so as to be parallel; then measure the distance between the rulers, and from it subtract the thickness of the staves on both sides.

2. To twice the square of the diameter D F add the square of the head-diameter L M, multiply the sum by the wet inches, and divide the product by the triple of the divisors for circular parts, viz. by 1077.15 for ale-gallons, and by 882.36 for wine-gallons.

EX-

E X A M P L E.

| | Inches. |
|---------------------|-------------|
| Length B G | 32.5 |
| Bung-diameter H I | 27 |
| Head-diameter L M | 23 |
| Wet E G | 8.5 |
| Diameter D F | 26.14 |
| Content of the cask | 59.95 A. G. |

$$DF \ 26.14 \times 26.14 \times 2 = 1366.6$$

$$\text{Add } 23 \times 23 = 529$$

$$\hline 1895.6$$

$$\text{Wet inches } 8.5$$

$$\hline 94780$$

$$\hline 151648$$

$$1077.15)16112.60 \quad (14.95 \text{ A. G.}$$

$$\text{Cask's content} \quad \hline 59.95$$

$$\text{Wanted to fill the cask } 45$$

If you work with the dry inches, the result will be the quantity of liquor that would fill the vacuity; which, subtracted from the content of the whole cask, will leave the quantity of liquor in the cask.

P R O B. IX.

To gauge malt.

R U L E.

If the malt be in a cistern, or lie on a floor in a rectangular form, find the area of the base in bushels, by multiplying the length into the breadth, and dividing the product by 2150.42; then multiply this quot by the mean depth; and the product is the content in bushels.

If the base be circular or elliptical, divide the square of the diameter, or product of the two elliptical diameters, by 2738, and multiply the quot by the mean depth.

Ex. 1. There is a cistern, whose length is 84 inches, and breadth 54 inches, and the mean depth 43.6 inches: Required the content.

$$84 \times 54 = 4536 \quad \text{And } 2150.42)4536.00(2.1093$$

$$\text{And } 2.1093 \times 43.6 = 91.965 \text{ bushels.}$$

Ex. 2. There is a malt-floor, whose length is 245 inches, and the breadth 184 inches, and the mean depth 5.6 inches: Required the content.

$$245 \times 184 = 45080 \quad \text{And } 2150.42)45080.00(20.963$$

$$\text{And } 20.963 \times 5.6 = 117.3928 \text{ bushels.}$$

By the sliding rule.

Set the depth on the line MD to the length on the line N upon the slider; and against the breadth on the line A you have the content on the line marked B upon the slider.

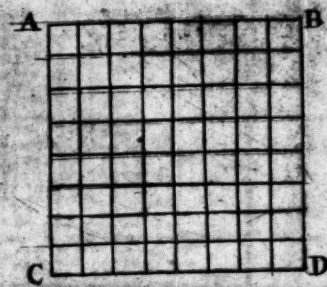


Direction to the BOOKBINDER.

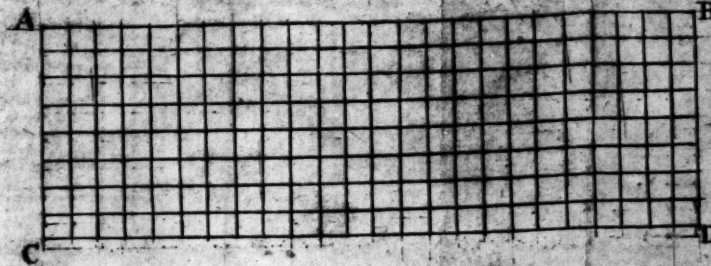
Place the four copper-plates at the end of the volume, in successive order; and sew them so as that every plate may, when opened out, be wholly without the book.

PLATE I

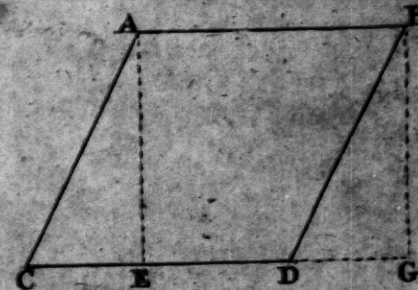
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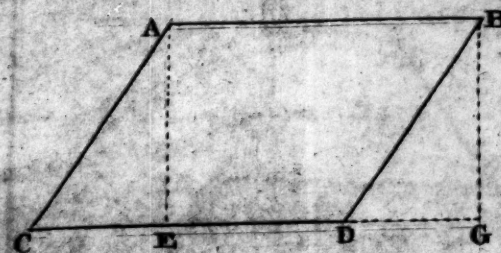
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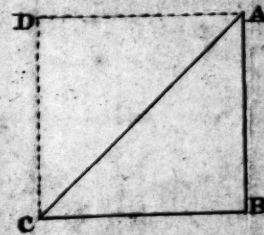
PROB. III



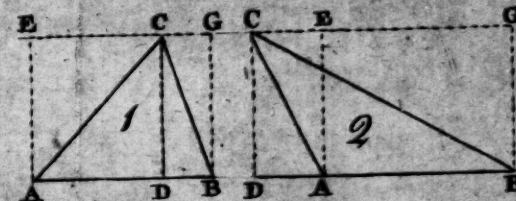
PROB. IV



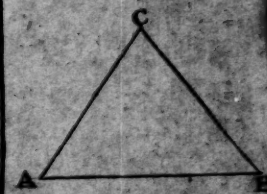
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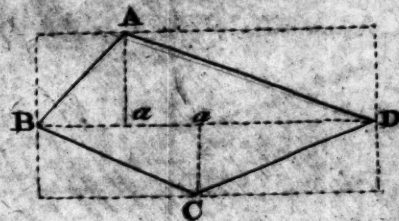
PROB. VI



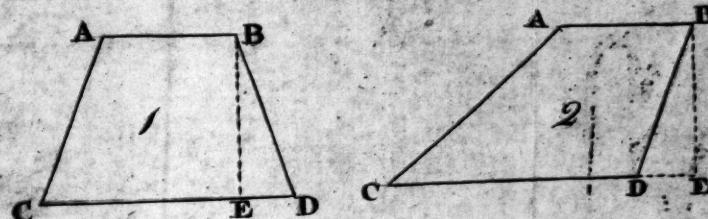
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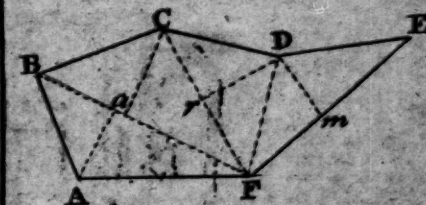
PROB. VIII



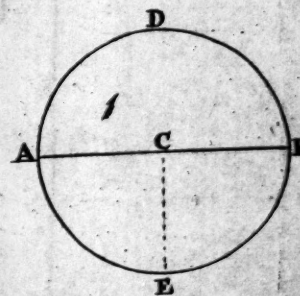
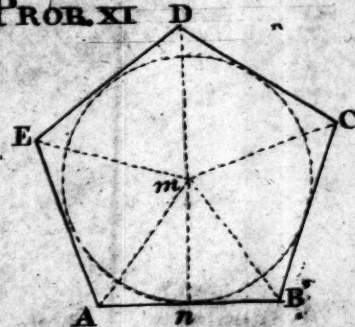
PROB. IX



PROB. X



PROB. XI



PROB. XII

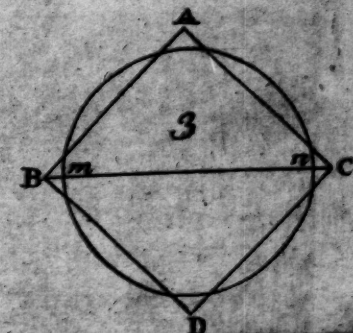
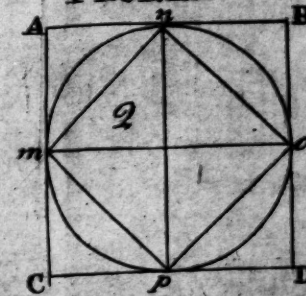
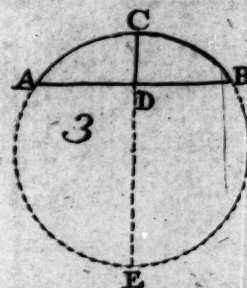
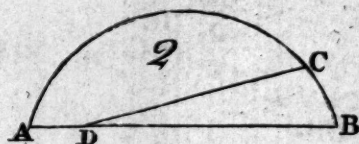
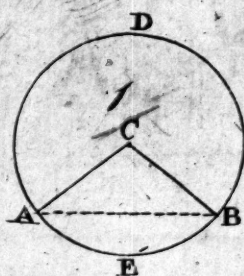
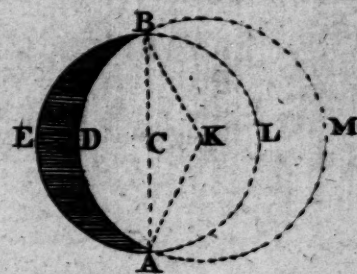


PLATE II

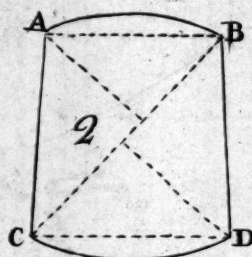
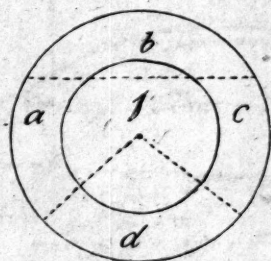
PROB. XIII



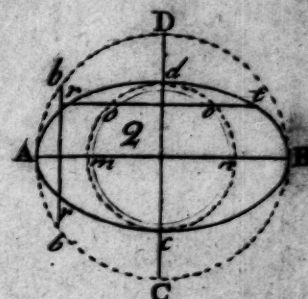
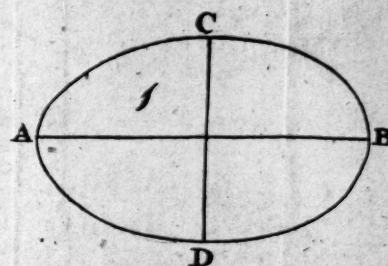
PROB. XIV



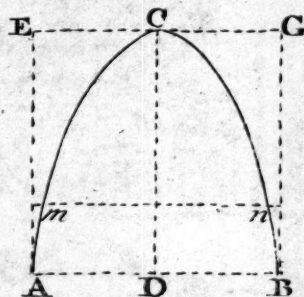
PROB. XV



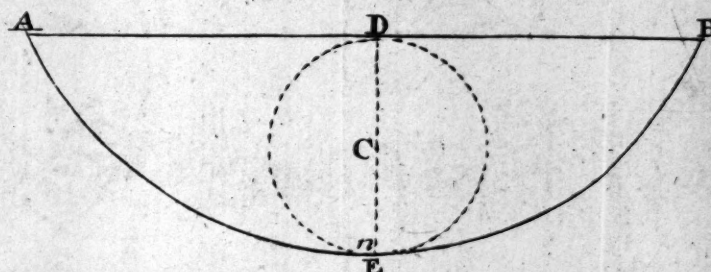
PROB. XVI



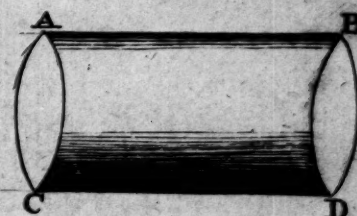
PROB. XVII



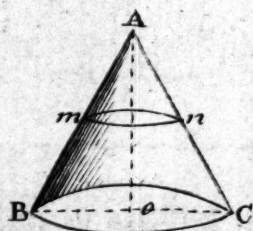
PROB. XVIII



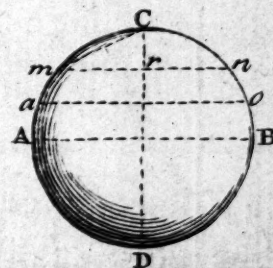
PROB. XIX



PROB. XX



PROB. XXI



PROB. XXII

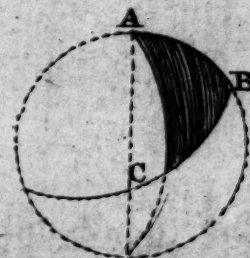
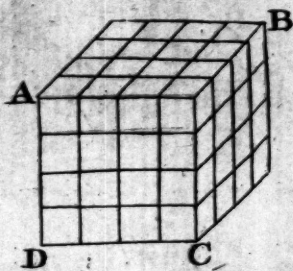
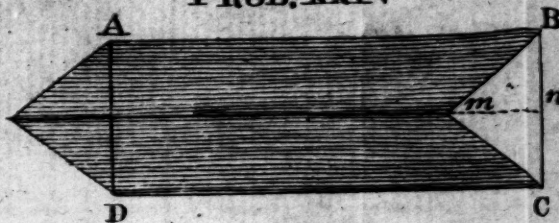


PLATE III

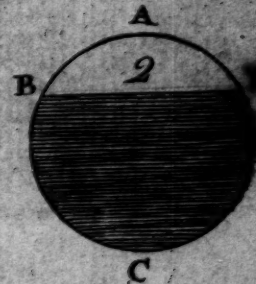
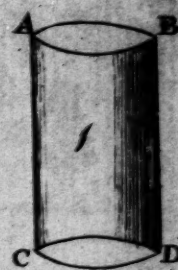
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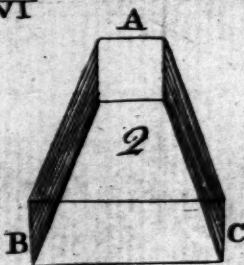
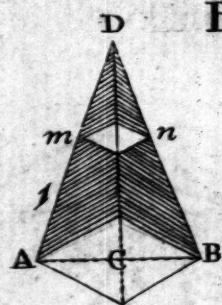
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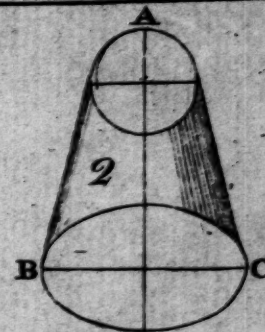
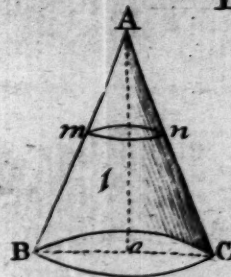
PROB. XXV



PROB. XXVI



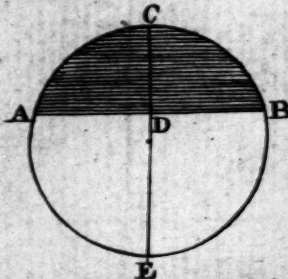
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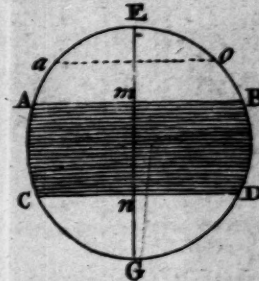
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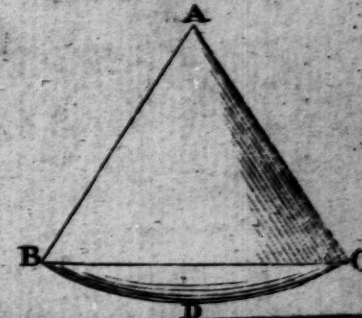
PROB. XXIX



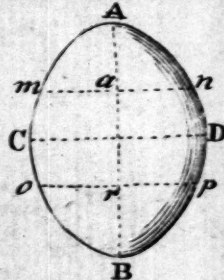
PROB. XXX



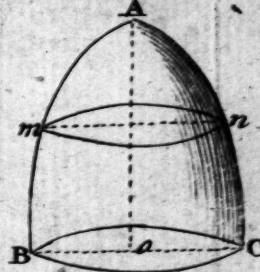
PROB. XXXI



PROB. XXXII



PROB. XXXIII



PROB. XXXIV

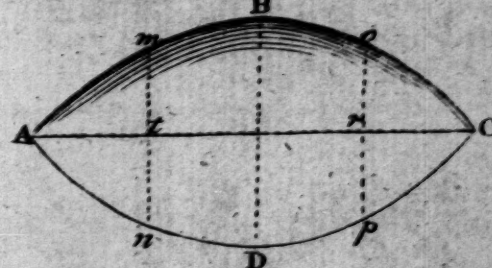
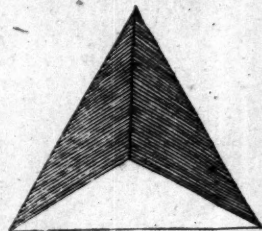
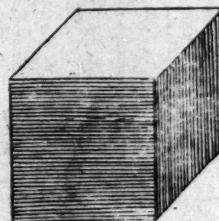


PLATE IV.

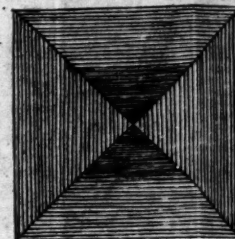
Tetraedron



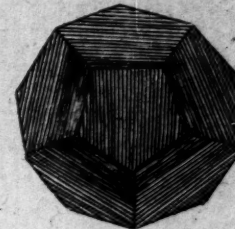
Hexaedron



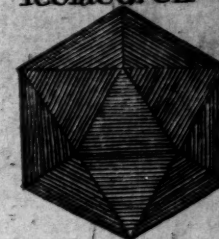
PROB. XXXV.
Octaedron



Dodecaedron



Icosaedron



SURVEYING

FIG. 1.

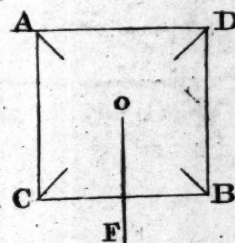
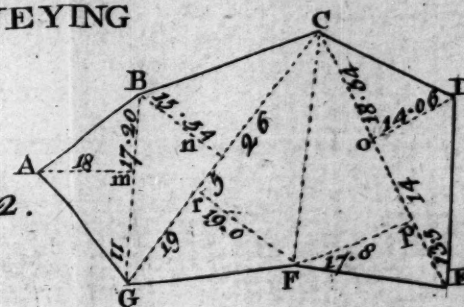
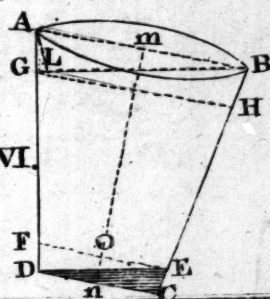


FIG. 2.

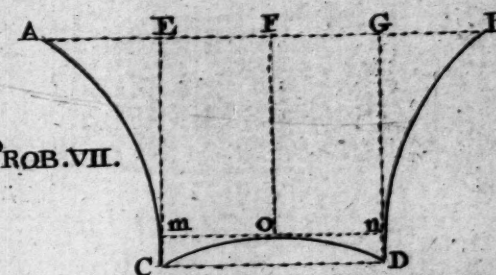


GAUGING

PROB. VI.



PROB. VII.



PROB. VIII.

